Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name: Solutions

1. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx$$

$u = x-1$, $du = dx$ $\quad$ $dv = \sin(x)\,dx$, $v = -\cos(x)$ $\quad$ $uv - \int v\,du = -(x-1)\cos x\Big]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)\,dx$

$$= -(x-1)\cos x\Big]_0^{\pi/2} + \sin(x)\Big]_0^{\pi/2}$$

$$= \cancel{-\left(\frac{\pi}{2}-1\right)\cos\left(\frac{\pi}{2}\right)}^{\,0} - \left[-(0-1)\cos(0)\right] + \sin\left(\frac{\pi}{2}\right) - \cancel{\sin(0)}^{\,0}$$

$$= -1 + 1 = 0$$

$\int_0^{\pi/2} (x-1)\sin(x)\,dx = \boxed{0}$

2. Evaluate

$$\int 3x\ln(x^7)\,dx$$

Rewrite $\int 3x(7\ln(x))\,dx = \int 21x\ln x\,dx$

$u = 21\ln(x)$, $du = \frac{21}{x}\,dx$ $\quad$ $dv = x\,dx$, $v = \frac{x^2}{2}$ $\quad$ $uv - \int v\,du$

$$= \frac{21x^2\ln x}{2} - \int \frac{x^2}{2}\cdot\frac{21}{x}\,dx$$

$$= \frac{21x^2\ln x}{2} - \int \frac{21}{2}x\,dx$$

$$= \frac{21x^2\ln x}{2} - \frac{21}{2}\cdot\frac{x^2}{2} + C$$

$$= \frac{21x^2\ln x}{2} - \frac{21x^2}{4} + C$$

$\int 3x\ln(x^7)\,dx = \boxed{\frac{21x^2\ln x}{2} - \frac{21x^2}{4} + C}$

3. Evaluate

$$\int x^3 \ln(2x)\, dx$$

$u = \ln(2x)$ $\quad$ $dv = x^3 dx$ $\quad$ $uv - \int v\,du = \dfrac{x^4 \ln(2x)}{4} - \int \dfrac{x^4}{4} \cdot \dfrac{1}{x} dx$

$du = \frac{1}{2x} \cdot 2dx$ $\quad$ $v = \frac{x^4}{4}$

$du = \frac{1}{x} dx$

$$= \frac{x^4 \ln(2x)}{4} - \frac{1}{4} \int x^3 dx$$

$$= \frac{x^4 \ln(2x)}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + c$$

$$\int x^3 \ln(2x)\, dx = \boxed{\frac{x^4}{4} \ln(2x) - \frac{x^4}{16} + c}$$

4. Evaluate the definite integral.

$$\int_0^3 5xe^{3x}\, dx$$

$u = 5x$ $\quad$ $dv = e^{3x} dx$ $\quad$ $uv - \int v\,du$

$du = 5dx$ $\quad$ $v = \frac{1}{3} e^{3x}$

$$= \frac{5x}{3} e^{3x} - \int \frac{5}{3} e^{3x} dx$$

$$= \left(\frac{5x}{3} e^{3x} - \frac{5}{3} \cdot \frac{e^{3x}}{3}\right)\Big]_0^3$$

$$= \frac{15}{3} e^9 - \frac{5}{9} e^9 - \left[0 - \frac{5}{9}\right]$$

$$= \frac{40}{9} e^9 + \frac{5}{9}$$

$$\int_0^3 5xe^{3x}\, dx = \boxed{\frac{40}{9} e^9 + \frac{5}{9}}$$

5. Evaluate the indefinite integral.

$$\int 4x\sin(7x)\,dx$$

$u = 4x$ $\quad dv = \sin(7x)\,dx$ $\quad uv - \int v\,du$

$du = 4dx$ $\quad v = -\frac{\cos(7x)}{7}$

$= -\frac{4}{7}x\cos(7x) + \int \frac{4}{7}(+\cos(7x))dx$

$= -\frac{4}{7}x\cos(7x) + \frac{4}{7}\int \cos(7x)\,dx$

$= -\frac{4}{7}x\cos(7x) + \frac{4}{7}\frac{\sin(7x)}{7} + C$

$$\boxed{-\frac{4}{7}x\cos(7x) + \frac{4}{7}\frac{\sin(7x)}{7} + C}$$

$\int 4x\sin(7x)\,dx =$ ____________________

6. Evaluate the indefinite integral.

$$\int 6t\sqrt{2t+5}\,dt$$

$2t = u - 5$

$\int 3\cdot 2t\sqrt{2t+5}\,dt$ $\quad u = 2t+5$ $\quad \int 3(u-5)u^{1/2}\frac{du}{2} = \frac{3}{2}\int\left(u^{3/2} - 5u^{1/2}\right)du$

$du = 2dt$

$\frac{du}{2} = dt$

$= \frac{3}{2}\left(\frac{2}{5}\cdot u^{5/2} - 5\cdot\frac{2}{3}u^{3/2}\right) + C$

$$= \boxed{\frac{3}{5}(2t+5)^{5/2} - 5(2t+5)^{3/2} + C}$$

$\int 6t\sqrt{2t+5}\,dt =$ ____________________

7. After t days, the growth of a plant is measured by the function $2000te^{-20t}$ inches per day. What is the change in the height of the plant (in inches) after the first 14 days?

$$\int_0^{14} 2000te^{-20t}\,dt$$

$u = 2000t$ $\quad dv = e^{-20t}dt$ $\quad uv - \int v\,du$

$du = 2000dt$ $\quad v = \dfrac{e^{-20t}}{-20}$

$$= 2000t\left(\frac{e^{-20t}}{-20}\right) + \int\left(\frac{e^{-20t}}{+20}\right)2000dt$$

$$= -100te^{-20t} + 100\int e^{-20t}dt$$

$$= -100te^{-20t} + 100\left(\frac{e^{-20t}}{-20}\right)$$

$$= \left(-100te^{-20t} - 5e^{-20t}\right)\Big]_0^{14}$$

$$= 5$$

Answer: $\boxed{5}$

8. The velocity of a cyclist during an hour-long race is given by the function

$$v(t) = 166te^{-2.2t} \text{ mi/hr}, \qquad 0 \le t \le 1$$

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?

① $\int 166te^{-2.2t}\,dt$

$u = 166t \qquad dv = e^{-2.2t}\,dt \qquad uv - \int v\,du$

$du = 166\,dt \qquad v = \dfrac{e^{-2.2t}}{-2.2}$

$$= \frac{166t\,e^{-2.2t}}{-2.2} + \int \frac{e^{-2.2t}}{+2.2}\cdot 166\,dt$$

$$= -\frac{166te^{-2.2t}}{2.2} + \frac{166}{2.2}\cdot\frac{e^{-2.2t}}{-2.2} + C$$

$$= \frac{-166te^{-2.2t}}{2.2} - \frac{166e^{-2.2t}}{(2.2)^2} + C$$

② $s(0) = 0$. Find C.

$$0 = 0 - \frac{166}{(2.2)^2} + C \longrightarrow C = \frac{166}{(2.2)^2}$$

③ $$s(t) = \frac{-166te^{-2.2t}}{2.2} - \frac{166e^{-2.2t}}{(2.2)^2} + \frac{166}{(2.2)^2}$$

$$s(1) = -\frac{166}{22}e^{-2.2} - \frac{166}{(2.2)^2}e^{-2.2} + \frac{166}{(2.2)^2}$$

≈ 22.137

Answer: $\boxed{22.137}$

9. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{3x+1}{x^2(x+1)^2(x^2+1)}$$

(A) $\dfrac{A}{x^2} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x^2+1}$

(B) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2} + \dfrac{E}{x^2+1}$

(C) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2} + \dfrac{Ex+F}{x^2+1}$

(D) $\dfrac{A}{x} + \dfrac{Bx+C}{x^2} + \dfrac{D}{x+1} + \dfrac{Ex+F}{(x+1)^2} + \dfrac{Gx+H}{x^2+1}$

(E) $\dfrac{A}{x} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x^2+1}$

10. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)}$$

(A) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx+E}{x^2+4}$

(B) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{D}{x^2+4}$

(C) $\dfrac{A}{x-1} + \dfrac{Bx+C}{(x-1)^2} + \dfrac{D}{x-2} + \dfrac{E}{x^2+4}$

(D) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx}{x^2+4}$

(E) $\dfrac{A}{x-1} + \dfrac{Bx}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx+E}{x^2+4}$

11. Determine the partial fraction decomposition of

$$\frac{7x^2+9}{x(x^2+3)}$$

$$\frac{A}{x} + \frac{Bx+C}{x^2+3} = \frac{A(x^2+3)+x(Bx+C)}{x(x^2+3)}$$

$$= \frac{Ax^2+3A+Bx^2+Cx}{x(x^2+3)}$$

$$= \frac{(A+B)x^2+Cx+3A}{x(x^2+3)}$$

$$(A+B)x^2+Cx+3A = 7x^2+0x+9$$

$$\begin{cases} A+B=7 \\ C=0 \\ 3A=9 \rightarrow A=3 \end{cases}$$

So $B=4$

Answer: $\boxed{\frac{3}{x} + \frac{4x}{x^2+3}}$

12. Determine the partial fraction decomposition of

$$\frac{4x-11}{x^2-7x+10}$$

Factor $x^2-7x+10=(x-2)(x-5)$

$$\frac{4x-11}{(x-2)(x-5)}=\frac{A}{x-2}+\frac{B}{x-5}$$

$$=\frac{A(x-5)+B(x-2)}{(x-2)(x-5)}$$

$$=\frac{(A+B)x+(-5A-2B)}{(x-2)(x-5)}$$

So $4x-11=(A+B)x+(-5A-2B)$

$$\begin{cases} 4=A+B & \text{(i)} \\ -11=-5A-2B & \text{(ii)} \end{cases}$$

Multiply (i) by 5 and add (i) + (ii).

$$\begin{array}{rl} & 20=\cancel{5A}+5B \\ + & -11=\cancel{-5A}-2B \\ \hline & 9=3B \end{array}$$

$B=3$

Plug $B=3$ into (i)

$4=A+B$

$4=A+3$

$A=1$

Answer: $\boxed{\frac{1}{x-2}+\frac{3}{x-5}}$

13. Evaluate $\int \frac{5x^2+9}{x^2(x+3)}\,dx$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3} = \frac{Ax(x+3) + B(x+3) + Cx^2}{x^2(x+3)}$$

$$= \frac{Ax^2 + 3Ax + Bx + 3B + Cx^2}{x^2(x+3)}$$

$$= \frac{(A+C)x^2 + (3A+B)x + 3B}{x^2(x+3)}$$

$$(A+C)x^2 + (3A+B)x + 3B = 5x^2 + 0x + 9$$

$$\begin{cases} A + C = 5 \\ 3A + B = 0 \\ 3B = 9 \rightarrow B = 3 \end{cases}$$

$3A + B = 0$
$3A + 3 = 0$
$3A = -3$
$A = -1$

$A + C = 5$
$-1 + C = 5$
$C = 6$

$$\int -\frac{1}{x}\,dx + \int \frac{3}{x^2}\,dx + \int \frac{6}{x+3}\,dx = -\ln|x| - \frac{3}{x} + 6\ln|x+3| + c$$

$$\boxed{-\ln|x| - \frac{3}{x} + 6\ln|x+3| + c}$$

$\int \frac{5x^2+9}{x^2(x+3)}\,dx =$ ______________________

14. Evaluate $\int \frac{x^2+2}{x^3+3x^2+2x}\,dx$

Factor $x^3+3x^2+2x = x(x^2+3x+2) = x(x+1)(x+2)$

So $\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x+2} = \frac{A(x+1)(x+2)+Bx(x+2)+Cx(x+1)}{x(x+1)(x+2)}$

$= \frac{A(x^2+3x+2)+B(x^2+2x)+C(x^2+x)}{x(x+1)(x+2)}$

$= \frac{(A+B+C)x^2+(3A+2B+C)x+2A}{x(x+1)(x+2)}$

So $x^2+2 = (A+B+C)x^2+(3A+2B+C)x+2A$

$1\cdot x^2+0x+2 = (A+B+C)x^2+(3A+2B+C)x+2A$

$$\begin{cases} 1 = A+B+C & \text{(i)} \\ 0 = 3A+2B+C & \text{(ii)} \\ 2 = 2A & \text{(iii)} \end{cases}$$

Solve (iii).

$2=2A$

$A=1$

Plug $A=1$ into (i) and (ii).

$$\begin{cases} 1 = 1+B+C & \text{(i)} \\ 0 = 3+2B+C & \text{(ii)} \end{cases}$$

Subtract the eqns.

$1 = 1+B+\cancel{C}$

$-(0 = 3+2B+\cancel{C})$

$1 = -2-B$

$+2 \quad +2$

$3=-B$

$B=-3$

Plug $B=-3$ into (i).

$1 = 1+B+C$

$1 = 1-3+C$

$1 = -2+C$

$3 = C$

Plug $A=1$, $B=-3$, $C=3$ into decomposition.

$\frac{1}{x}+\frac{-3}{x+1}+\frac{3}{x+2}$

So $\int \frac{1}{x}\,dx + \int \frac{-3}{x+1}\,dx + \int \frac{3}{x+2}\,dx$

$\int \frac{x^2+2}{x^3+3x^2+2x}\,dx = \boxed{\ln|x| - 3\ln|x+1| + 3\ln|x+2| + C}$

15. Evaluate $\displaystyle\int \frac{9x^2-4x+5}{(x-1)(x^2+1)}\,dx$

	Bx	C
x	Bx^2	Cx
-1	$-Bx$	$-C$

So $\dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+1} = \dfrac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$

$$= \frac{Ax^2+A+Bx^2-Bx+Cx-C}{(x-1)(x^2+1)}$$

$$= \frac{(A+B)x^2+(C-B)x+(A-C)}{(x-1)(x^2+1)}$$

So
$$\begin{cases} A+B=9 & \text{(i)} \\ C-B=-4 & \text{(ii)} \\ A-C=5 & \text{(iii)} \end{cases}$$

Add (i) and (ii)

$$\begin{array}{rl} A+\cancel{B} & =9 \\ +\quad -\cancel{B}+C & =-4 \\ \hline A+C & =5 \quad \text{(iv)} \end{array}$$

Add (iii) and (iv)

$$\begin{array}{rl} A-\cancel{C} & =5 \\ +\ A+\cancel{C} & =5 \\ \hline 2A & =10 \\ A & =5 \end{array}$$

Plug $A=5$ into (i)

$A+B=9$

$5+B=9$

$B=4$

Plug $A=5$ into (iii)

$A-C=5$

$5-C=5$

$C=0$

So $\dfrac{5}{x-1} + \dfrac{4x}{x^2+1}$

$$\int \frac{5}{x-1}\,dx + \underbrace{\int \frac{4x}{x^2+1}\,dx}_{\substack{u=x^2+1 \\ du=2x\,dx}}$$

$$= 5\ln|x-1| + 2\ln|x^2+1| + C$$

$\displaystyle\int \frac{x^2+2}{x^3+3x^2+2x}\,dx =$ $\boxed{5\ln|x-1| + 2\ln|x^2+1| + C}$

16. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1-\cos x}\,dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$1 - \cos x = 0$

$1 = \cos x$

$x = 0, \pi, 2\pi$

17. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \tan(x)\,dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$\tan x = \frac{\sin x}{\cos x}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$

18. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \cos(x)\,dx \rightarrow \cos(x) \text{ is defined everywhere.}$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

Bonus do this question w/ all trig functions

19. Evaluate the following integral;

$$\int_1^{\infty} \frac{5}{\sqrt{x}} dx$$

$$\int_1^{\infty} \frac{5}{\sqrt{x}} dx = \lim_{N\to\infty} \int_1^N 5x^{-1/2} dx = \lim_{N\to\infty} \left(5\cdot 2x^{1/2}\right)\Big]_1^N$$

$$= \lim_{N\to\infty} \left(10(N)^{1/2} - 10\right) = \infty$$

$$\int_1^{\infty} \frac{5}{\sqrt{x}} dx = \boxed{\infty}$$

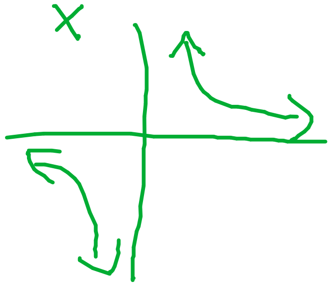
20. Evaluate the following integral;

$$\int_1^{\infty} \frac{3}{x^2} dx$$

$$\int_1^{\infty} \frac{3}{x^2} dx = \lim_{N\to\infty} \int_1^N 3x^{-2} dx = \lim_{N\to\infty} \left(\frac{3x^{-1}}{-1}\right)\Big]_1^N$$

$y = \frac{1}{x}$

$$= \lim_{N\to\infty} \left(-\frac{3}{x}\right)\Big]_1^N = \lim_{N\to\infty} \left(-\frac{3}{x} + \frac{3}{1}\right)$$



$$\int_1^{\infty} \frac{3}{x^2} dx = \boxed{3}$$

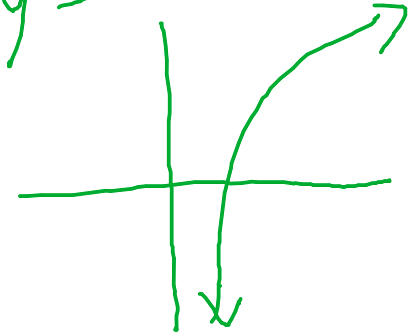
21. Evaluate the following integral;

$$\int_1^{\infty} \frac{10}{x} dx$$

$$\int_1^{\infty} \frac{10}{x} dx = \lim_{N\to\infty} \int_1^N \frac{10}{x} dx = \lim_{N\to\infty} \left(10 \ln|x|\right)\Big]_1^N$$

$$= \lim_{N\to\infty} \left(10 \ln|N| - 0\right)$$

$y = \ln|x|$



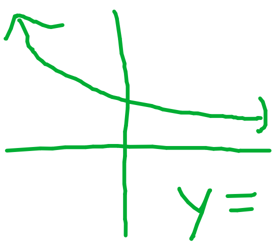
$$\int_1^{\infty} \frac{10}{x} dx = \boxed{\infty}$$

22. Evaluate the following integral;

$$\int_0^\infty \frac{7}{e^{10x}}dx$$

$$\int_0^\infty 7e^{-10x}\,dx = \lim_{N\to\infty}\int_0^N 7e^{-10x}\,dx = \lim_{N\to\infty}\left(7\frac{e^{-10x}}{-10}\right)\Big]_0^N$$

$$= \lim_{N\to\infty}\left(\frac{7e^{-10N}}{-10} + \frac{7}{10}\right) = 0 + \frac{7}{10}$$



$y = e^{-10x}$

$$\int_0^\infty \frac{7}{e^{10x}}dx = \boxed{7/10}$$

23. Evaluate the definite integral

$$\int_2^\infty \frac{dx}{5x+2}$$

$$\lim_{N\to\infty}\int_2^N \frac{dx}{5x+2}$$

$u = 5x+2$

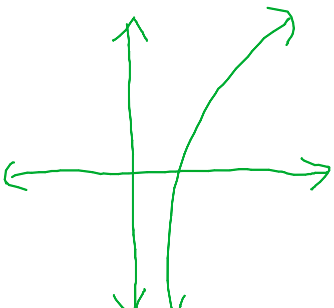
$du = 5\,dx$

$\frac{du}{5} = dx$

$$\lim_{N\to\infty}\int \frac{1}{5}\frac{1}{u}\,du$$

$$= \lim_{N\to\infty}\frac{1}{5}\ln|u| = \lim_{N\to\infty}\frac{1}{5}\ln|5x+2|\Big]_2^N$$

$$= \lim_{N\to\infty}\left(\frac{1}{5}\ln|5N+2| - \frac{1}{5}\ln|12|\right) = \infty$$



ln-function

$$\int_2^\infty \frac{dx}{5x+2} = \boxed{\infty}$$

$P(t) = 3000e^{-0.080t}$

24. The rate at which a factory is dumping pollution into a river at any time t is given by $P(t) = P_0 e^{-kt}$, where P_0 is the rate at which the pollution is initially released into the river. If $P_0 = 3000$ and $k = 0.080$, find the total amount of pollution that will be released into the river into the indefinite future.

$$\int_0^\infty P(t)\,dt = \int_0^\infty 3000e^{-0.080t}\,dt$$
$$= \lim_{N\to\infty} \int_0^N 3000e^{-0.080t}\,dt$$
$$= \lim_{N\to\infty} \frac{3000}{-0.080} e^{-0.080t}\Big]_0^N$$
$$= \lim_{N\to\infty} \left(-37500e^{-0.080N} + 37500\right) = \boxed{37500}$$

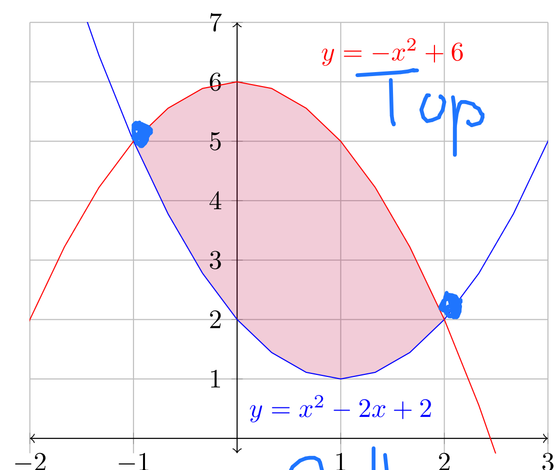
Answer:____________________

25. Set up the integral that computes the **AREA** shown to the right with respect to x.

DON'T COMPUTE IT!!!

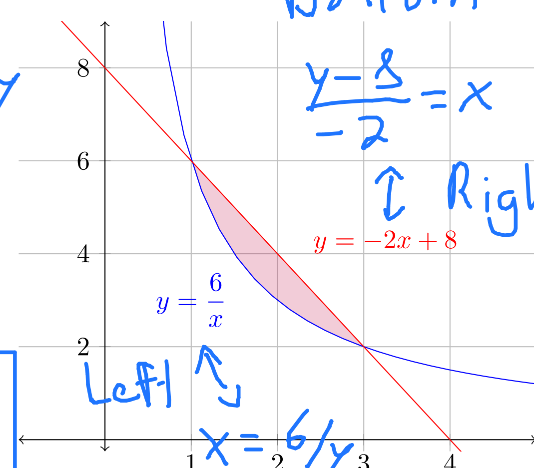
$$\boxed{\int_{-1}^{2} (-x^2+6)-(x^2-2x+2)\,dx}$$

Area $=$ ____________________



26. Set up the integral that computes the **AREA** shown to the right with respect to y. $\to dy$

DON'T COMPUTE IT!!!



Area $= \boxed{\int_2^6 \left(\frac{y-8}{-2}\right) - \frac{6}{y}\,dy}$

27. Set up the integral that computes the **AREA** with respect to x of the region bounded by

$$y = \frac{2}{x} \quad \text{and} \quad y = -x + 3$$

→ dx problem

Bounds:

$\frac{2}{x} = -x+3$

$2 = -x^2 + 3x$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

Test Pt: $x = 1.5$

$y = \frac{2}{x} \Rightarrow y = \frac{2}{1.5} = \frac{4}{3} \approx 1.33 \rightarrow$ Bottom

$y = -x+3 \Rightarrow y = -1.5 + 3 = 1.5 \rightarrow$ Top

Area = $\boxed{\int_1^2 \left(-x + 3 - \frac{2}{x}\right) dx}$

28. Find the area of the region bounded by $y = 6x^2$ and $y = 12x$.

Bounds: $6x^2 = 12x$

$6x^2 - 12x = 0$

$6x(x-2) = 0$

$x = 0, 2$

Test Pt: $x = 1$

$y = 6x^2 \rightarrow y = 6 \rightarrow$ Bottom

$y = 12x \rightarrow y = 12 \rightarrow$ Top

$A = \int_0^2 (12x - 6x^2)\, dx$

$= \left(\frac{12x^2}{2} - \frac{6x^3}{3}\right)\Big]_0^2$

$= \left(6x^2 - 2x^3\right)\Big]_0^2$

$= 8$

Area = $\boxed{8}$

29. Find the area of the region bounded by $y = 6x - x^2$ and $y = 2x^2$.

Bounds:

$6x - x^2 = 2x^2$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0, 2$

Test Pt: $x = 1$

$y = 6x - x^2 \Rightarrow y = 5 \to$ Top

$y = 2x^2 \Rightarrow y = 2 \to$ Bottom

$A = \int_0^2 \left[(6x - x^2) - 2x^2\right] dx$

$= \int_0^2 (6x - 3x^2)\, dx$

$= \left(3x^2 - x^3\right)\Big]_0^2 = 4$

Area = $\boxed{4}$

30. Calculate the **AREA** of the region bounded by the following curves.

$$x = 100 - y^2 \quad \text{and} \quad x = 2y^2 - 8$$

Bounds:

$100 - y^2 = 2y^2 - 8$

$108 = 3y^2$

$36 = y^2$

$y = \pm 6$

Test Pt: $y = 0$

$x = 100 - y^2 \to x = 0 \to$ Right

$x = 2y^2 - 8 \to x = -8 \to$ Left

$A = \int_{-6}^{6} (100 - y^2) - (2y^2 - 8)\, dy$

$= \int_{-6}^{6} (108 - 3y^2)\, dy$

$= \left(108y - y^3\right)\Big]_{-6}^{6}$

$= 864$

Area = $\boxed{864}$

31. Calculate the **AREA** of the region bounded by the following curves.

$$y = x^3 \quad \text{and} \quad y = x^2$$

Bounds:

$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0, 1$

Test Pt: $x = \frac{1}{2}$

$y = x^3 \rightarrow y = \frac{1}{8} \rightarrow$ Bottom

$y = x^2 \rightarrow y = \frac{1}{4} \rightarrow$ Top

$A = \int_0^1 (x^2 - x^3)\,dx$

$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big]_0^1$

$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

Area = $\boxed{1/12}$

32. After t hours studying, one student is working $Q_1(t) = 25 + 9t - t^2$ problems per hour, and a second student is working on $Q_2(t) = 5 - t + t^2$ problems per hour. How many more problems will the first student have done than the second student after 10 hours?

$\int_0^{10} Q_1(t) - Q_2(t)\,dt$

$= \int_0^{10} (25 + 9t - t^2) - (5 - t + t^2)\,dt$

$= \int_0^{10} (20 + 10t - 2t^2)\,dt$

$= \left(20t + 5t^2 - \frac{2}{3}t^3\right)\Big]_0^{10}$

$= \frac{100}{3}$

Answer: $\boxed{100/3}$

33. The birthrate of a particular population is modeled by $B(t) = 1000e^{0.036t}$ people per year, and the death rate is modeled by $D(t) = 725e^{0.019t}$ people per year. How much wll the population increase in the span of 10 years? $(0 \leq t \leq 20)$ Round to the nearest whole number.

$$\int_0^{10} B(t) - D(t)\,dt = \int_0^{10} 1000e^{0.036t} - 725e^{0.019t}\,dt$$

$$= \left(\frac{1000}{0.036} e^{0.036t} - \frac{725}{0.019} e^{0.019t}\right)\Bigg]_0^{10}$$

$$\approx 4052$$

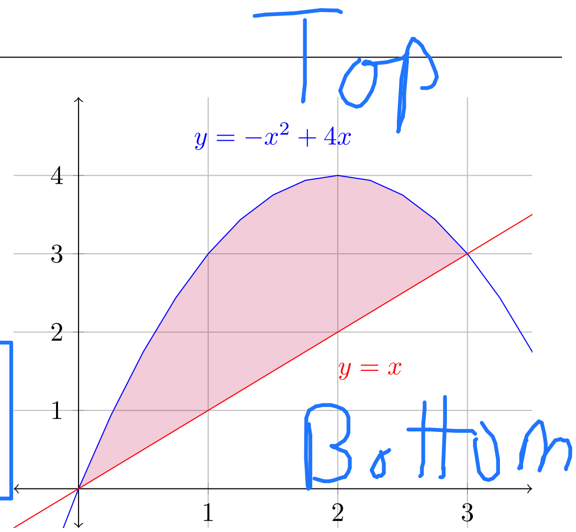
Answer: $\boxed{4052}$

34. Let R be the region shown below. Set up the integral that computes the **VOLUME** as R is rotated around the x-axis.

DON'T COMPUTE IT!!!

$$\boxed{\pi \int_0^3 \left[(-x^2+4x)^2 - (x)^2\right] dx}$$

Volume = ____________________



35. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{16 - x}, \quad y = 0 \quad \text{and} \quad x = 0$$

about the y-axis $\Rightarrow$ dy problem

Disk

$y = \sqrt{16-x}$

$y^2 = 16 - x$

$x = 16 - y^2$

Bounds: Given $y=0$

Plug $x=0$ into $y = \sqrt{16-x}$

$y = \sqrt{16-x}$

$y = \sqrt{16}$

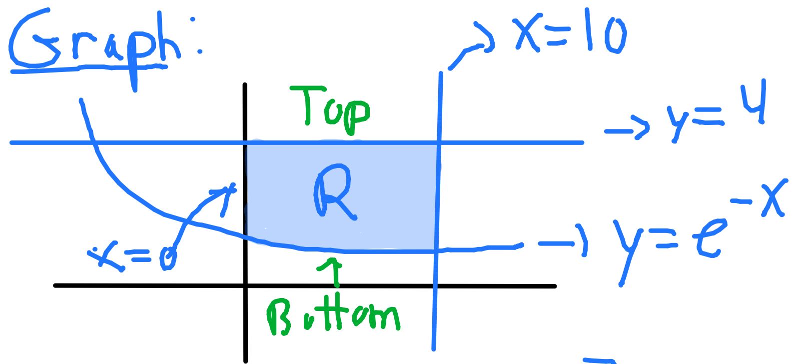
$y = 4$

Volume = $\boxed{\pi \int_0^4 (16-y^2)^2 \, dy}$

36. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = e^{-x}, \quad y = 4 \quad x = 0 \quad \text{and} \quad x = 10$$

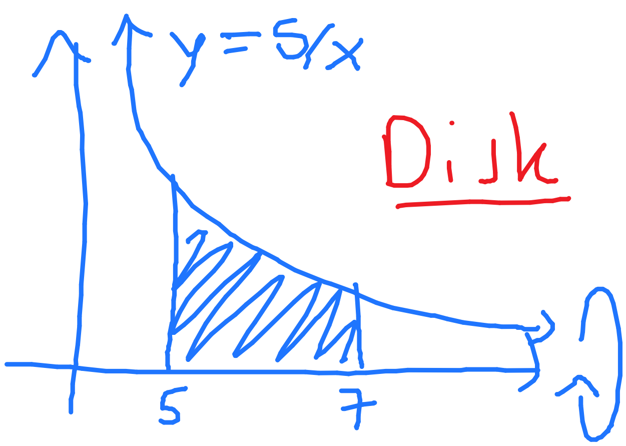
about the x-axis $\rightarrow dx$



$$V = \pi \int_0^2 \left[4^2 - (e^{-x})^2 \right] dx$$

Volume = $\boxed{\pi \int_0^2 (16 - e^{-2x})\, dx}$

37. Find the volume of the solid that results by revolving the region enclosed by the curves $y = \dfrac{5}{x}$, $y = 0$, $x = 5$, and $x = 7$ about the x-axis. $\Rightarrow dx$



$$V = \pi \int_5^7 \left(\frac{5}{x}\right)^2 dx$$

$$= \pi \int_5^7 \frac{25}{x^2}\, dx$$

$$= 25\pi \int_5^7 x^{-2}\, dx$$

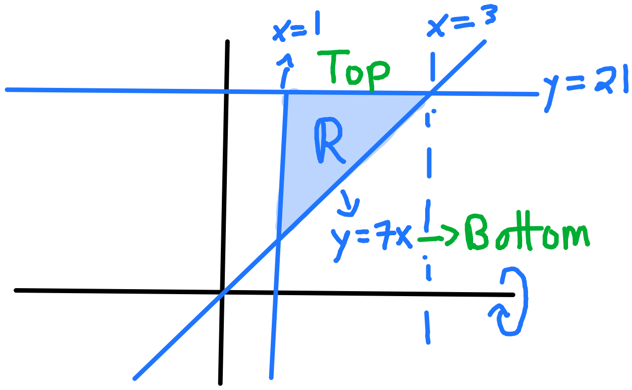
$$= 25\pi \left(-\frac{1}{x}\right)\Big]_5^7$$

Volume = $\boxed{\dfrac{10\pi}{7}}$

38. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 21 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis $\to dx$



Washer

$$V = \pi \int_1^3 \left[21^2 - (7x)^2\right] dx$$

$$= \pi \int_1^3 (441 - 49x^2)\, dx$$

$$= \pi \left(441x - \frac{49x^3}{3}\right)\Big]_1^3$$

$$= \frac{1274}{3}\pi$$

Volume $=$ $\boxed{\dfrac{1274\pi}{3}}$

39. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 0 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis $\to dx$

Disk

$$V = \pi \int_1^3 (7x)^2 dx$$

$$= \pi \int_1^3 49x^2 dx$$

$$= \pi \left(\frac{49x^3}{3}\right)\Big]_1^3$$

$$= \frac{49\pi}{3}(3^3 - 1)$$

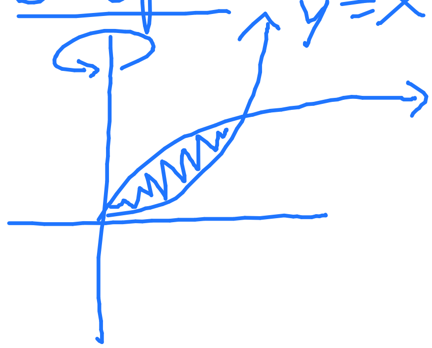
Volume $=$ $\boxed{\dfrac{1274}{3}\pi}$

40. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y = \sqrt{x}$$

about the y-axis .

Graph:



Bounds:

$\sqrt{y} = y^2$

$y = y^4$

$0 = y^4 - y$

$0 = y(y^3 - 1)$

$y = 0, 1$

But y-axis $\Rightarrow dy$

Right $\rightarrow y = x^2 \rightarrow x = \sqrt{y}$

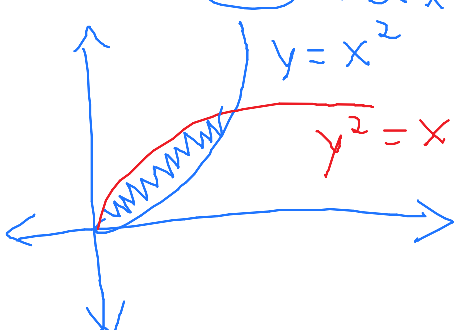
Left $\rightarrow y = \sqrt{x} \rightarrow x = y^2$

Volume $=$ $\boxed{\pi \int_0^1 \left[(\sqrt{y})^2 - (y^2)^2\right] dy}$

41. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y^2 = x$$

about the x-axis $\rightarrow$ dx problem



$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$$

Bounds:

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

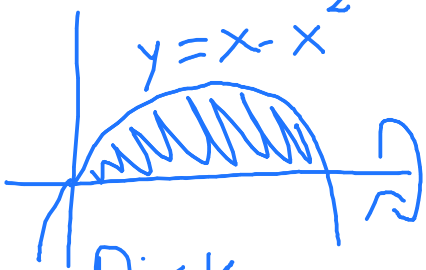
$x = 0, 1$

Volume $=$ $\boxed{\pi \int_0^1 (x - x^4) \, dx}$

42. Find the **VOLUME** of the region bounded by

$$y = x - x^2, \quad \text{and} \quad y = 0$$

around the x-axis



Disk

Bounds:

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$

$V = \pi \int_0^1 (x - x^2)^2 \, dx$

$= \pi \int_0^1 (x^2 - 2x^3 + x^4) \, dx$

$= \pi \left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \Big]_0^1$

$= \frac{\pi}{30}$

Volume $=$ $\boxed{\pi/30}$

43. Find the **VOLUME** of the solid generate by revolving the given region about the x-axis: $\rightarrow dx$

$$y = 8\sqrt{x}, \quad y = 0, \quad x = 3, \quad x = 6$$

$V = \pi \int_3^6 (8\sqrt{x})^2 \, dx$

$= \pi \int_3^6 64x \, dx$

$= \pi \frac{64x^2}{2} \Big]_3^6$

$= \pi \, 32x^2 \Big]_3^6$

$= 864\pi$

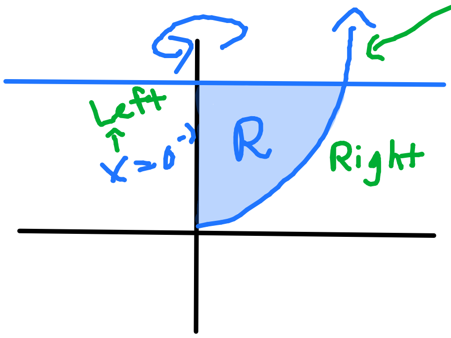
Volume $=$ $\boxed{864\pi}$

44. Find the **VOLUME** of the region bounded by

$$y = 4x^2, \quad x = 0, \quad y = 4$$

around the y-axis → dy

$$x = \sqrt{\frac{y}{4}}$$



Disk

$$V = \pi \int_0^4 \left(\sqrt{\frac{y}{4}}\right)^2 dy$$

$$= \pi \int_0^4 \frac{y}{4}\, dy$$

$$= \frac{\pi}{4} \cdot \frac{y^2}{2}\Big]_0^4$$

$$= \frac{\pi}{8} \cdot 16$$

$$= 2\pi$$

Volume $=$ $\boxed{2\pi}$

45. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x + 8, \quad \text{and} \quad y = (x-4)^2$$

about the x-axis

Bounds:

$$x + 8 = (x-4)^2$$

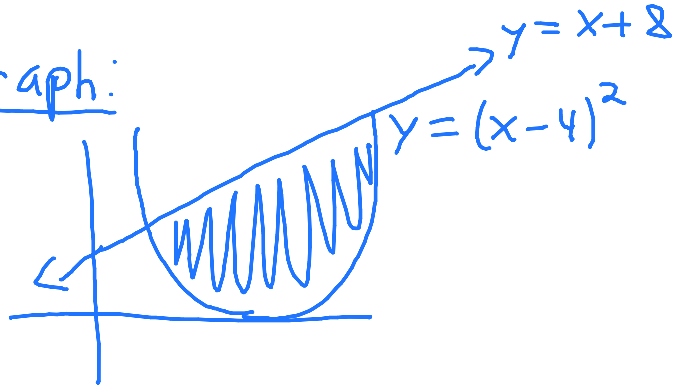
$$x + 8 = x^2 - 8x + 16$$

$$0 = x^2 - 9x + 8$$

$$0 = (x-8)(x-1)$$

$$x = 1, 8$$

Graph:



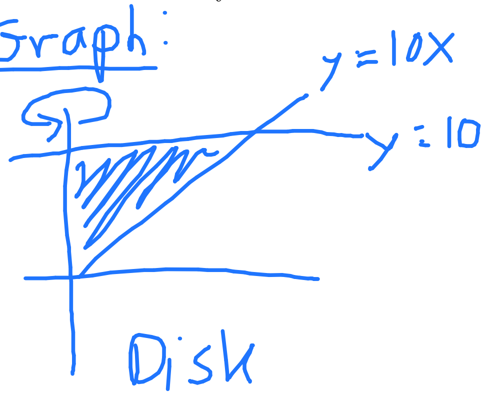
Volume $=$ $\boxed{\pi \int_1^8 \left[(x+8)^2 - (x-4)^4\right] dx}$

46. Find the **VOLUME** of the region bounded by

$$y = 10x, \quad x = 0, \quad y = 10$$

around the y-axis

Graph:



Disk

But y-axis $\Rightarrow$ dy problem

$y = 10x$

$\frac{y}{10} = x$

$V = \pi \int_0^{10} \left(\frac{y}{10}\right)^2 dy$

$= \pi \int_0^{10} \frac{y^2}{100} dy$

$= \frac{\pi}{100}\left(\frac{y^3}{3}\right)\Big]_0^{10}$

$= \frac{10\pi}{3}$

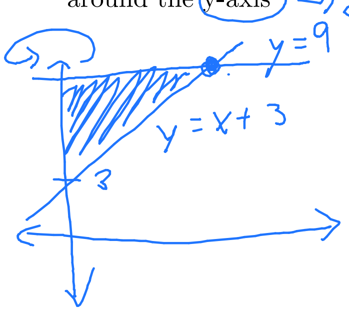
Volume $=$ $\boxed{10\pi/3}$

47. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 3, \quad x = 0, \quad y = 9$$

$\rightarrow x = y - 3$

around the y-axis $\rightarrow$ dy problem.



$V = \pi \int_3^9 (y-3)^2 dy$

$= \pi \int_3^9 (y^2 - 6y + 9)\, dy$

$= \pi\left(\frac{y^3}{3} - 3y^2 + 9y\right)\Big]_3^9$

Volume $=$ $\boxed{72\pi}$

48. Find the **VOLUME** of the region bounded by

$$x + 3y = 9, \quad x = 0, \quad y = 0$$

around the y-axis

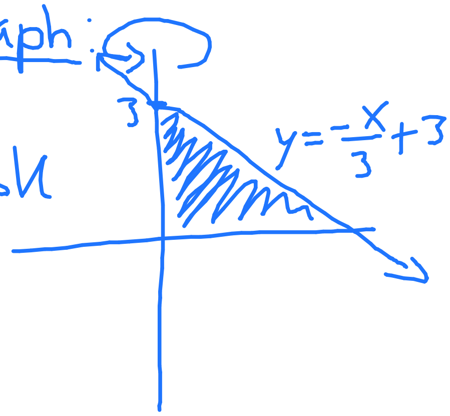
$x + 3y = 9$

$3y = -x + 9$

$y = -\frac{x}{3} + 3$

Graph:

Disk



But y-axis $\Rightarrow dy$

So $x + 3y = 9$

$x = 9 - 3y$

$$V = \pi \int_0^3 (9 - 3y)^2 \, dy$$

$$= \pi \int_0^3 (81 - 54y + 9y^2) \, dy$$

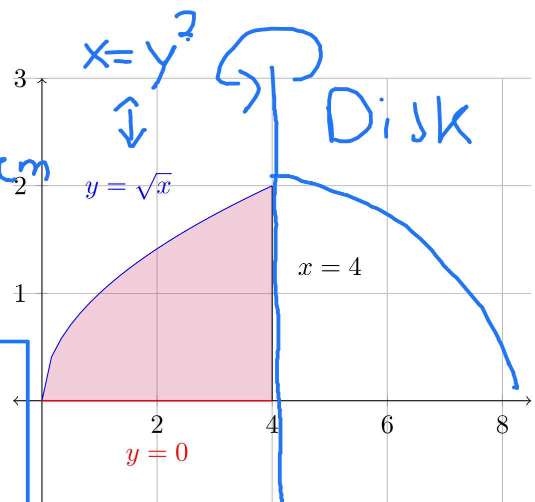
$$= \pi \left(81y - 27y^2 + 3y^3 \right) \Big]_0^3$$

$$= 81\pi$$

Volume $=$ $\boxed{81\pi}$

49. Let R be the region shown to the right. Set up the integral that computes the **VOLUME** as R is rotated around the line $x = 4$. $\Rightarrow$ dy problem

DON'T COMPUTE IT!!!



Volume $=$ $\boxed{\pi \int_0^2 (y^2 - 4)^2 \, dy}$

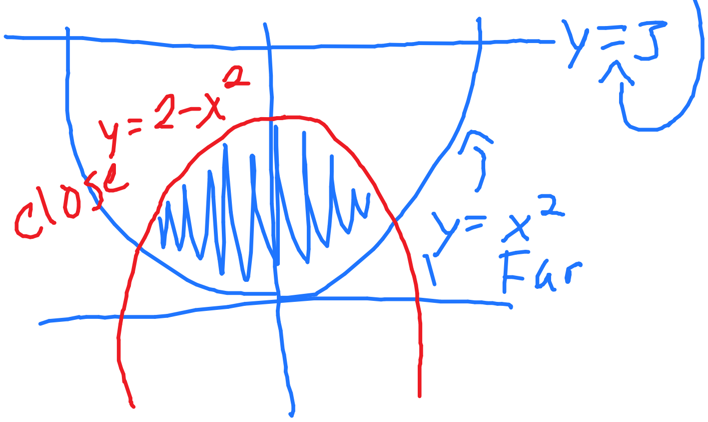
50. Set up the integral needed to find the volume of the solid obtained when the region bounded by

$$y = 2 - x^2 \quad \text{and} \quad y = x^2$$

is rotated about the line $y = 3$.

$y = 3 \Rightarrow dx$ problem

Graph Washer



Bounds: $2 - x^2 = x^2$
$2 = 2x^2$
$1 = x^2$
$x = \pm 1$

Volume = $\boxed{\pi \int_{-1}^{1} (2-x^2-3)^2 - (x^2-3)^2\,dx}$

51. **SET-UP using the disk/washer method.** the **VOLUME** of the region bounded by

$$y = 3x, \quad x = 0, \quad y = 27$$

Disk

around the line $y = 27$ $\rightarrow dx$

(A) $\pi \displaystyle\int_0^{27} (729 - 162x + 9x^2)\,dx$

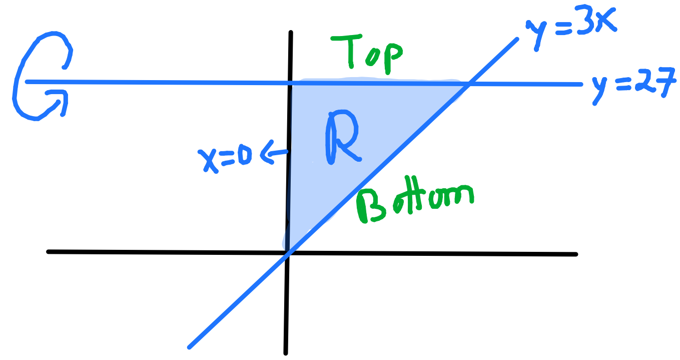
(B) $\pi \displaystyle\int_0^{27} 9x^2\,dx$

(C) $\pi \displaystyle\int_0^{9} 9x^2\,dx$

(D) $\pi \displaystyle\int_0^{9} (9x^2 - 162x)\,dx$

(E) $\pi \displaystyle\int_0^{27} (729 - 9x^2)\,dx$

(F) $\boxed{\pi \displaystyle\int_0^{9} (729 - 162x + 9x^2)\,dx}$



Bound: $3x = 27$
$x = 9$

$V = \pi \int_0^9 (3x-27)^2\,dx$

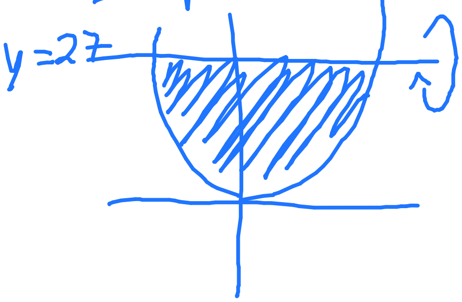
$= \pi \int_0^9 (9x^2 - 162x + 729)\,dx$

52. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Graph Disk



$$V = \pi \int_0^3 (3x^2 - 27)^2 \, dx$$

$$= \pi \int_0^3 (9x^4 - 162x^2 + 729) \, dx$$

$$= \pi \left(\frac{9x^5}{5} - 54x^3 + 729x \right]_0^3$$

$$= 11664.4\pi$$

$y = 27 \Rightarrow dx$ problem

Bounds: Given $x = 0$

$27 = 3x^2$

$9 = x^2 \rightarrow x = 3$

$$\boxed{\frac{8322}{5}\pi}$$

Volume $=$ ____________________

dx

53. **SET-UP using the Shell method**, the integral that computes the **VOLUME** of the region in quadrant I enclosed by the region defined by a triangle with vertices at (0,0), (0,5), and (4,0) about the y-axis.

(A) $\pi \int_0^5 \left(8x - \frac{5}{4}x^2\right) dx$

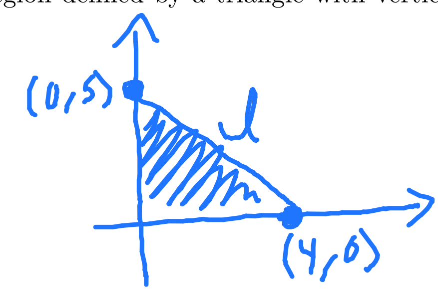
(B) $\pi \int_0^5 \frac{5}{4}x^2 \, dx$

(C) $\pi \int_0^4 4x^2 \, dx$

(D) $\pi \int_0^4 \left(8x - \frac{5}{4}x^2\right) dx$

$\boxed{\text{(E) } \pi \int_0^4 \left(10x - \frac{5}{2}x^2\right) dx}$

(F) $\pi \int_0^5 \left(10x - \frac{5}{2}x^2\right) dx$



$$V = 2\pi \int_0^4 x \cdot l \, dx$$

Find the eqn of the line, l.

$$m = \frac{0-5}{4-0} = -\frac{5}{4}$$

y-intercept is @ 5 b/c (0,5)

$$l = -\frac{5}{4}x + 5$$

$$V = 2\pi \int_0^4 x\left(-\frac{5}{4}x + 5\right) dx$$

$$= \pi \int_0^4 \left(10x - \frac{5}{2}x^2\right) dx$$

54. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$x = 2y - y^2, \quad \text{and} \quad x = 0$$

about the x-axis. $\rightarrow dy$

+

Bounds: $0 = 2y - y^2$

$0 = y(2-y)$

$y = 0, 2$

$V = 2\pi \int_0^2 y(2y - y^2)\,dy$

$\boxed{2\pi \int_0^2 y(2y - y^2)\,dy}$

Volume = ______________________

55. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 2 - x^2, \quad \text{and} \quad y = x^2$$

about the y-axis. $\rightarrow dx$

+

$V = 2\pi \int_{-1}^{1} x(2 - x^2 - x^2)\,dx$

Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

Test Pt: $x = 0$

$y = 2 - x^2 \rightarrow y = 2 \rightarrow$ Top

$y = x^2 \rightarrow y = 0 \rightarrow$ Bottom

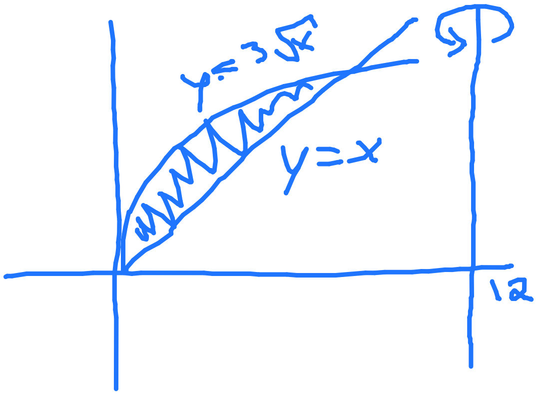
Volume = $\boxed{2\pi \int_{-1}^{1} x(2 - 2x^2)\,dx}$

56. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

+ ↓

$$y = 3\sqrt{x}, \quad \text{and} \quad y = x$$

about the $x = 12 \rightarrow dx$



Bounds

$3\sqrt{x} = x$

$9x = x^2$

$9x - x^2 = 0$

$x(9-x) = 0$

$x = 0, 9$

★ Note x=12 is on the right of our region.

$$\boxed{V = 2\pi \int_0^9 \underbrace{(12-x)}_{★}(3\sqrt{x} - x)\,dx}$$

Volume = ____________________

57. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

+ ↓

$$y = x, \quad \text{and } y = x^2$$

about the line $x = -2. \rightarrow dx$

Since x=-2 is on the left of our region

Bounds:

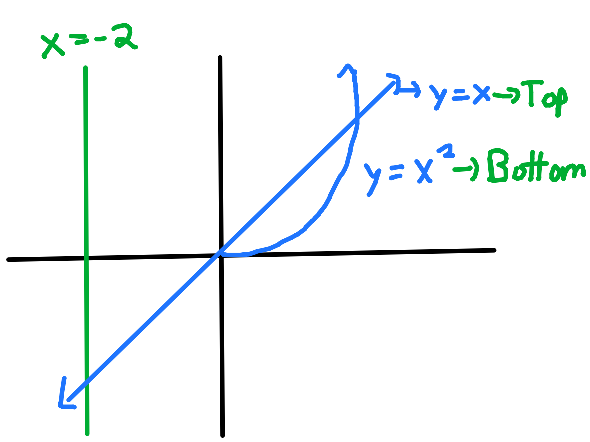
$x = x^2$

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$

$$V = 2\pi \int_0^1 (x-(-2))[x - x^2]\,dx$$



Volume = $\boxed{2\pi \int_0^1 (x+2)(x-x^2)\,dx}$

58. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$+ \quad dx$

$$y = 7x^2, \quad y = 0 \text{ and } \quad x = 2$$

about the line $x = 3$. $\to dx$

$V = 2\pi \int_0^2 (\_\_\_\_)(7x^2)\,dx$

Since $x=3$ is larger than the bounds,

$V = 2\pi \int_0^2 (3-x)(7x^2)\,dx$

Volume $=$ $\boxed{2\pi \int_0^2 (3-x)(7x^2)\,dx}$

59. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$+ \quad dy$

$$x = y^2 + 1, \text{ and } \quad x = 2$$

about the line $y = -2$. $\to dy$

Since $y=-2$ is smaller than the bounds

$V = 2\pi \int_{-1}^{1} (y-(-2))(2-(y^2+1))\,dy$

Bounds: $y^2+1=2$

$y^2=1$

$y=\pm 1$

Test Pt: $y=0$

$x=y^2+1 \to x=1 \to$ Left

$x=2 \to x=2 \to$ Right

Volume $=$ $\boxed{2\pi \int_{-1}^{1} (y+2)(2-(y^2+1))\,dy}$