Lesson R1: Review of Basic Differentiation

Constant Rule: $\frac{d}{dx}(c)=0$ where c is a constant

Power Rule: $\frac{d}{dx}(x^n)=nx^{n-1}$ where n is any real #

Constant Multiple Rule: $\frac{d}{dx}[cf(x)]=c\frac{d}{dx}[f(x)]$ where c is a constan

Sum/Difference Rule: $\frac{d}{dx}(f(x)\pm g(x))=\frac{d}{dx}(f(x))\pm\frac{d}{dx}(g(x))$

Example 1: Find the derivative for the following:

ⓐ $f(x)=x^3$

$f'(x)=3x^2$

ⓑ $f(x)=1/x$

$f(x)=x^{-1}$

$f'(x)=-x^{-2}$

ⓒ $f(x)=\sqrt{x}$

$f(x)=x^{1/2}$

$f'(x)=\frac{1}{2}x^{-1/2}$

ⓓ $f(x)=\frac{3}{x^4}-2x^2+6x-7$

$f(x)=3x^{-4}-2x^2+6x-7$

$f'(x)=3(-4)x^{-5}-2(2)x+6$

$=-12x^{-5}-4x+6$

$=\frac{-12}{x^5}-4x+6$

ⓔ $f(x)=\frac{x^{2.5}-2x^{-3}}{x}$

$f(x)=x^{1.5}-2x^{-4}$

$f'(x)=1.5x^{0.5}-2(-4)x^{-5}$

$=1.5x^{0.5}+8x^{-5}$

Position/Velocity/Acceleration

Position	$s(t)$
Velocity	$v(t) = s'(t)$
Acceleration	$a(t) = v'(t) = s''(t)$

Other Differentiation Rules

$\frac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \tan x \sec x$ $\qquad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(e^x) = e^x$ $\qquad$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Product Rule: $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)}$

Chain Rule: Let $h(x) = f(g(x))$. Then
$h'(x) = f'(g(x))\, g'(x)$

Example 2: Find the derivative of the following:

(a) $y = (3x+1)^2$

$f(x) = x^2 \qquad g(x) = 3x+1$

$f'(x) = 2x \qquad g'(x) = 3$

$y' = f'(3x+1) \cdot 3$

$= 2(3x+1) \cdot 3 = 6(3x+1) = 18x + 6$

(b) $y = 2\cos^3 x$

$y = 2(\cos x)^3$

$f(x) = 2x^3 \qquad g(x) = \cos x$

$f'(x) = 6x \qquad g'(x) = -\sin x$

$y' = f'(\cos x)(-\sin x)$

$= -6(\cos x)^2 \sin x = -6\cos^2 x \sin x$

ⓒ $y = \left(\dfrac{2x}{3x^2+x}\right)^3$

$$y = \left(\frac{2x}{x(3x+1)}\right)^3 = \left(\frac{2}{3x+1}\right)^3 = \frac{8}{(3x+1)^3} = 8(3x+1)^{-3}$$

$f(x) = 8x^{-3}$ $\quad$ $g(x) = 3x+1$

$f'(x) = -24x^{-4}$ $\quad$ $g'(x) = 3$

$$y' = f'(3x+1)\cdot 3$$
$$= -24(3x+1)^{-4}\cdot 3 = \frac{-72}{(3x+1)^4}$$

ⓓ $y = \dfrac{15}{\sqrt[3]{x^2+1}}$

$$y = 15(x^2+1)^{-1/3}$$

$f(x) = 15x^{-1/3}$ $\quad$ $g(x) = x^2+1$

$f'(x) = 15(-\tfrac{1}{3})x^{-4/3}$ $\quad$ $g'(x) = 2x$

$= -5x^{-4/3}$

$$y' = f'(x^2+1)\cdot(2x)$$
$$= -5(x^2+1)^{-4/3}(2x) = \frac{-10x}{(x^2+1)^{4/3}}$$

ⓔ $y = \sin(x^2)$

$f(x) = \sin x$ $\quad$ $g(x) = x^2$

$f'(x) = \cos x$ $\quad$ $g'(x) = 2x$

$$y' = f'(x^2)\cdot(2x)$$
$$= \cos(x^2)\cdot(2x) = 2x\cos(x^2)$$

ⓕ $y = \sec(-2x+1)\tan(3x)$

$f(x) = \sec(-2x+1)$ $\quad$ $g(x) = \tan(3x)$

$f'(x) = \sec(-2x+1)\tan(-2x+1)\cdot(-2)$ $\quad$ $g'(x) = \sec^2(3x)\cdot 3$

$$y' = f'(x)g(x) + g'(x)f(x)$$
$$= -2\sec(-2x+1)\tan(-2x+1)\tan(3x)$$
$$+ 3\sec^2(3x)\sec(-2x+1)$$

⑨ $y = \exp[(1-2x)^4]$

$f(x) = e^x \qquad g(x) = (1-2x)^4$

$u(x) = x^4 \qquad v(x) = 1-2x$

$u'(x) = 4x^3 \qquad v'(x) = -2$

$f'(x) = e^x \qquad g'(x) = u'(1-2x) \cdot (-2)$

$= 4(1-2x)^3 \cdot (-2) = -8(1-2x)^3$

$y' = f'((1-2x)^4) \cdot [-8(1-2x)^3]$

$= \exp[(1-2x)^4] \cdot [-8(1-2x)^3]$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/4$
sin	$0 = 0/2$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2 = 1$
Cos	$1 = \sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	$0/2 = 0$