

MA 16010 LESSONS 20+21: OPTIMIZATION

Optimization Problems: Often this includes finding the maximum or minimum value of some function.

- e.g. The minimum time to make a certain journey,
- e.g. The minimum cost for constructing some object,
- e.g. The maximum profit to gain for a business, and so on.

How do we solve an optimization problem?

- Determine a function (**known as objective function**) that we need to maximize or minimize.
- Determine if there are some constraints on the variables. (The equations that describe the constraints are called **the constraint equations**.)
 - If there are constraint equations, rewrite the **objective function** as a function of only one variable.
- Then we can solve for absolute maximum or minimum like we did before.
 - Using either the First Derivative Test or Second Derivative Test.

Recipe for Solving an Optimization Problem

Step 1: Identify what quantity you are trying to optimize.

Step 2: Draw a picture (if applicable), corresponding to the problem, and label it with your variables.

Step 3: Express the variable to be optimized as a function of the variables you used in Step 2.

Step 4: Find relations among the variables from Step 2 and express the variable to be optimized a function of just one of the variables from Step 2.

Step 5: Find the absolute extrema of the variable to be optimized on this domain.

Step 6: Reread the question and be sure you have answered exactly what was asked.

Example 1: Of all the numbers whose sum is 50, find the two that have the maximum product.

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Example 2: A carpenter is building a rectangular room with a fixed perimeter of 54 feet. What are the dimensions of the largest room that can be built? What is its area?

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Example 3: An open-top box with a square base is to have a volume of 8 cubic feet. Find the dimensions of the box that can be made with the least amount of material.

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Example 4: From a thin piece of cardboard 20 in by 20 in, square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume?

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Example 5: A rectangular box has a square base. If the sum of the height and the perimeter of the square base is 20 in, what is the maximum possible volume?

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Example 6: A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2400 - 200p$ units. Each unit costs \$5 to make.

a) What price, p , should the company charge to maximize their revenue?

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Example 6: A company's marketing department has determined that if their product is sold at the price of p dollars per unit, they can sell $q = 2400 - 200p$ units. Each unit costs \$5 to make.

b) What price, p , should the company charge to maximize their profit?

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Example 7: A rectangular box is to have a square base and a volume of 800 ft^3 . If the material for the base costs \$2 per square foot, the material for the sides costs \$4 per square foot, and the material for the top costs \$1 per square foot, determine the minimum cost for constructing such a box.

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Example 8: Find the point on the graph of $f(x) = 2x + 4$ that is the closest to the point $(1,3)$.

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