Lesson 4: Basic Rules of Differentiation

Note all the following rules can be proven via the limit definition of the derivative.

① Constant Rule: For any constant c,

$$\frac{d}{dx}[c]=0$$

② Power Rule: For any real #, n,

$$\frac{d}{dx}[x^n]=nx^{n-1}$$

③ Constant Multiple Rule: $\frac{d}{dx}[cf(x)]=c\frac{d}{dx}[f(x)]$

④+⑤ Sum/Difference Rule:

$$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$$

Before we do some examples, recall the following equations for powers:

$$\frac{1}{x^m}=x^{-m} \quad \text{and} \quad \sqrt[q]{x^p}=x^{p/q}$$

Example 1: Find the derivative of the following:

ⓐ $f(x)=x^2$

By ②, $f'(x)=2x^{2-1}=2x$

ⓑ $f(x)=\frac{1}{x^4}$

Let's first rewrite the function: $f(x)=x^{-4}$

By ②, $f'(x)=-4x^{-4-1}=-4x^{-5}=-\frac{4}{x^5}$

ⓒ $f(x) = \sqrt[6]{x^7}$

Let's first rewrite the function: $f(x) = x^{7/6}$

By ②, $f'(x) = \frac{7}{6} x^{7/6 - 1} = \frac{7}{6} x^{1/6}$

ⓓ $f(x) = \frac{2}{\sqrt[5]{x}}$

Let's first rewrite the function: $f(x) = \frac{2}{x^{1/5}} = 2x^{-1/5}$

By ②, $f'(x) = 2\left(\frac{-1}{5}\right) x^{-1/5 - 1} = \frac{-2}{5} x^{-6/5}$

ⓔ $f(x) = x^5 + 5x^2$

By ④, $f'(x) = \frac{d}{dx}(x^5) + \frac{d}{dx}(5x^2)$

By ③, $f'(x) = \frac{d}{dx}(x^5) + 5\frac{d}{dx}(x^2)$

By ②, $f'(x) = 5x^{5-1} + 5 \cdot 2x^{2-1} = 5x^4 + 10x$

ⓕ $f(x) = x^6 + x^{-5/7}$

By ④, $f'(x) = \frac{d}{dx}(x^6) + \frac{d}{dx}(x^{-5/7})$

By ②, $f'(x) = 6x^{6-1} + \left(\frac{-5}{7}\right) x^{-5/7 - 1}$

$= 6x^5 - \frac{5}{7} x^{-12/7}$

ⓖ $f(x) = x^3(-2x^2 + 8x - 2)$

Let's expand out the multiplication

$f(x) = -2x^2x^3 + 8x x^3 - 2x^3$

$= -2x^5 + 8x^4 - 2x^3$

By ④, ③, ②, $f'(x) = -2(5)x^4 + 8(4)x^3 - 2(3)x^2$

$= -10x^4 + 32x^3 - 6x^2$

(b) $f(x) = \dfrac{x^2 - 8x^{1.8}}{\sqrt{x}}$

Let's first simplify the function:

$$f(x) = \frac{x^2 - 8x^{1.8}}{x^{1/2}} = \frac{x^2}{x^{1/2}} - \frac{8x^{1.8}}{x^{1/2}} = x^{3/2} - 8x^{1.3}$$

Then $f'(x) = \dfrac{3}{2}x^{3/2-1} - 8(1.3)x^{1.3-1}$

$$= \frac{3}{2}x^{1/2} - 10.4x^{0.3}$$

Derivative of Sine and Cosine

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

Example 2: Find y' when $y = 3\sin x - 4\cos x$

$$y' = 3\frac{d}{dx}[\sin x] - 4\frac{d}{dx}[\cos x]$$

$$= 3\cos x - 4(-\sin x)$$

$$= 3\cos x + 4\sin x$$

Example 3: Find the equation of the tangent line to the graph of $f(x) = 8\cos x$ @ $x = \pi/2$.

Using the game plan from Lesson 3,

1. Find $f'(x)$.

$$f'(x) = -8\sin x$$

2. Find $f'(\pi/2)$

$$f'(\pi/2) = -8\sin(\pi/2) = -8 \cdot 1 = -8$$

3. Find $f(\pi/2)$

$$f(\pi/2) = 8\cos(\pi/2) = 8 \cdot 0 = 0$$

4. Plug 2 and 3 into

$$y - f(\pi/2) = f'(\pi/2)(x - \pi/2)$$

$$y - 0 = -8(x - \pi/2)$$

$$y = -8x + 4\pi$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x)$$

Example 4: Find y' of $y = \pi e^x$

Note π is just a constant. So

$$y' = \pi \frac{d}{dx}(e^x) = \pi e^x$$

Example 5: Find the x-value at which the derivative of $y = 10e^x$ is 1.

i.e. Solve $y'(x) = 1$ for x.

First find $y' = 10 \frac{d}{dx}(e^x) = 10e^x$

Set that equal to 1, and then solve for x.

$$10e^x = 1$$
$$e^x = 1/10$$
$$\ln(e^x) = \ln(1/10)$$
$$x = \ln\left(\frac{1}{10}\right)$$