MATH 16020: Applied Calculus 2 Quiz 9 (Solutions) Fri., Mar. 4, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:____

1. (a) **[2 pts]** Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \dots$$

   (b) **[2 pts]** Use the sum from (a) and find it's sum.

Solution:

(a) Rewrite each decimal to a fraction. So

$$1 - 0.6 + 0.36 - 0.216 + \cdots = 1 - \frac{6}{10} + \frac{36}{100} - \frac{216}{1000} + \dots \quad \textbf{[1 pt]}$$

Seeing the pattern we get,

$$1 - 0.6 + 0.36 - 0.216 + \cdots = \sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n \quad \textbf{[1 pt]}$$

(b) Let's rewrite the sum so we can apply the Geometric Series Formula.

$$\sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n = \sum_{n=0}^{\infty} \left(-\frac{6}{10}\right)^n = \underbrace{\frac{1}{1-(-6/10)}}_{\textbf{1 pt}} = \frac{1}{16/10} = \underbrace{\frac{10}{16}}_{\textbf{1 pt}}$$

2. **[2 pts]** If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Solution: **[2 pts]** Since our sum starts at $n = 0$, we can check whether our r satisfies the condition:

$$0 < |r| < 1$$

If $r = 3/2$, well it doesn't satisfy the condition. Hence the series diverges.

3. [**4 pts**] Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

Solution: Let's do a bit of rewriting:

$$\sum_{n=1}^{\infty} \frac{5^n \cdot 5^2}{6^n} = \sum_{n=1}^{\infty} 25 \left(\frac{5}{6}\right)^n \quad \textbf{[1 pt]}$$

Now, notice that the sum starts at 1 and not 0. So let's add and subtract the

$$a_0 = 25 \left(\frac{5}{6}\right)^0 = 25$$

$$\sum_{n=1}^{\infty} \frac{5^n \cdot 5^2}{6^n} = -25 + 25 + \sum_{n=1}^{\infty} 25 \left(\frac{5}{6}\right)^n = -25 + \sum_{n=0}^{\infty} 25 \left(\frac{5}{6}\right)^n \quad \textbf{[1 pt]}$$

Now let's use the Geometric Series Formula

$$\sum_{n=0}^{\infty} 25 \left(\frac{5}{6}\right)^n = \frac{25}{1 - 5/6} = \frac{25}{1/6} = 25 \cdot 6 = 150 \quad \textbf{[1 pt]}$$

Hence

$$\sum_{n=1}^{\infty} \frac{5^n \cdot 5^2}{6^n} = -25 + 150 = 125 \quad \textbf{[1 pt]}$$