MA 16010 QUIZ 3 (SOLUTIONS) WED., JUNE 22, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:____

1. **[3 pts]** The derivative of a function is found by

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{3\sin(x+h)}{\sqrt{x+h}} - \dfrac{3\sin x}{\sqrt{x}}}{h}$$

What is $f(x)$?

Solution: Recall that

$$f'(x) = \lim h \to 0 \frac{f(x+h) - f(x)}{h}$$

It is enough to focus our attention to the purple portion to determine $f(x)$. So

$$f(x) = \frac{3\sin(x)}{\sqrt{x}} \qquad \textbf{[3 pts]}$$

2. Find the derivative of the following functions:

(a) **[1 pt]** $f(x) = 3e^x$

Solution: Recall that

$$\frac{d}{dx}(e^x) = e^x$$

So $f'(x) = 3e^x$

(b) **[1 pt]** $g(x) = 7\cos(x)$

Solution: Recall that

$$\frac{d}{dx}(\cos x) = -\sin x$$

So $g'(x) = -7\sin x$

(c) **[3 pts]** $h(x) = \sqrt[3]{x^2} + \dfrac{3}{x^4} - x$

Solution: Rewrite $h(x)$.

$$h(x) = x^{2/3} + 3x^{-4} - x$$

By Power Rule,

$$h'(x) = \frac{2}{3}x^{-1/3} + 3(-4)x^{-5} - 1 = \frac{2}{3}x^{-1/3} - 12x^{-5} - 1$$

3. **[4 pts]** Let $w(x) = 4\sin x \left(\sqrt[3]{x^2} + \frac{3}{x^4} - x\right)$. Find $w'(x)$. **(Don't Simplify.)**

Solution:

$$\text{Let} \quad u(x) = 4\sin(x) \qquad v(x) = \sqrt[3]{x^2} + \frac{3}{x^4} - x$$

$$u'(x) = 4\cos(x) \qquad v'(x) = \frac{2}{3}x^{-1/3} - 12x^{-5} - 1$$

By Product Rule,

$$w'(x) = u'(x)v(x) + u(x)v'(x)$$

$$= (4\cos(x))\left(\sqrt[3]{x^2} + \frac{3}{x^4} - x\right) + (4\sin(x))\left(\frac{2}{3}x^{-1/3} - 12x^{-5} - 1\right)$$