Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: Solutions

1. Given $f(x) = 2x^{5/2} - \cos(3\pi x)$, evaluate $f'(4)$.

$$f'(x) = 2\cdot\frac{5}{2}x^{3/2} - \left[-\sin(3\pi x)\right]\cdot(3\pi)$$

$$= 5x^{3/2} + 3\pi\sin(3\pi x)$$

$$f'(4) = 5(4)^{3/2} + 3\pi\underbrace{\sin(3\pi\cdot 4)}_{0} = 40$$

$f'(4) =$ $\boxed{40}$

2. Evaluate the definite integral

$$\int_0^{\pi/6} (3\cos(x) - 6)\,dx$$

$$= \left(3\sin(x) - 6x\right)\Big]_0^{\pi/6}$$

$$= 3\sin\left(\frac{\pi}{6}\right) - 6\left(\frac{\pi}{6}\right) - \left(\cancel{3\sin(0) - 6(0)}\right) \to 0$$

$$= \frac{3}{2} - \pi$$

$\int_0^{\pi/6} (3\cos(x) - 6)\,dx =$ $\boxed{\frac{3}{2} - \pi}$

3. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate $r(t)$ is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

$\left.\begin{array}{l} \text{10:00am} \Rightarrow 1 \text{ hr} \\ \text{1:00pm} \Rightarrow 4 \text{ hrs} \end{array}\right\} \Rightarrow \int_1^4 6t^{1/2}\,dt$

$= 6 \cdot \frac{2}{3} t^{3/2} \Big]_1^4$

$= 4t^{3/2} \Big]_1^4$

$= 28$

Answer: $\boxed{28}$

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Solve $\int_0^t 6t^{1/2}\,dt = 121$

$4t^{3/2} = 121$

$t^{3/2} = \frac{121}{4}$

$t = \left(\frac{121}{4}\right)^{2/3}$

Answer: $\boxed{\left(\frac{121}{4}\right)^{2/3}}$

4. Which derivative rule is undone by integration by substitution?
   (A) Power Rule
   (B) Quotient Rule
   (C) Product Rule
   (D) Chain Rule
   (E) Constant Rule
   (F) None of these

5. Which derivative rule is undone by integration by parts?
   (A) Power Rule
   (B) Quotient Rule
   (C) Product Rule
   (D) Chain Rule
   (E) Constant Rule
   (F) None of these

6. What would be the best substitution to make the solve the given integral?

$$\int e^{2x} \cos(e^{2x}) \sin^3(e^{2x})\, dx$$

$$\int e^{2x} \cos(e^{2x}) [\sin(e^{2x})]^3\, dx$$

$u =$ $\boxed{\sin(e^{2x})}$

Always check du is in integral

7. What would be the best substitution to make the solve the given integral?

$$\int \sec^2(5x) e^{\tan(5x)}\, dx$$

$u =$ $\boxed{\tan(5x)}$

Always check du is in integral

8. What would be the best substitution to make the solve the given integral?

$$\int \tan(5x) \sec(5x) e^{\sec(5x)}\, dx$$

$u =$ $\boxed{\sec(5x)}$

Always check du is in integral

9. Find the area under the curve $y = 14e^{7x}$ for $0 \leq x \leq 4$.

$$A = \int_0^4 14e^{7x}\,dx \quad \frac{u = 7x}{du = 7dx} \quad \int 2e^u\,du$$

$$= 2e^u = 2e^{7x}\Big]_0^4 = 2e^{28} - 2$$

Area = $\boxed{2e^{28} - 2}$

10. Evaluate the definite integral.

$$\int_0^2 (5e^{2x} + 8)\,dx$$

$$\underbrace{\int_0^2 5e^{2x}\,dx}_{u\text{-sub}} + \int_0^2 8\,dx = \frac{5}{2}e^{2x}\Big]_0^2 + 8x\Big]_0^2$$

$$= \frac{5}{2}(e^4 - e^0) + 8(2-0)$$

$$= \frac{5}{2}e^4 - \frac{5}{2} + 16$$

$$= \frac{5}{2}e^4 - \frac{27}{2}$$

$\int_0^2 (5e^{2x} + 8)\,dx =$ $\boxed{\frac{5}{2}e^4 + \frac{27}{2}}$

11. Evaluate the indefinite integral.

$$\int 64x^7 \sin(x^8)\, dx$$

$u = x^8$

$du = 8x^7 dx$

$\frac{du}{8x^7} = dx$

$\int \cancel{64x^7}^{\,8} \sin(u) \frac{du}{\cancel{8x^7}} = \int 8\sin(u)du$

$= -8\cos(u) + C$

$= -8\cos(x^8) + C$

$$\int 64x^7 \sin(x^8)\, dx = \boxed{-8\cos(x^8) + C}$$

12. Evaluate the indefinite integral.

$$\int 9x^3 e^{-x^4}\, dx$$

$u = -x^4$

$du = -4x^3 dx$

$\frac{du}{-4x^3} = dx$

$\int 9x^3 e^u \frac{du}{-4x^3} = -\frac{9}{4}\int e^u du$

$= -\frac{9}{4}e^u = -\frac{9}{4}e^{-x^4} + C$

$$\int 9x^3 e^{-x^4}\, dx = \boxed{-\frac{9}{4}e^{-x^4} + C}$$

13. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that t hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$L'(t) = \sqrt{3t+2} \quad \text{gallows per hour.}$$

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.

i.e. $\int_0^4 (3t+2)^{1/2}\,dt$ $\quad \dfrac{u = 3t+2}{du = 3\,dt}$ $\quad \int u^{1/2} \dfrac{du}{3}$

$\dfrac{du}{3} = dt$

$= \dfrac{1}{3} \cdot \dfrac{2}{3} u^{3/2} = \dfrac{2}{9}(3t+2)^{3/2} \Big]_0^4$

≈ 11.0122

Answer: $\boxed{11.0122}$

14. It is estimated that t-days into a semester, the average amount of sleep a college math student gets per day $S(t)$ changes at a rate of

$$\frac{-4t}{e^{t^2}} = -4te^{-t^2}$$

hours per day. When the semester begins, math students sleep an average of 8.2 hours per day. What is $S(t)$, 2 days into the semester?

① $\int -4te^{-t^2}dt$ $\quad \frac{u=-t^2}{du=-2t\,dt}$ $\quad \int -\overset{2}{\cancel{4t}}e^u \frac{du}{\cancel{-2t}}$

$\frac{du}{-2t} = dt$

$= \int 2e^u du = 2e^u + C$

$= 2e^{-t^2} + C$

② $S(0) = 8.2$ Find C.

$8.2 = 2e^0 + C$

$8.2 = 2 + C$

$C = 6.2$

③ $S(t) = 2e^{-t^2} + 6.2$

$S(2) = 2e^{-4} + 6.2$

≈ 6.237

Answer: $\boxed{6.237}$

15. A biologist determines that, t hours after a bacterial colony was established, the population of bacteria in the colony is changing at a rate given by

$$P'(t) = \frac{5e^t}{1+e^t}$$

million bacteria per hour, $0 \leq t \leq 5$.

If the bacterial colony started with a population of 1 million, how many bacteria, in millions are present in the colony after the 5-hour experiment?

① $\int \frac{5e^t}{1+e^t}\,dt$ $\quad \frac{u = 1+e^t}{du = e^t dt}$ $\quad \int \frac{5\cancel{e^t}}{u} \frac{du}{\cancel{e^t}} = \int \frac{5}{u}\,du$

$\frac{du}{e^t} = dt$

$= 5\ln|u| + c$

$= 5\ln|1+e^t| + c$

② $P(0) = 1$ Find c.

$1 = 5\ln|1+e^0| + c$

$1 = 5\ln|1+1| + c$

$1 = 5\ln 2 + c$

$1 - 5\ln 2 = c$

③ $P(t) = 5\ln|1+e^t| + 1 - 5\ln 2$

$P(5) = 5\ln|1+e^5| + 1 - 5\ln 2$

≈ 22.57

Answer: $\boxed{22.57}$

16. Evaluate the indefinite integral

$$\int x(x^2+4)^3\,dx$$

$u = x^2+4$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int \cancel{x}\,u^3\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int u^3\,du = \frac{1}{2}\cdot\frac{u^4}{4} + C$$

$$= \frac{1}{8}(x^2+4)^4 + C$$

$$\int x(x^2+4)^3\,dx = \boxed{\frac{1}{8}(x^2+4)^4 + C}$$

17. Evaluate the definite integral.

$$\int_0^{\pi/4} 3\sin(2x)\,dx$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$\int 3\sin(u)\,\frac{du}{2} = \frac{3}{2}\int \sin(u)\,du = -\frac{3}{2}\cos(u)$$

$$= -\frac{3}{2}\cos(2x)\Big]_0^{\pi/4}$$

$$= \cancel{-\frac{3}{2}\cos\left(\frac{2\pi}{4}\right)}^{\,0} - \left(-\frac{3}{2}\cos(0)\right)$$

$$= 3/2$$

$\int_0^{\pi/4} 3\sin(2x)\,dx =$ $\boxed{3/2}$

18. Evaluate the indefinite integral.

$$\int (x+4)\sqrt{x^2+8x}\,dx$$

$u = x^2 + 8x$

$du = (2x+8)\,dx$

$du = 2(x+4)\,dx$

$\dfrac{du}{2(x+4)} = dx$

$$\int \cancel{(x+4)}\sqrt{u}\,\frac{du}{2\cancel{(x+4)}}$$

$$= \frac{1}{2}\int u^{1/2}\,du$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{3/2} + C$$

$$= \frac{1}{3}(x^2+8x)^{3/2} + C$$

$\int (x+4)\sqrt{x^2+8x}\,dx =$ $\boxed{\frac{1}{3}(x^2+8x)^{3/2} + C}$

19. Evaluate the definite integral.

$$\int_0^9 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)}$$

$u = \sqrt{x} + 1$

$u = x^{1/2} + 1$

$du = \frac{1}{2}x^{-1/2}\,dx$

$du = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$$\int \frac{\cancel{2\sqrt{x}}\,du}{\cancel{2\sqrt{x}}\cdot u} = \int \frac{du}{u} = \ln|u|$$

$$= \ln|\sqrt{x}+1|\Big]_0^9$$

$$= \ln|\sqrt{9}+1| - \ln|\sqrt{0}+1|$$

$$= \ln(4)$$

$\int_0^9 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)} = \ln(4)$

20. A tree is transplanted and after t years is growing at a rate

$$r'(t) = 1 + \frac{1}{(t+1)^2} \qquad \text{meters per year.}$$

After 2 years it has reached a height of 5 meters. How tall was the tree when it was originally transplanted? Round to one decimal place.

$$r(t) = \int \left(1 + \frac{1}{(t+1)^2}\right) dt$$

$$= \int \left(1 + (t+1)^{-2}\right) dt$$

$$= t + \frac{(t+1)^{-1}}{-1} + c$$

$$= t - \frac{1}{t+1} + C$$

Find c w/ $r(2) = 5$

$$5 = 2 - \frac{1}{2+1} + c$$

$$3 + \frac{1}{3} = C$$

$$\frac{10}{3} = c$$

So $r(t) = t - \frac{1}{t+1} + \frac{10}{3}$

$$r(0) = 0 - 1 + \frac{10}{3}$$

$$= 7/3 \approx 2.3$$

Height = $\boxed{2.3}$

21. The marginal revenue from the sale of x units of a particular product is estimated to be $R'(x) = 50 + 350xe^{-x^2}$ dollars per unit, and where $R(x)$ is revenue in dollars. What revenue should be expected from the sale of 100 units? Assume that $R(0) = 0$.

$$R(x) = \int 50 + 350xe^{-x^2}\,dx$$

$$= \int 50\,dx + \underbrace{\int 350xe^{-x^2}\,dx}$$

$u = -x^2$

$du = -2x\,dx$

$\frac{du}{-2x} = dx$

$$= \int 50\,dx + \int 350\cancel{x}e^{u}\,\frac{du}{-2\cancel{x}}$$

$$= \int 50\,dx - 175\int e^{u}\,du$$

$$= 50x - 175e^{u} + C$$

$$= 50x - 175e^{-x^2} + C$$

$R(0) = 0$

$0 = 0 - 175 + C$

$C = 175$

$$R(x) = 50x - 175e^{-x^2} + 175$$

$R(100) \approx 5175$

$R(100) =$ $\boxed{5175}$

22. Evaluate the indefinite integral

$$\int \frac{\ln(5x)}{x}\,dx$$

$u = \ln(5x)$

$du = \frac{1}{5x} \cdot 5\,dx$

$du = \frac{1}{x}\,dx$

$\int u\,du = \frac{u^2}{2} = \frac{(\ln(5x))^2}{2} + C$

$\int \frac{\ln(5x)}{x}\,dx = \boxed{\frac{(\ln(5x))^2}{2} + C}$

23. Evaluate

$$\int_1^e \frac{\ln(x^4)}{x}\,dx$$

Rewrite $\int_1^e \frac{4\ln x}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$ $\quad \int 4u\,du = \frac{4u^2}{2} = 2u^2 = 2(\ln x)^2\Big]_1^e$

$= \underbrace{2(\ln e)^2}_{2} - \underbrace{2(\ln 1)^2}_{0}$

$= 2$

$\int_1^e \frac{\ln(x^4)}{x}\,dx = \boxed{2}$

24. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx$$

$\dfrac{u=x-1}{du=dx}$ $\dfrac{dv=\sin(x)\,dx}{v=-\cos(x)}$ $uv-\int v\,du = -(x-1)\cos x\Big]_0^{\pi/2} - \int_0^{\pi/2}(-\cos x)\,dx$

$= -(x-1)\cos x\Big]_0^{\pi/2} + \sin(x)\Big]_0^{\pi/2}$

$= \cancel{-\left(\frac{\pi}{2}-1\right)\cos\left(\frac{\pi}{2}\right)}^{\to 0} - \left[-(0-1)\cos(0)\right]$

$+ \sin\left(\frac{\pi}{2}\right) \cancel{-\sin(0)}^{\to 0}$

$= -1+1 = 0$

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx = \boxed{0}$$

25. Evaluate

$$\int 3x\ln(x^7)\,dx$$

Rewrite $\int 3x(7\ln(x))\,dx = \int 21x\ln x\,dx$

$\dfrac{u=21\ln(x)}{du=\frac{21}{x}\,dx}$ $\dfrac{dv=x\,dx}{v=\frac{x^2}{2}}$ $uv-\int v\,du$

$= \dfrac{21x^2\ln x}{2} - \int \dfrac{x^2}{2}\cdot\dfrac{21}{x}\,dx$

$= \dfrac{21x^2\ln x}{2} - \int \dfrac{21}{2}x\,dx$

$= \dfrac{21x^2\ln x}{2} - \dfrac{21}{2}\cdot\dfrac{x^2}{2} + C$

$= \dfrac{21x^2\ln x}{2} - \dfrac{21x^2}{4} + C$

$$\int 3x\ln(x^7)\,dx = \boxed{\frac{21x^2\ln x}{2} - \frac{21x^2}{4} + C}$$

26. Evaluate

$$\int x^3 \ln(2x)\, dx$$

$u = \ln(2x)$ $\quad$ $dv = x^3 dx$ $\quad$ $uv - \int v\,du = \frac{x^4 \ln(2x)}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$

$du = \frac{1}{2x} \cdot 2dx$ $\quad$ $v = \frac{x^4}{4}$

$du = \frac{1}{x} dx$

$$= \frac{x^4 \ln(2x)}{4} - \frac{1}{4}\int x^3 dx$$

$$= \frac{x^4 \ln(2x)}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C$$

$$\int x^3 \ln(2x)\, dx = \boxed{\frac{x^4}{4} \ln(2x) - \frac{x^4}{16} + C}$$

27. Evaluate the definite integral.

$$\int_0^3 5xe^{3x}\, dx$$

$u = 5x$ $\quad$ $dv = e^{3x} dx$ $\quad$ $uv - \int v\,du$

$du = 5dx$ $\quad$ $v = \frac{1}{3}e^{3x}$

$$= \frac{5x}{3}e^{3x} - \int \frac{5}{3}e^{3x} dx$$

$$= \left(\frac{5x}{3}e^{3x} - \frac{5}{3} \cdot \frac{e^{3x}}{3}\right)\Big]_0^3$$

$$= \frac{15}{3}e^9 - \frac{5}{9}e^9 - \left[0 - \frac{5}{9}\right]$$

$$= \frac{40}{9}e^9 + \frac{5}{9}$$

$$\int_0^3 5xe^{3x}\, dx = \boxed{\frac{40}{9}e^9 + \frac{5}{9}}$$

28. The population of pink elephants in Dumbo's dreams, in hundreds, t years after the year 1980 is given by

$$P(t) = \frac{e^{5t}}{1+e^{5t}}$$

What is the average population during the decade between 1980 and 2000?

i.e. $\frac{1}{2000-1980}\int_0^{20} \frac{e^{5t}}{1+e^{5t}}\,dt$ $\quad u = 1+e^{5t}$, $du = 5e^{5t}dt$, $\frac{du}{5e^{5t}} = dt$ $\quad \frac{1}{20}\int \frac{e^{5t}}{u}\cdot\frac{du}{5e^{5t}}$

$= \frac{1}{100}\int \frac{du}{u} = \frac{1}{100}\ln|u|$

$= \frac{1}{100}\ln|1+e^{5t}|\Big]_0^{20}$

≈ 0.9931

Answer: $\boxed{\text{0.9931 hundreds or 993}}$

29. Evaluate the indefinite integral.

$$\int 4x\sin(7x)\,dx$$

$u = 4x$, $du = 4dx$ $\quad dv = \sin(7x)\,dx$, $v = -\frac{\cos(7x)}{7}$ $\quad uv - \int v\,du$

$= -\frac{4}{7}x\cos(7x) + \int \frac{4}{7}(+\cos(7x))\,dx$

$= -\frac{4}{7}x\cos(7x) + \frac{4}{7}\int\cos(7x)\,dx$

$= -\frac{4}{7}x\cos(7x) + \frac{4}{7}\frac{\sin(7x)}{7} + C$

$\boxed{-\frac{4}{7}x\cos(7x) + \frac{4}{7}\frac{\sin(7x)}{7} + C}$

$\int 4x\sin(7x)\,dx =$ ________________

30. The velocity of a cyclist during an hour-long race is given by the function

$$v(t) = 166te^{-2.2t} \text{ mi/hr}, \quad 0 \le t \le 1$$

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?

① $\int 166te^{-2.2t}\,dt$

$u = 166t$ $\quad dv = e^{-2.2t}\,dt$ $\quad uv - \int v\,du$

$du = 166\,dt$ $\quad v = \dfrac{e^{-2.2t}}{-2.2}$

$$= \frac{166te^{-2.2t}}{-2.2} + \int \frac{e^{-2.2t}}{+2.2} \cdot 166\,dt$$

$$= -\frac{166te^{-2.2t}}{2.2} + \frac{166}{2.2} \cdot \frac{e^{-2.2t}}{-2.2} + C$$

$$= -\frac{166te^{-2.2t}}{2.2} - \frac{166e^{-2.2t}}{(2.2)^2} + C$$

② $s(0) = 0$. Find C.

$$0 = 0 - \frac{166}{(2.2)^2} + C \longrightarrow C = \frac{166}{(2.2)^2}$$

③ $s(t) = -\dfrac{166te^{-2.2t}}{2.2} - \dfrac{166e^{-2.2t}}{(2.2)^2} + \dfrac{166}{(2.2)^2}$

$$s(1) = -\frac{166}{22}e^{-2.2} - \frac{166}{(2.2)^2}e^{-2.2} + \frac{166}{(2.2)^2}$$

≈ 22.137

Answer: $\boxed{22.137}$