Lesson R2: Review of Integration

Indefinite Integration

$\int f(x)\,dx = F(x) + C$ where C is a constant

Basic Integration Rules:

$$\int 0\,dx = C$$

$$\int k\,dx = kx + C$$

$$\int kf(x)\,dx = k\int f(x)\,dx$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

$$\boxed{\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad \text{for } n \neq -1}$$ ← Power Rule

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Recall you can check your answer by taking the derivative of it and seeing if it matches the original function.

Example 1: Evaluate the following.

ⓐ $\int 6\sec^2 x - 5e^x dx = 6\int \sec^2 x\,dx - 5\int e^x dx$
$= 6\tan x - 5e^x + C$

ⓑ $\int (x^2 + 2\sqrt{x})dx = \int (x^2 + 2x^{1/2})dx$
$= \frac{x^3}{3} + 2\frac{x^{3/2}}{3/2} + C$
$= \frac{x^3}{3} + \frac{4}{3}x^{3/2} + C$

ⓒ $\int \left(\frac{3}{x} + \sqrt[3]{x^2}\right)dx = 3\int \frac{dx}{x} + \int x^{2/3}dx$
$= 3\ln|x| + \frac{3}{5}x^{5/3} + C$

Differential Equations

Example 2: Solve the differential equation $y' = 3x$.

$y = \int y' dx = \int 3x\,dx = \frac{3}{2}x^2 + C$

This is called the general solution.

What if we are given an initial condition?

Example 3: Solve the initial value problem (IVP) $y' = 3x$ with $y(0) = 2$.

From Ex 2, $y = \frac{3}{2}x^2 + C$. Using $y(0) = 2$, we can find C.

$2 = \frac{3}{2}(0)^2 + C$

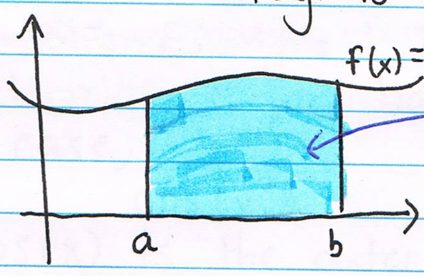
$2 = C \Rightarrow y = \frac{3}{2}x^2 + 2$

Definite Integral $\int_a^b f(x)\,dx$

Fundamental Theorem of Calculus (FTC)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

We can use definite integrals to find area under a curve.



Bounded by
$y=f(x)$, $y=0$
$x=a$, $x=b$

Example 4: Find the area of the region bounded by
$y=2x+1$; $y=0$; $x=1$; $x=3$

$$\int_1^3 (2x+1)\,dx = \left(\frac{2x^2}{2}+x\right)\Big]_1^3 = (x^2+x)\Big]_1^3 = (3^2+3)-(1^2+1)$$

$$= 9+3-2 = 10$$

Example 5: Evaluate the following

ⓐ $$\int_1^4 \frac{x^2+x}{\sqrt{x}}\,dx = \int_1^4\left(\frac{x^2}{x^{1/2}}+\frac{x}{x^{1/2}}\right)dx = \int_1^4 \left(x^{3/2}+x^{1/2}\right)dx$$

$$= \left(\frac{2}{5}x^{5/2}+\frac{2}{3}x^{3/2}\right)\Big]_1^4$$

$$= \left(\frac{2}{5}(4)^{5/2}+\frac{2}{3}(4)^{3/2}\right) - \left(\frac{2}{5}(1)^{5/2}+\frac{2}{3}(1)^{3/2}\right)$$

$$= \frac{2\cdot 2^5}{5}+\frac{2\cdot 2^3}{3}-\frac{2}{5}-\frac{2}{3}$$

$$= \frac{256}{15}$$

ⓑ $$\int_0^{\pi/4} \sec^2 x\,dx = \tan x\Big]_0^{\pi/4} = \tan\left(\frac{\pi}{4}\right) - \tan(0)$$

$$= 1 - 0 = 1$$