Lesson 18: Limits at Infinity Pt 2

Slant Asymptotes

The line $y=ax+b$, where a and b are constants and $a \neq 0$, is a slant asymptote of $f(x)$ if $f(x)$ gets closer and closer to $y=ax+b$ as $x \to \pm\infty$.

To find the slant asymptotes of a rational function, we use
- Synthetic Division
- Long Division

Synthetic Division Slide 1

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

To use Synthetic Division, find the zeros of the denominator of y.

i.e. $x-2=0 \implies x=2$

$2 \,\big|$

Synthetic Division Slide 2

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Next, list the coefficients of the polynomial on the numerator.

$$\begin{array}{c|ccc} 2 & 1 & 2 & 8 \\ & & & \\ \hline \end{array}$$

Synthetic Division Slide 3

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Drag down the first # inside the $\llcorner$

$$\begin{array}{c|ccc} 2 & 1 & 2 & 8 \\ & \downarrow & & \\ \hline & 1 & & \end{array}$$

Synthetic Division Slide 4

Ex5: Find the slant asymptote of $y=\dfrac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Multiply 2 by the value just written on the bottom row.

$$\begin{array}{c|ccc} 2 & 1 & 2 & 8 \\ & \downarrow & 2 & \\ \hline & 1 & & \end{array}$$

Synthetic Division Slide 5

Ex5: Find the slant asymptote of $y=\dfrac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Add the column created.

$$\begin{array}{c|ccc} 2 & 1 & 2 & 8 \\ & \downarrow & 2 & \\ \hline & 1 & 4 & \end{array}$$

Synthetic Division Slide 6

Ex5: Find the slant asymptote of $y=\dfrac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Repeat: Multiply 2 by the value just written on the bottom row.

$$\begin{array}{r|rrr} 2 & 1 & 2 & 8 \\ & \downarrow & 2 & 8 \\ \hline & 1 & 4 & \end{array}$$

Synthetic Division Slide 7

Ex5: Find the slant asymptote of $y=\dfrac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Repeat: Add the column created.

$$\begin{array}{r|rrr} 2 & 1 & 2 & 8 \\ & \downarrow & 2 & 8\downarrow \\ \hline & 1 & 4 & 16 \end{array}$$

Synthetic Division Slide 8

Ex5: Find the slant asymptote of $y=\frac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Rewrite $y=\frac{x^2+2x+8}{x-2}=x+4+\frac{16}{x-2}$

$$\begin{array}{c|ccc} 2 & 1 & 2 & 8 \\ & \downarrow & 2 & 8 \\ \hline & 1 & 4 & 16 \end{array}$$

16 ← remainder

Synthetic Division Slide 9

Ex5: Find the slant asymptote of $y=\frac{x^2+2x+8}{x-2}$

Method 1: Synthetic Division

Rewrite $y=\frac{x^2+2x+8}{x-2}=\boxed{x+4}+\frac{16}{x-2}$

$\Downarrow$

Slant Asymptote

i.e. Slant Asymptote $y=x+4$

Long Division Slide 1

Ex5: Find the slant asymptote of $y = \frac{x^2+2x+8}{x-2}$

Method 2: Long Division

Long Division works w/o knowing the zeros of y's denominator. But involves alot of work.

$$\underbrace{x-2}_{\text{Divisor}} \overline{\big)\, \underbrace{x^2+2x+8}_{\text{Dividend}}}$$

Long Division Slide 2

Ex5: Find the slant asymptote of $y = \frac{x^2+2x+8}{x-2}$

Method 2: Long Division

Divide the first term of the dividend by first term of the divisor.

① $\frac{x^2}{x} = x$

$$x-2 \overline{\big)\, x^2+2x+8}$$

Long Division Slide 3

Ex 5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Place what's found in ① above $\sqrt{}$

$$x-2\ \overset{x}{\overline{\big)\,x^2+2x+8}}$$

① $\dfrac{x^2}{x} = x$

Long Division Slide 4

Ex 5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Multiply that term in ① with divisor.

$$x-2\ \overset{x}{\overline{\big)\,x^2+2x+8}}$$

① $\dfrac{x^2}{x} = x$

② $x(x-2) = x^2-2x$

Long Division Slide 5

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Line up what you found in ② under the dividend

$$\begin{array}{r|l} & \quad x \\ \hline x-2 & x^2+2x+8 \\ & -(x^2-2x) \\ \hline \end{array}$$

① $\dfrac{x^2}{x} = x$

② $x(x-2) = x^2 - 2x$

Long Division Slide 6

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Subtract and bring down the next term.

$$\begin{array}{r|l} & \quad x \\ \hline x-2 & \cancel{x^2}+2x+8 \\ & -(\cancel{x^2}-2x) \downarrow \\ \hline & \quad 4x+8 \end{array}$$

① $\dfrac{x^2}{x} = x$

② $x(x-2) = x^2 - 2x$

Long Division Slide 7

Ex 5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Repeat: Divide the first term of the dividend by first term of the divisor.

$$
\begin{array}{r|l}
 & \quad\quad\;\; x \\ \hline
x-2 & \cancel{x^2}+2x+8 \\
 & -(\cancel{x^2}-2x)\;\downarrow \\ \hline
 & \quad\quad 4x+8
\end{array}
$$

① $\dfrac{4x}{x} = 4$

Long Division Slide 8

Ex 5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Repeat: Place what's found in ① above $\sqrt{}$

$$
\begin{array}{r|l}
 & \quad\quad\;\; x+4 \\ \hline
x-2 & \cancel{x^2}+2x+8 \\
 & -(\cancel{x^2}-2x)\;\downarrow \\ \hline
 & \quad\quad 4x+8
\end{array}
$$

① $\dfrac{4x}{x} = 4$

Long Division Slide 9

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Repeat: Multiply that term in ① with divisor.

$$\begin{array}{r l}
 & \phantom{x^2+2x+{}}x+4 \\
x-2 & \overline{\big)\, \cancel{x^2}+2x+8} \\
 & -(\cancel{x^2}-2x) \downarrow \\
\hline
 & \phantom{x^2+{}}4x+8
\end{array}$$

① $\dfrac{4x}{x} = 4$

② $4(x-2) = 4x-8$

Long Division Slide 10

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Repeat: Line up what you found in ② under the dividend

$$\begin{array}{r l}
 & \phantom{x^2+2x+{}}x+4 \\
x-2 & \overline{\big)\, \cancel{x^2}+2x+8} \\
 & -(\cancel{x^2}-2x) \downarrow \\
\hline
 & \phantom{x^2+{}}4x+8 \\
 & -(4x-8) \\
\hline
\end{array}$$

① $\dfrac{4x}{x} = 4$

② $4(x-2) = 4x-8$

Long Division Slide 11

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Repeat: Subtract.

$$
\begin{array}{r|l}
 & \quad\; x + 4 \\
\hline
x-2 & \cancel{x^2} + 2x + 8 \\
 & -(\cancel{x^2} - 2x) \downarrow \\
\hline
 & \qquad \cancel{4x} + 8 \\
 & \quad -(\cancel{4x} - 8) \\
\hline
 & \qquad\quad 16
\end{array}
$$

① $\dfrac{4x}{x} = 4$

② $4(x-2) = 4x - 8$

Long Division Slide 12

Ex5: Find the slant asymptote of $y = \dfrac{x^2+2x+8}{x-2}$

Method 2: Long Division

Rewrite $y = \dfrac{x^2+2x+8}{x-2}$

$= \boxed{x+4} + \dfrac{16}{x-2}$

$\Downarrow$

Slant Asymptote

$$
\begin{array}{r|l}
 & \quad\; x + 4 \\
\hline
x-2 & \cancel{x^2} + 2x + 8 \\
 & -(\cancel{x^2} - 2x) \downarrow \\
\hline
 & \qquad \cancel{4x} + 8 \\
 & \quad -(\cancel{4x} - 8) \\
\hline
 & \qquad\quad 16
\end{array}
$$

i.e. Slant Asymptote $y = x + 4$

Ex 6: Find the slant asymptote of

$$y=\frac{x^3+x^2+1}{x+3} = \frac{x^3+x^2+0x+1}{x+3}$$

Sythetic Division: zeros of $x+3 \Rightarrow x=-3$

$$\begin{array}{r|rrrr} -3 & 1 & 1 & 0 & 1 \\ & \downarrow & -3 & 6 & -18 \\ \hline & 1 & -2 & 6 & -17 \end{array}$$

$g(x)=\boxed{x^2-2x+6}-\frac{17}{x+3}$

But $g(x)$ has no slant asymptote

Recall a slant asymptote has the form $y=ax+b$.

Is $y=x^2-2x+6$ of that form? No

Hence no slant asymptote.

Note: You can avoid all that work by checking the difference b/w the power of the leading terms of numerator and denominator.

- If difference $=0$, then there horizontal asymptote
- If difference $=1$, then there slant asymptote
- If difference >1, then nothing