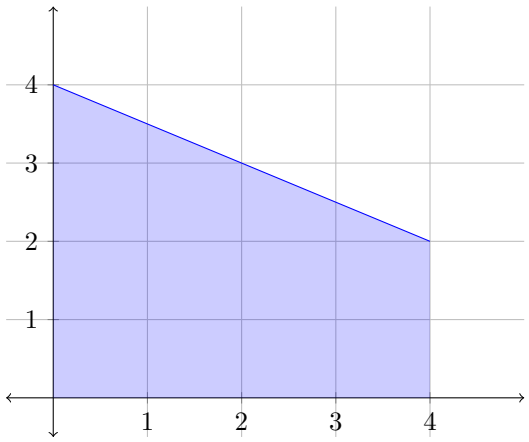
MA 16010 Quiz 8 (SOLUTIONS) Thurs., July 28, 2022

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:_____

1. Given the following graph, answer the following questions:



(a) **[3 pts]** Write the definite integral that represents the shaded area.

Solution: We can see the bounds of the integral will be 0 to 4. i.e.

$$\int_0^4 \boxed{}\, dx \quad \textbf{[1 pt]}$$

Next we need to determine the equation of the dark blue line. Note that we have two points (0,4) and (4,2). So the slope of those two points is

$$m = \frac{2-4}{4-0} = \frac{-1}{2} \quad \textbf{[1 pt]}$$

Note we also are given the y-intercept. So the equation of the line is

$$y = -\frac{1}{2} + 4$$

Hence,

$$\int_0^4 \left(-\frac{1}{2}x + 4\right) dx \quad \textbf{[1 pt]}$$

(b) Using **GEOMETRIC FORMULAS**, evaluate the integral found in (a).

Solution: Using the graph above, we can find the area by using the trapezoid area formula or by splitting the graph into a rectangle and a triangle. Either way the

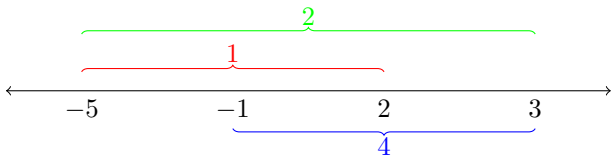
$$A = 12 \quad \textbf{[3 pts]}$$

2. **[6 pts]**

Given $\int_{-5}^{2} f(x)\,dx = 1$, $\int_{-1}^{3} f(x)\,dx = 4$ and $\int_{-5}^{3} f(x)\,dx = 2$. Find $\int_{-1}^{2} f(x)\,dx$

.

Solution: Start by creating a number line with all the information given.



[2 pts]

Note there are two ways of getting the correct answer.

(i) First find

$$\int_{-5}^{-1} f(x)\,dx = \int_{-5}^{3} f(x) - \int_{-1}^{3} f(x)\,dx = 2 - 4 = -2 \qquad \textbf{[2 pts]}$$

Then

$$\int_{-1}^{2} f(x)\,dx = \int_{-5}^{2} f(x)\,dx - \int_{-5}^{-1} f(x)\,dx = 1 - (-2) = 3 \qquad \textbf{[2 pts]}$$

(ii) **OR....** First find

$$\int_{2}^{3} f(x)\,dx = \int_{-5}^{3} f(x) - \int_{-5}^{2} f(x)\,dx = 2 - 1 = 1 \qquad \textbf{[2 pts]}$$

Then

$$\int_{-1}^{2} f(x)\,dx = \int_{-1}^{3} f(x)\,dx - \int_{2}^{3} f(x)\,dx = 4 - 1 = 3 \qquad \textbf{[2 pts]}$$