MA 16010 LESSON 11+12: RELATED RATES HANDOUT

Related Rates are word problems that use implicit differentiation.

We will be taking the derivative of equations with respect to time, t.

Recipe for Solving a Related Rates Problem

Step 1: Draw a good picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you **WANT** to find.

Step 3: Find an equation relating the relevant variables. This usually involves a formula from geometry, similar triangles, the Pythagorean Theorem, or a formula from trigonometry. **Use your picture!**

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you **WANT**. Do **NOT** substitute before this step!

Some Useful Formulas

Right Triangle *Pythagorean Theorem:* $a^2 + b^2 = c^2$	Triangle $A = \frac{1}{2}bh$ $P = a + b + c$	Trapezoid $A = \frac{1}{2}(a + b)h$	Rectangular Box $V = lwh$ $S = 2(hl + lw + hw)$	Cone $V = \frac{1}{3}\pi r^2 h$ $SA = \pi rl + \pi r^2$
Rectangle $A = lw$ $P = 2l + 2w$	Circle $A = \pi r^2$ $C = 2\pi r$	Sphere $A = \frac{4}{3}\pi r^3$ $S = 4\pi r^2$	Right Circular Cylinder $V = \pi r^2 h$ $SA = 2\pi rh$	

Example 1: If x and y are both functions of t and $x + y^3 = 2$.

a) Find $\frac{dy}{dt}$ when $\frac{dx}{dt} = -2$ and $y = 1$

b) Find $\frac{dx}{dt}$ when $\frac{dy}{dt} = 3$ and $x = 1$

<u>Example 2:</u> A spherical balloon is being deflated at a constant rate of 20 cubic cm per second. How fast is the radius of the balloon changing at the instant when the balloon's radius is 12 cm?

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you WANT to find.

KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

Example 3: A cylindrical tank standing upright (with one circular base on the ground) has a radius of 22 cm for the base. How fast does the water level in the tank drop when the water is being drained at 28 cm^3/sec? Note: The formula right circular cylinder is $V = \pi r^2 h$.

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you WANT to find.

KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

<u>Example 4:</u> A plane is flying directly away from you at 500 mph at an altitude of 3 miles.

a) How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you?

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you WANT to find.

KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

<u>Example 4:</u> A plane is flying directly away from you at 500 mph at an altitude of 3 miles.

b) How fast is the angle of elevation changing when it is $\pi/3$?

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you WANT to find.

KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

<u>Example 5:</u> A ladder 5 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.3 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 3 m from the wall?

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you WANT to find.

KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t*.

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**

<u>Example 6:</u> A baseball diamond is a square 90 ft on a side. A player runs from first base to second base at 14 ft/sec. At what rate is the player's distance from home base increasing when he is halfway from first to second base?

Step 1: Draw a picture. Label all constant values and give variable names to any changing quantities.

Step 2: Determine what information you **KNOW** and what you WANT to find.

KNOW: **WANT:**

Step 3: Find an equation relating the relevant variables.

Step 4: Use implicit differentiation to differentiate the equation with respect to time *t.*

Step 5: Substitute in what you **KNOW** from **Step 2** and any information that your equation in **Step 3** can give you and solve for the quantity you WANT**.**