MA 16020 LESSON 30: EXTREMA OF FUNCTIONS OF TWO VARIABLES (WORKSHEET)

How do we solve Optimization Problems via Extrema?

- Determine an **objective function** that we need to maximize or minimize.

- Determine if there are some constraints on the variables which yields **constraint equations.**
  - If there are constraint equations, rewrite the objective function as a function of only TWO variables.

- Then we can solve for the maximum or minimum via **the Second Derivative Test for Multivariable Functions** from Last Class.
  1. Find all the critical points.
     i.e. All (x_0, y_0) such that $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$
  2. Compute f_{xx}, f_{xy}, f_{yy} and
     $$D = f_{xx}f_{yy} - \left(f_{xy}\right)^2$$
     where D is known as the discriminant.
  3. For every given critical point (x_0, y_0), evaluate D and f_{xx} at (x_0, y_0)
  4. Apply the Second Derivative Test for Multivariable Functions.
     i. If $D(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) > 0$ $\Rightarrow$ relative min
     ii. If $D(x_0, y_0) > 0$ and $f_{xx}(x_0, y_0) < 0$ $\Rightarrow$ relative max
     iii. If $D(x_0, y_0) < 0$ $\Rightarrow$ saddle point
     iv. If $D(x_0, y_0) = 0$ $\Rightarrow$ Test is inconclusive

- Reread the question and be sure you have answered exactly what was asked.

<u>Example 1:</u> We are tasked with constructing a rectangular box with a volume of 13 cubic feet. The material for the top costs 9 dollars per square foot, the material for the 4 sides cost 2 dollars per square foot, and the material for the bottom costs 1 dollar per square foot. To the nearest cent, what is the minimum cost for such a box?

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<u>Example 2:</u> The post office will accept packages whose combined length and girth is at most 71 inches. (The girth is the perimeter/distance around the package perpendicular to the length; for a rectangular box, the length is the largest of the three dimensions.) What is the largest volume that can be sent in a rectangular box?

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<u>Example 3:</u> A manufacturer is planning to sell a new product at the price of 350 dollars per unit and estimates that if x thousand dollars is spent on development and y thousand dollars is spent on promotion, consumers will buy approximately

$$\frac{110y}{y+4}+\frac{190x}{x+9}$$

Units of the product. If the manufactor costs for the product are 170 dollars per unit, how much should the manufactor spend on development and how much on promotion to generate max profit?

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