MA 16010 Quiz 7 (SOLUTIONS) Wed., July 20, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. **[4 pts]** Determine all the asymptotes of

$$f(x) = \frac{x^2 + 4x + 3}{x^2 - 1}$$

Solution: **Vertical Asymptote(s):** First we need to simplify $f(x)$ just in case there are holes.

$$f(x) = \frac{x^2 + 4x + 3}{x^2 - 1} = \frac{(x+1)(x+3)}{(x+1)(x-1)} = \frac{x+3}{x-1}$$

Hence we have a VA at $x = 1$.

Horizontal/Slant Asymptote: Recall that you can only have a Horizontal Asymptote or a Slant Asymptote. Not both.

By checking the leading terms of the numerator and denominator of $f(x)$, we see that their degrees match. i.e. We have a Horizontal Asymptote. Hence with their coefficients we see that we have a HA at $y = 1$ and Slant NONE.

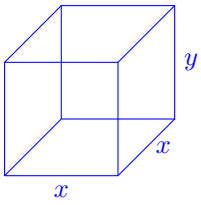
2. **[8 pts]** Vann is building an open top wooden box for his kitten, Misu, who likes to play with boxes. Vann is using wood on all four sides and the bottom, and is covering the wood on the bottom with carpet.

Vann would like the bottom to have a square base. The cost of wood is \$2 per ft^2 and the cost of the carpet is \$ 1 per ft^2. Misu requires 48 ft^3 to have a satisfactory play in the box. What is the minimum total cost of this box?

Solution: After reading the problem, we have a open top box with the following conditions: Wood for 4 sides and the Bottom - \$2 and Carpet for the Bottom - \$2.

(1) **[1 pt]** Cost, C

(2) **[1 pt]** Open-Top Box



(3) **[1 pt]**

$$\begin{aligned} C &= \$2(4xy + x^2) + \$1(x^2) \\ &= 8xy + 2x^2 + x^2 \\ &= 8xy + 3x^2 \end{aligned}$$

(4) **[1 pt]** $48 = x^2 y$

(5) **[3 pts]** Solve (4) for y.

$$48 = x^2 y$$
$$y = \frac{48}{x^2}$$

Plug y into (3).

$$\begin{aligned} C &= 8x\left(\frac{48}{x^2}\right) + 3x^2 \\ &= \frac{384}{x} + 3x^2 \\ &= 384x^{-1} + 3x^2 \end{aligned}$$

Take the derivative and set $= 0$.

$$\begin{aligned} C' &= -384x^{-2} + 6x = 0 \\ -\frac{384}{x^2} + 6x &= 0 \\ \frac{6x}{1} &= \frac{384}{x^2} \\ 6x^3 &= 384 \\ x^3 &= 64 \\ x &= 4 \end{aligned}$$

(6) **[1 pt]** We want the Cost of this box. With $x = 4$,

$$y = \frac{48}{x^2} = \frac{48}{4^2} = 3$$

Hence the Cost of such box is $C = \$144$