

MA 16020 LESSONS 9+10: PARTIAL FRACTIONS

METHOD OF DECOMPOSING INTO PARTIAL FRACTIONS

Given a rational function $\frac{N(x)}{D(x)}$

1. Factor the denominator as much as possible.

2. Write the fraction into decomposition form.

a) Distinct linear terms like $x - a$ decompose to

$$\frac{A}{x - a}$$

b) Repeated linear terms like $(x - a)^3$ decompose to

$$\frac{A}{x - a} + \frac{B}{(x - a)^2} + \frac{C}{(x - a)^3}$$

c) Distinct irreducible quadratic terms like $x^2 + a^2$ decompose to

$$\frac{Ax + B}{x^2 + a^2}$$

d) Repeated irreducible quadratic terms like $(x^2 + a^2)^2$ decompose to

$$\frac{Ax + B}{x^2 + a^2} + \frac{Cx + D}{(x^2 + a^2)^2}$$

3. Combine your decomposition from (2) as 1 fraction.

4. Set the original numerator, $N(x)$, equal to the numerator from (3).

5. Equate the coefficients of the terms, to yield a system of equations. Then solve the constants.

i.e. Find A, B, C, \dots

6. Plug the values found in (5) in (2).