MA 16010 QUIZ 5 (SOLUTIONS) TUES., JULY 12, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:______________________________________

1. **[5 pts]** Find $x$ value at which the function $g(x) = \frac{1}{5}x^5 - 3x^3$ has a relative minimum.

**Solution:** Recall that there are 2 ways to find a relative minimum. Below I will show it using the First Derivative Test.

First solve $g'(x) = 0$.

$$\begin{aligned} g'(x) = x^4 - 9x^2 &= 0 \qquad \textbf{[1pt]} \\ x^2(x^2 - 9) &= 0 \\ x^2(x-3)(x+3) &= 0 \\ x &= -3, 0, 3 \qquad \textbf{[1pt]} \end{aligned}$$

Next draw a number line with $x = -3, 0, 3$. With that number line determine test points and plug them into $f'$ to determine whether $f'$ is positive or negative in that interval.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $+$ | | $-$ | | $-$ | | $+$ |
| $f'$ | | $-3$ | | $0$ | | $3$ | |

**[2 pts]**

Recall that we have a relative minimum if on the sign chart, we have + and then −. Hence we have a relative mininum when $x = 3$ **[1 pt]**.

2. **[7 pts]** Given

$$f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + 7$$

Determine the largest open interval(s) on which $f(x)$ is decreasing and concave down.

**Solution:** Remember to determine increasing/decreasing we need to create a sign chart (i.e. number line) using $f'$. So,

$$\begin{aligned} f'(x) = x^3 + 2x^2 + x &= 0 \qquad \textbf{1 pt]} \\ x(x^2 + 2x + 1) &= 0 \\ x(x+1)^2 &= 0 \\ x &= -1, 0 \qquad \textbf{[1 pt]} \end{aligned}$$

Next draw a number line with $x = -1, 0$. With that number line determine test points and plug them into $f'$ to determine whether $f'$ is positive or negative in that interval.

| | $(-\infty,-1)$ | $-1$ | $(-1,0)$ | $0$ | $(0,\infty)$ | |
|---|---|---|---|---|---|---|
| $f'$ | $-$ | | $-$ | | $+$ | **[1 pt]** |

Remember to determine concavity we need to create a sign chart (i.e. number line) using $f''$. So,

$$
\begin{aligned}
f''(x) = 3x^2 + 4x + 1 &= 0 \qquad \textbf{1 pt}] \\
(3x+1)(x+1) &= 0 \\
x &= -1, -1/3 \qquad [\textbf{1 pt}]
\end{aligned}
$$

Next draw a number line with $x = -1, -1/3$. With that number line determine test points and plug them into $f''$ to determine whether $f''$ is positive or negative in that interval.

| | $(-\infty,-1)$ | $-1$ | $\left(-1,-\frac{1}{3}\right)$ | $-\frac{1}{3}$ | $\left(-\frac{1}{3},\infty\right)$ | |
|---|---|---|---|---|---|---|
| $f''$ | $+$ | | $-$ | | $+$ | **[1 pt]** |

Recall we want decreasing (- for $f'$) and concave down (- for $f''$).

Hence based on both number lines, that only occurs in the interval $\left(-1, -\frac{1}{3}\right)$ [**1 pt**].