

⑫ After 10 mins in Jean-Luc's room, his tea has cooled to 48°C from 100°C . The room temperature is 21°C . How much longer will it take to cool to 40°C ?

Solution: Newton's Cooling Formula: $T' = k(T - S)$

T - temp of tea

S - temp of room

$$\left(\frac{dT'}{T-21} \right) = k dt$$

$$\ln |T-21| = kt + C$$

$$|T-21| = Ce^{kt}$$

$$T-21 = Ce^{kt}$$

$$T = Ce^{kt} + 21$$

When $t=0$, $T=100$

$$100 = T(0) = C + 21$$

$$79 = C \Rightarrow T = 79e^{kt} + 21$$

When $t=10$, $T=48$

$$48 = T(10) = 79e^{10k} + 21$$

$$27 = 79e^{10k}$$

$$\frac{1}{10}\ln(27/79) = k \Rightarrow T = 79 \exp\left[\frac{1}{10}\ln(27/79)t\right] + 21$$

Find t when $40 = T(t)$

$$40 = 79 \exp\left[\frac{1}{10}\ln(27/79)t\right] + 21$$

$$\ln(19/79) = \frac{1}{10}\ln(27/79)t$$

$$t = \frac{10\ln(19/79)}{\ln(27/79)} \approx 13.273 \text{ min}$$

So time between 48°C to 40°C is $13.273 - 10 = 3.273$ mins