MATH 16020: Applied Calculus 2 Quiz 8 (Solutions) Fri., Mar. 2, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. **[3 pt]** Evaluate

$$\int_0^\infty 5xe^{-2x}\,dx$$

Solution:

$$\begin{aligned}
\int_0^\infty 5xe^{-2x}\,dx &= \lim_{N\to\infty}\int_0^N 5xe^{-3x^2+2}\,dx \\
&\overset{\substack{u=5x \\ du=5\,dx}}{=\!=\!=}\overset{\substack{dv=e^{-2x}\,dx \\ v=\frac{1}{2}e^{-2x}}}{=\!=\!=} \lim_{N\to\infty}\left(-\frac{5}{2}xe^{-2x}\Big]_0^N - \left(-\frac{5}{2}\right)\int_0^N e^{-2x}\,dx\right) \\
&= \lim_{N\to\infty}\left(-\frac{5}{2}xe^{-2x}\Big]_0^N + \frac{5}{2}\int_0^N e^{-2x}\,dx\right) \\
&= \lim_{N\to\infty}\left(-\frac{5}{2}xe^{-2x}\Big]_0^N - \frac{5}{4}e^{-2x}\Big]\right) \\
&= \lim_{N\to\infty}\left(-\frac{5}{2}Ne^{-2N} + 0 - \frac{5}{4}e^{-2N} + \frac{5}{4}\right) \\
&= \lim_{N\to\infty}\left(-\frac{5}{2}Ne^{-2N} + 0 - \frac{5}{4}e^{-2N} + \frac{5}{4}\right) \\
&= \frac{5}{4}
\end{aligned}$$

How I graded?

- **1 pt** for Integration
- **1 pt** for Limit
- **1 pt** for Final Answer

2. **[3 pt]** Evaluate

$$\int_6^\infty \frac{dx}{x \ln x}$$

Solution:

$$\begin{aligned}
\int_6^\infty \frac{dx}{x \ln x} &= \lim_{N\to\infty} \int_6^N \frac{dx}{x \ln x} \\
&\overset{\substack{u=\ln x \\ du=\frac{1}{x}\,dx}}{=} \lim_{N\to\infty} \int \frac{du}{u} \\
&= \lim_{N\to\infty} \ln|u| \\
&= \lim_{N\to\infty} \ln|u| \\
&= \lim_{N\to\infty} \left(\ln|\ln(x)|\big]_6^N \right) \\
&= \lim_{N\to\infty} \left(\ln|\ln(N)| - \ln|\ln(6)| \right) \\
&= \infty
\end{aligned}$$

How I graded?

- **1 pt** for Integration
- **1 pt** for Limit
- **1 pt** for Final Answer

3. **[4 pt]** Evaluate

$$\int_0^3 \frac{x}{\sqrt{9-x^2}}\,dx$$

Solution: Note that the function is discontinuous at $x = \pm 3$. So

$$\begin{aligned}
\int_0^3 \frac{x}{\sqrt{9-x^2}}\,dx &= \lim_{N\to 3} \int_0^N \frac{x}{\sqrt{9-x^2}}\,dx \\
&\overset{\substack{u=9-x^2 \\ du=-2x\,dx}}{=\!=\!=} \lim_{N\to 3} \int \frac{-1}{2}\frac{du}{\sqrt{u}} \\
&= \lim_{N\to 3} \int \frac{-1}{2} u^{-1/2}\,du \\
&= \lim_{N\to 3} \frac{-1}{2} \cdot \frac{2}{1} u^{1/2} \\
&= \lim_{N\to 3} \left(-\sqrt{9-x^2}\Big]_0^N \right) \\
&= \lim_{N\to 3} \left(-\sqrt{9-N^2} + \sqrt{9} \right) \\
&= -\sqrt{9-3^2} + \sqrt{9} \\
&= 3
\end{aligned}$$

How I graded?

- **1 pt** for finding the discontinuity
- **1 pt** for Limit
- **1 pt** for Integration
- **1 pt** for Final Answer