| Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided. |
|---|

Name:______________________________

1. Find the derivative of the following functions:

(a) **[3 pts]** $f(x) = (3x-1)^{100}$

**Solution:**

Let $u(x) = x^{100}$ and $v(x) = 3x - 1$
Then $u'(x) = 100x^{99}$ and $v'(x) = 3$

By Chain Rule,

$$\begin{aligned} f'(x) &= u'(v(x)) \cdot v'(x) \\ &= u'(3x-1) \cdot 3 \\ &= 100(3x-1)^{99} \cdot 3 \\ &= \boxed{300(3x-1)^{99}} \end{aligned}$$

(b) **[4 pts]** $g(x) = 20\tan^2(5x+1)$

**Solution:** First rewrite $g(x)$. So

$$g(x) = 20(\tan(5x+1))^2$$

Note to find the derivative of this function, we need to take apply Chain Rule **twice**.

Let $u(x) = 20x^2$ and $v(x) = \tan(5x+1)$

$a(x) = \tan(x)$ $\quad b(x) = 5x+1$
$a'(x) = \sec^2(x)$ $\quad b'(x) = 5$

Then $u'(x) = 40x$ and $v'(x) = a'(b(x)) \cdot a(x)$
$v'(x) = a'(5x+1) \cdot 5$
$v'(x) = sec^2(5x+1) \cdot 5$

By Chain Rule,

$$\begin{aligned} g'(x) &= u'(v(x)) \cdot v'(x) \\ &= u'(\tan(5x+1)) \cdot 5\sec^2(5x+1) \\ &= 40(\tan(5x+1)) \cdot 5\sec^2(5x+1) \\ &= \boxed{200\tan(5x+1)\sec^2(5x+1)} \end{aligned}$$

2. **[5 pts]** Find the second derivative of $h(x) = \ln\sqrt{x^2+2}$

**Solution:** First let's rewrite $h(x)$ so that taking the derivative can be easier. Using logarithmic properties,

$$h(x) = \ln\sqrt{x^2+2} = \ln(x^2+2)^{1/2} = \frac{1}{2}\ln(x^2+2)$$

Now let's apply Chain Rule.

$$\begin{array}{llll} \text{Let} & u(x) = \frac{1}{2}\ln(x) & \text{and} & v(x) = x^2+2 \\ \text{Then} & u'(x) = \frac{1}{2}\cdot\frac{1}{x} & \text{and} & v'(x) = 2x \end{array}$$

By Chain Rule,

$$\begin{aligned} h'(x) &= u'(v(x))\cdot v'(x) \\ &= u'(x^2+2)\cdot(2x) \\ &= \frac{1}{2}\cdot\frac{1}{x^2+2}\cdot(2x) \\ &= \frac{x}{x^2+2} \end{aligned}$$

Note that we want the **SECOND DERIVATIVE.** Notice that now we have a quotient so let's use Quotient Rule on $h'(x)$.

$$\begin{array}{llll} \text{Let} & u(x) = x & \text{and} & v(x) = x^2+2 \\ \text{Then} & u'(x) = 1 & \text{and} & v'(x) = 2x \end{array}$$

By Quotient Rule,

$$\begin{aligned} h''(x) &= \frac{u'(x)v(x) - u(x)v'(x)}{v^2(x)} \\ &= \frac{1\cdot(x^2+2) - x\cdot(2x)}{(x^2+2)^2} \\ &= \frac{x^2+2-2x^2}{(x^2+2)^2} \\ &= \frac{2-x^2}{(x^2+2)^2} \end{aligned}$$