MA 16020 LESSONS 17 + 18: VOLUME BY REVOLUTION – SHELL METHOD (SUPPEMENTAL HOMEWORK)

Formulas:

- **Rotating around y-axis:**

$$V = 2\pi \int_a^b x \cdot (Top - Bottom)\, dx$$

- **Rotating around x-axis:**

$$V = 2\pi \int_c^d y \cdot (Right - Left)\, dy$$

- **Rotating around x = #**
  - If $x = \#$ is on the left of your region, then

$$V = 2\pi \int_a^b (x - \#) \times (Top - Bottom)\ dx$$

  - If $x = \#$ is on the right of your region, then

$$V = 2\pi \int_a^b (\# - x) \times (Top - Bottom)\ dx$$

- **Rotating around y = #**
  - If $y = \#$ is below your region, then

$$V = 2\pi \int_a^b (y - \#) \times (Right - Left)\ dy$$

  - If $y = \#$ is above your region, then

$$V = 2\pi \int_a^b (\# - y) \times (Right - Left)\ dy$$

(OPTIONAL HOMEWORK): Set up the integral using the Shell Method that represents the volume of the following solids about the given line:

1. $x = y,$ $x = 0,$ $y = 2$ about the x-axis

2. $x = 2y - y^2, \quad x = 0$ **about the x-axis**

3. $y = \sqrt{x}, \qquad y = -x$ **about the y-axis**

4. $y = 2 - x^2, \qquad y = x^2$ **about the y-axis**

5. $y = x,$ $\qquad y = x^2$ $\qquad\qquad$ about $x = -2$

6. $y = \sqrt{x},$ $\qquad y = 0,$ $\qquad x = 1$ $\qquad$ about $x = -1$

7. $y = x,$ $\qquad y = x^2$ $\qquad\qquad$ about $x = 3$

8. $y = 4x - x^2$, $\quad y = 3$ $\qquad\qquad\qquad\qquad$ about $x = 1$

9. $x = 2y - y^2$, $\quad x = 1$, $\qquad\qquad y = 0$ $\qquad$ about $y = -1$

10. $x = y^2 + 1$, $\quad x = 2$ $\qquad\qquad\qquad\qquad$ about $y = -2$

11. $x = 4y^2 - y^3, \quad x = 0$ **about** $y = 6$

12. $x = (y - 3)^2, \quad x = 4$ **about** $y = 1$