Lesson 0: Review

Exponential Rules

1. $x^a x^b = x^{a+b}$
2. $\frac{x^a}{x^b} = x^{a-b}$
3. $(x^a)^b = x^{a \cdot b}$
4. $x^1 = x$
5. $x^0 = 1$
6. $x^{-1} = \frac{1}{x}$

Logarithmic Rules

1. $\ln 1 = 0$
2. $\ln(e^x) = x$
3. $e^{\ln x} = x$
4. $\ln(xy) = \ln x + \ln y$
5. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
6. $\ln(x^m) = m \ln x$

Rational Powers

1. $\sqrt{x} = x^{1/2}$
2. $\sqrt[3]{x} = x^{1/3}$
3. $\sqrt[q]{x^p} = x^{p/q}$

Trigonometry

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	$0 = \frac{0}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
cos	$1 = \frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{0}{2} = 0$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

Lesson 1: Review of Differentiation

Constant Rule: $\frac{d}{dx}(c) = 0$ where c is a constant

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$ where n is any real #

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$ where c is a constant

Sum/Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Example 1: Find the derivative for the following:

ⓐ $f(x) = x^3$

$f'(x) = 3x^2$

ⓑ $f(x) = 1/x$

$f(x) = x^{-1}$

$f'(x) = -x^{-2} = -\frac{1}{x^2}$

ⓒ $f(x) = \sqrt{x^3}$

$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2}x^{1/2} = \frac{3}{2}\sqrt{x}$

ⓓ $f(x) = \frac{3}{x^4} - 2x^2 + 6x - 7$

$f(x) = 3x^{-4} - 2x^2 + 6x - 7$

$f'(x) = 3(-4)x^{-5} - 2(2)x + 6$

$= -12x^{-5} - 4x + 6$

$= -\frac{12}{x^5} - 4x + 6$

ⓔ $f(x) = \dfrac{x^{2.5} - 2x^{-3}}{x}$

$$f(x) = \frac{x^{2.5}}{x} - \frac{2x^{-3}}{x} = x^{1.5} - 2x^{-4}$$

$$f'(x) = 1.5\,x^{0.5} - 2(-4)\,x^{-5}$$
$$= 1.5\,x^{0.5} + 8x^{-5}$$

Position / Velocity / Acceleration

Position	$s(t)$
Velocity	$v(t) = s'(t)$
Acceleration	$a(t) = v'(t) = s''(t)$

Other Differentiation Rules

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \tan x \sec x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

Product Rule: $\dfrac{d}{dx}(u(x)v(x)) = u'(x)\,v(x) + u(x)\,v'(x)$

Quotient Rule: $\dfrac{d}{dx}\left[\dfrac{u(x)}{v(x)}\right] = \dfrac{u'(x)v(x) - u(x)v'(x)}{v^2(x)}$

Chain Rule: Let $h(x) = f(g(x))$. Then
$$h'(x) = f'(g(x)) \cdot g'(x)$$

Example 2: Find the derivative of the following

ⓐ $y = (3x+1)^2$

$f(x) = x^2 \qquad g(x) = 3x+1$

$f'(x) = 2x \qquad g'(x) = 3$

$y' = f'(g(x))\,g'(x)$

$= f'(3x+1) \cdot 3$

$= 2(3x+1) \cdot 3 = 6(3x+1) = 18x+6$

ⓑ $y = 2\cos^3 x$

$y = 2(\cos x)^3$

$f(x) = 2x^3 \qquad g(x) = \cos x$

$f'(x) = 6x^2 \qquad g'(x) = -\sin x$

$y' = f'(g(x)) \cdot g'(x)$

$= f'(\cos x) \cdot (-\sin x)$

$= 6(\cos x)^2 \cdot (-\sin x) = -6\cos^2 x \sin x$

ⓒ $y = \dfrac{15}{\sqrt[3]{x^2+1}}$

$y = 15(x^2+1)^{-1/3}$

$f(x) = 15x^{-1/3} \qquad g(x) = x^2+1$

$f'(x) = 15(-\tfrac{1}{3})x^{-4/3} \qquad g'(x) = 2x$

$= -5x^{-4/3}$

$y' = f'(g(x))\,g'(x)$

$= f'(x^2+1) \cdot 2x$

$= -5(x^2+1)^{-4/3} \cdot 2x = \dfrac{-10x}{(x^2+1)^{4/3}}$

ⓓ $y = \sin(x^3)$

$f(x) = \sin x \qquad g(x) = x^3$

$f'(x) = \cos x \qquad g'(x) = 3x^2$

$y' = f'(g(x)) \cdot g'(x)$

$= f'(x^3) \cdot (3x^2)$

$= \cos(x^3) \cdot 3x^2 = 3x^2\cos(x^3)$