MA 16020 EXAM 2 STUDY GUIDE: CALCULUS II

When to use substitution to integrate?

- **When you have something containing a function (which we call u) and that something is multiplied by the derivative of u.**

  Ex. $\int f\big(u(x)\big) \cdot u'(x)\, dx = \int f(u)\, du$

- **How do you use substitution?**
  - **Determine if there is an inner function and call that u.**
  - **Take the derivative of u. So you have**

    $$du = u'(x)\, dx$$
  - **Solve for dx.**
  - **Transform the integral using u and dx.**

When to use partial fraction decomposition to integrate?

- **When you have a fraction with polynomials on the numerator and denominator, and substitution doesn't work.**

- **How do you use partial fraction decomposition?**
  - **Decompose the fraction using the steps outlined in the Handout, METHOD OF DECOMPOSING INTO PARTIAL FRACTIONS.**

- **Note: Some integrals will yield $ln|?\,|$ and others will need a substitution.**

When to use by parts to integrate?

- **When all else fails**

- **How do you use by parts?**
  - **Choose u to be the one to differentiate**
    - **Recall the acronym that tells how to choose u.**

      L – Logarithmic

      A – Algebraic (like polynomials)

      T – Trigonometric

      E – Exponential
  - **Choose dv to be integrated**
  - **Determine du and v and apply the following formula:**

$$u \cdot v - \int v\, du$$

- **Note:**
  1. **You may have to do a substitution within your problem.**
  2. **You may have to apply by parts more than once.**

MA 16020 Exam 2 Study Guide: Cal 2

An <u>improper integral</u> is when
(1) we have $\pm\infty$ in the bounds, or
(2) we have a discontinuity within the bounds
$\downarrow$
Check if the integrand is undefined and check if that value is in the interval.

When computing them, rewrite with a limit

<u>ex.</u> $\int_0^\infty e^{-x}\,dx = \lim_{N\to\infty} \int_0^N e^{-x}\,dx$

To review limit check MA 16020 Exam 2 Study Guide: Cal 1.

<u>Area Between Two Curves</u>

The area between two curves can be described two ways:

$A = \int_a^b (\text{Top} - \text{Bottom})\,dx \rightarrow$ You want $y = \text{something}_x$ for Top and Bottom

or $A = \int_c^d (\text{Right} - \text{Left})\,dy \rightarrow$ You want $x = \text{something}_y$ for Right and Left

<u>Volume of Solids of Revolution</u>

Read the problem to see if a particular method is asked for. Plus try to draw the regions.

When the region "hugs" the line of rotation ⇒ Disk

- x-axis ⇒ dx problem ⇒ $V = \int_a^b \pi (f(x))^2 \, dx$
- y-axis ⇒ dy problem ⇒ $V = \int_c^d \pi (g(y))^2 \, dy$
- the line $y = \#$ ⇒ dx problem ⇒ $V = \int_a^b \pi (f(x) - \#)^2 \, dx$
- the line $x = \#$ ⇒ dy problem ⇒ $V = \int_c^d \pi (g(y) - \#)^2 \, dy$

When there is a "gap" between the region and the line of rotation ⇒ Washer

- x-axis ⇒ dx problem ⇒ $V = \int_a^b \pi (R^2 - r^2) \, dx$
- y-axis ⇒ dy problem ⇒ $V = \int_c^d \pi (R^2 - r^2) \, dy$
- the line $y = \#$ ⇒ dx problem ⇒ $V = \int_a^b \pi \left[(R - \#)^2 - (r - \#)^2\right] dx$
- the line $x = \#$ ⇒ dy problem ⇒ $V = \int_c^d \pi \left[(R - \#)^2 - (r - \#)^2\right] dy$

Where R is the farthest from the line of rotation and r is the closest to the line of rotation

---

But if you find solving for x or y, in either method, is hard ⇒ Shell

MA 16020 LESSON 18: VOLUME BY REVOLUTION – SHELL METHOD (SUPPEMENTAL HOMEWORK)

Formulas:

- **Rotating around y-axis:**

$$V = 2\pi \int_a^b x \cdot (Top - Bottom)\, dx$$

- **Rotating around y = #**
  - **If $a \geq \#$, then**

$$V = 2\pi \int_a^b (x - \#) \times (Top - Bottom)\ dx$$

  - **If b $\leq$ #, then**

$$V = 2\pi \int_a^b (\# - x) \times (Top - Bottom)\ dx$$

- **Rotating around x-axis:**

$$V = 2\pi \int_c^d y \cdot (Right - Left)\, dy$$

- **Rotating around x = #**
  - **If $a \geq \#$, then**

$$V = 2\pi \int_a^b (y - \#) \times (Right - Left)\ dy$$

  - **If b $\leq$ #, then**

$$V = 2\pi \int_a^b (\# - y) \times (Right - Left)\ dy$$
