Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name:____

1. The rate of change of the population $n(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate k in terms of minutes. Round to four decimal places.

$k=$ ______________________________

2. Let y denote the mass of a radioactive substance at time t. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time t in hours does half the original mass remain? Round your answer to 3 decimal places.

$t \quad =$ ______________________________

3. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

$y \quad = \quad$ ______________________________

4. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

$y \quad = \quad$ ______________________________

5. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

$y \quad = \quad$ ________________________________

6. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

$y =$ ________________________________

7. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$y =$ ______________________________

8. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2 y$$

$y =$ ______________________________

9. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$y =$ ______________________________

10. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$y =$ ______________________________

11. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$y =$ ______________________________

12. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2 e^{-x^3} \quad \text{where} \quad y = 10 \quad \text{when} \quad x = 2$$

Find the value of the integration constant, C.

$C =$ ______________________________

13. Calculate the constant of integration, C, for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \qquad y(1) = 2$$

$C=$ ________________________________

14. The volume of an object $V(t)$ in cubic millimeter at any time t in seconds changes according to the model

$$\frac{dV}{dt} = \cos\left(\frac{t}{10}\right),$$

where $V(0) = 5$. Find the volume of the object at $t = 3$ seconds. Round to 4 decimal places.

$V(3) =$ ________________________________

15. What is the **integrating factor** of the following differential equation?

$$y' + \left(\frac{2x+3}{x}\right) y = 10\ln(x)$$

$u(x) =$ ______________________

16. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right) y = 10\ln(x)$$

$u(x) =$ ______________________

17. What is the **integrating factor** of the following differential equation?

$$x^8 y' - 14x^7 y = 32e^{7x}$$

$u(x) =$ ______________________

18. What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = (x+1)e^{x^2}$$

$u(x) =$ ______________________

19. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) =$ ______________________________

20. What is the **integrating factor** of the following differential equation?

$$y' + \tan(x) \cdot y = \sec(x)$$

$u(x) =$ ______________________________

21. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + (4x - 1)y = 8x - 2$$

$y =$ ______________________________

22. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2(y+4) \text{ and } y(0) = 3$$

$y =$ ______________________________

23. Solve the initial value problem.

$$x^4 y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$y =$ ____________________

24. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \ldots$$

Answer:______________________________

(b) Use the sum from (a) and compute the sum.

Answer:______________________________

25. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n =$ ______________________________

26. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ ____________________________

27. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ ____________________________

28. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \underline{\hspace{10cm}}$$

29. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \underline{\hspace{10cm}}$$

30. Evaluate the sum of the following infinite series.

$$\sum_{n=0}^{\infty}(-1)^n \frac{5^{n+1}}{3^{2n}}$$

Answer:______________________________

31. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty}\left(\frac{3^{n-1}}{4^n} + \frac{(-1)^{n+1}}{9^n}\right)$$

Answer:______________________________

32. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

$R =$ ____________________

33. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

$R =$ ____________________

34. Use the first three terms of the powers series representation of the $f(x) = \dfrac{3x}{10+2x}$ to estimate $f(0.5)$. Round to 4 decimal places.

$f(0.5) \approx$ ______________________

35. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$\dfrac{3}{1+2x}$ = ________________________________

$R =$ ________________________________

36. Express $f(x) = \dfrac{5x}{3+2x^2}$ as a power series and determine it's radius of converge.

$\dfrac{5x}{3+2x^2}$ = ________________________________

$R =$ ________________________________

37. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$\int \sin(x^{3/2})\,dx =$ ______________________________

38. Evaluate the following indefinite integral as a series. [Leave your answer as a sum.]

$$\int 5e^{5x^3}\,dx$$

$\int 5e^{5x^3}\,dx =$ ______________________________

39. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5+x^6}\,dx$$

$\int_0^{0.24} \frac{x}{5+x^6}\,dx \approx$ ______________________

40. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx$$

$\int_0^{0.11} \frac{1}{1+x^4}\,dx \approx$ ______________________

41. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\,dx$$

$\int_0^{0.23} e^{-x^2}\,dx \approx$ ______________________

42. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ ______________________

43. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$\ln(1.56) \approx$ ______________________

44. Use the first 4 terms of the Macluarin series for $f(x) = \sin(x)$ to evaluate $\sin(0.75)$. Round to 5 decimal places.

$\sin(0.75) \approx$ ____________________

45. Find the domain of

$$f(x, y) = \frac{\sqrt{x + y - 1}}{\ln(y - 11) - 9}$$

Domain = ____________________

46. Find the domain of

$$f(x, y) = \frac{\ln(x^2 - y + 3)}{\sqrt{x - 6}}$$

Domain = ____________________

47. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \;\; C = \ln(36)$$

(a) Parabola with vertices at $(0, 0)$
(b) Circle with center at $(0, \ln(36))$ and radius 6
(c) Parabola with vertices at $(0, \ln(36))$
(d) Circle with center at $(0, 0)$ and radius 6
(e) Increasing Logarithm Function

48. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

49. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{x^2 + y^2}$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

50. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

51. For the following function $f(x,y)$, evaluate $f_y(-2,-3)$.

$$f(x,y) = 8x^4y^5 + 3x^3 - 12y^2$$

$f_y(-2,-3) =$ ______________________________

52. Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y^2-1}}$$

$f_x(6,5)=$ ______________________________

53. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$f_x(x,y) =$ ______________________________

$f_y(x,y) =$ ______________________________

54. Find the first order partial derivatives of

$$f(x,y) = x\sin(xy)$$

$f_x(x,y) =$ ______________________________

$f_y(x,y) =$ ______________________________

55. Find the first order partial derivatives of $f(x, y) = (xy - 1)^2$

$f_x(x, y) =$ ____________________

$f_y(x, y) =$ ____________________

56. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$f_x(x, y) =$ ____________________

$f_y(x, y) =$ ____________________

57. Find the first order partial derivatives of $f(x, y) = -7\tan(x^7y^8)$

$f_x(x, y) =$ ______________________

$f_y(x, y) =$ ______________________

58. Find the first order partial derivatives of $f(x, y) = y\cos(x^2y)$

$f_x(x, y) =$ ______________________

$f_y(x, y) =$ ______________________