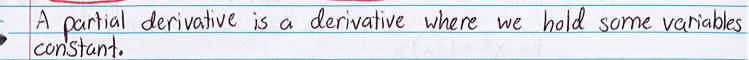
Lesson 27. Partial Derivatives



Let's think about a function of one variable. ex. $f(x)=x^2 \implies f'(x)=2x$

But what about a function of two variables? f(x,y)= x2+y

We find its partial derivative with respect to x by treating y as a constant. $f_x = 2x + 0^2 2x$

To find the partial clerivative with respect to y, we treat x as a constant.

fy = 0 + 3y² = 3y²

The partial clerivative with respect to y, we treat

Definition: The (first) partial derivative fx describes the rate of change of f as x changes, where y remains constant. i.e. Find the derivative with respect to x, where we treat y as a constant

· The (first) partial derivative fy describes the rate of change of f as y changes, where x remains constant. I.e. Find the derivative with respect to y, where we treat x as a constant.

Example 1: Compute the first order partial derivatives $(x,y) = x^3 + 3xy$

First order partials => We need to find fx and fy.

First find fx. i.e. Find the derivative w/ respect to x and treat y as a constant.

 $f = x^3 + (3y)x$ $f_{x} = 3x^2 + 3y$

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	Next find fur her Find the derivative w/ respect to y and
golden fov	treat x as a constant.
	Next find fy. i.e. Find the derivative w/ respect to y and treat x as a constant. $f= x^3 + (3x)y$ $f_y=0+3x=3x$
,	+y=0+3x = 3x
Chain	
Rule	First find fx, i.e. Find the derivative w/ respect to x
Problem	First find fx, i.e. Find the derivative w/ respect to x and treat y as a constant, $f_{x} = \frac{1}{x+2y} \cdot \frac{3}{2x} (x+2y) = \frac{1}{x+2y} \cdot (1+0) = \frac{1}{x+2y}$
	$\frac{1}{x} = \frac{1}{x + 2y} = \frac{1}{x + 2y} = \frac{1}{x + 2y} = \frac{1}{x + 2y}$
	and
V-2-24	Next find fy. i.e. Find the derivative w/ respect to y and treat x as a constant,
	treat x as a constant,
	$fy = \frac{1}{x + 2y} \cdot \frac{3}{2y} (x + 2y) = \frac{1}{x + 2y} \cdot (0 + 2) = \frac{2}{x + 2y}$
	$\bigcirc f(x,y) = \frac{qxy}{\sqrt{y-1}}$
ocs mes	First Rind fx, i.e. Find the derivative w/ respect to x and
where	First find fx. i.e. Find the derivative w/ respect to x and treat y as a constant.
	$f(x,y)^2 \frac{qy}{(x)}(x)$
s Hile	1y-1 Ry = 9y 2/1 - 9y
20	Jy-T' 3x (X) Jy-T'
avayna "	constant the first the derivative with magnet to
	Next find fy, i.e. Find the derivative w/ respect to y and treat x as a constant,
	$f(x,y) = q_x \left(\begin{array}{c} y \end{array} \right)$
	$\left(\sqrt{\sqrt{y-1}}\right)$
apply	$\frac{1}{1} = \frac{1}{2} \left(\frac{y}{y-1} \right) = \frac{1}{2} \left(\frac{1}{1} \cdot \frac{y-1}{y-1} - \frac{y \cdot \frac{y}{2}(y-1)^{2}}{(1-y-1)^{2}} \right)$
Rule	$= 9 \times (\sqrt{y-1} - \sqrt{2}) \times (\sqrt{y-1})^{-1/2}$
	4 material to an vy-1st
	$+ = x^2 + (3y) \times \dots \times (y^2 + y^2)$
	VC ↑ XC ₹XI

Example 2: Evaluate the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ at the given point $P_o(x_0,y_0)$, $f(x,y) = x^3y^2 + 6x^2$; $P_o(1,-1)$ First find fx, i.e. Find the derivative w/ respect to x and treat y as a constant,

fx = 3x2y2+12x2 Plug (1,-1) into fx. $f_X(1,-1) = 3(1)^2(-1)^2 + 12(1)^2 = 15$ Next find fy, i.e. Find the derivative w/ respect to y and treat x as a constant, fy = $x^3 \cdot 2y = 2x^3y$ Plug (1,-1) into fy. $f_y(1,-1) = 2(1)^3(-1) = -2$