MATH 16020 PRACTICE FINAL EXAM FRI., DECEMBER 15, 2023

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name: Solutions

1. Given $f(x) = 2x^{5/2} - \cos(3\pi x)$, evaluate $f'(4)$.

$$f'(x) = 2\cdot\frac{5}{2}x^{3/2} - \left[-\sin(3\pi x)\right]\cdot(3\pi)$$

$$= 5x^{3/2} + 3\pi\sin(3\pi x)$$

$$f'(4) = 5(4)^{3/2} + 3\pi\underbrace{\sin(3\pi\cdot 4)}_{0} = 40$$

$f'(4) =$ $\boxed{40}$

2. Evaluate the definite integral

$$\int_0^{\pi/6} (3\cos(x) - 6)\,dx$$

$$= \Big(3\sin(x) - 6x\Big)\Big]_0^{\pi/6}$$

$$= 3\sin\left(\frac{\pi}{6}\right) - 6\left(\frac{\pi}{6}\right) - \cancelto{0}{\Big(3\sin(0) - 6(0)\Big)}$$

$$= \frac{3}{2} - \pi$$

$\displaystyle\int_0^{\pi/6} (3\cos(x) - 6)\,dx =$ $\boxed{\frac{3}{2} - \pi}$

3. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate $r(t)$ is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

$$\left.\begin{array}{l} 10{:}00\text{am} \Rightarrow 1\text{ hr} \\ 1{:}00\text{pm} \Rightarrow 4\text{ hrs} \end{array}\right\} \Rightarrow \int_1^4 6t^{1/2}\,dt$$

$$= 6 \cdot \frac{2}{3} t^{3/2} \Big]_1^4$$

$$= 4t^{3/2} \Big]_1^4$$

$$= 28$$

Answer: $\boxed{28}$

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Solve $\int_0^t 6t^{1/2}\,dt = 121$

$$4t^{3/2} = 121$$

$$t^{3/2} = \frac{121}{4}$$

$$t = \left(\frac{121}{4}\right)^{2/3}$$

Answer: $\boxed{\left(\frac{121}{4}\right)^{2/3}}$

4. Which derivative rule is undone by integration by substitution?

   (A) Power Rule
   (B) Quotient Rule
   (C) Product Rule
   (D) Chain Rule
   (E) Constant Rule
   (F) None of these

5. Which derivative rule is undone by integration by parts?

   (A) Power Rule
   (B) Quotient Rule
   (C) Product Rule
   (D) Chain Rule
   (E) Constant Rule
   (F) None of these

6. What would be the best substitution to make the solve the given integral?

$$\int e^{2x}\cos(e^{2x})\sin^3(e^{2x})\,dx$$

$$\int e^{2x}\cos(e^{2x})[\sin(e^{2x})]^3\,dx$$

$u =$ $\boxed{\sin(e^{2x})}$

Always check du is in integral

7. What would be the best substitution to make the solve the given integral?

$$\int \sec^2(5x)e^{\tan(5x)}\,dx$$

$u =$ $\boxed{\tan(5x)}$

Always check du is in integral

8. What would be the best substitution to make the solve the given integral?

$$\int \tan(5x)\sec(5x)e^{\sec(5x)}\,dx$$

$u =$ $\boxed{\sec(5x)}$

Always check du is in integral

9. Find the area under the curve $y = 14e^{7x}$ for $0 \le x \le 4$.

$$A = \int_0^4 14e^{7x}\,dx \quad \frac{u = 7x}{du = 7dx} \quad \int 2e^u\,du$$

$$= 2e^u = 2e^{7x}\Big]_0^4 = 2e^{28} - 2$$

Area = $\boxed{2e^{28} - 2}$

10. Evaluate the definite integral.

$$\int_0^2 (5e^{2x} + 8)\,dx$$

$$\underbrace{\int_0^2 5e^{2x}\,dx}_{u\text{-sub}} + \int_0^2 8\,dx = \frac{5}{2}e^{2x}\Big]_0^2 + 8x\Big]_0^2$$

$$= \frac{5}{2}(e^4 - e^0) + 8(2-0)$$

$$= \frac{5}{2}e^4 - \frac{5}{2} + 16$$

$$= \frac{5}{2}e^4 - \frac{27}{2}$$

$\int_0^2 (5e^{2x} + 8)\,dx = \boxed{\frac{5}{2}e^4 + \frac{27}{2}}$

11. Evaluate the indefinite integral.

$$\int 18x\cos(x^2)\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int \cancel{18x}^{\,9} \cos(u)\,\frac{du}{\cancel{2x}} = \int 9\cos(u)\,du$

$= 9\sin(u) + C$

$= 9\sin(x^2) + C$

$$\int 18x\cos(x^2)\,dx = \boxed{9\sin(x^2) + C}$$

12. Evaluate the indefinite integral.

$$\int 9x^3 e^{-x^4}\,dx$$

u

$u = -x^4$

$du = -4x^3\,dx$

$\frac{du}{-4x^3} = dx$

$\int 9x^3 e^u \frac{du}{-4x^3} = -\frac{9}{4}\int e^u\,du$

$= -\frac{9}{4}e^u = -\frac{9}{4}e^{-x^4} + C$

$$\int 9x^3 e^{-x^4}\,dx = \boxed{-\frac{9}{4}e^{-x^4} + C}$$

13. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that t hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$L'(t) = \sqrt{3t+2} \quad \text{gallows per hour.}$$

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.

i.e. $\int_0^4 (3t+2)^{1/2}\,dt$ $\quad \dfrac{u = 3t+2}{du = 3dt}$ $\quad \int u^{1/2} \dfrac{du}{3}$

$\dfrac{du}{3} = dt$

$= \dfrac{1}{3} \cdot \dfrac{2}{3} u^{3/2} = \dfrac{2}{9}(3t+2)^{3/2} \Big]_0^4$

≈ 11.0122

Answer: $\boxed{11.0122}$

14. It is estimated that t-days into a semester, the average amount of sleep a college math student gets per day $S(t)$ changes at a rate of

$$\frac{-4t}{e^{t^2}} = -4te^{-t^2}$$

hours per day. When the semester begins, math students sleep an average of 8.2 hours per day. What is $S(t)$, 2 days into the semester?

① $\int -4te^{-t^2}\,dt$ $\quad \dfrac{u=-t^2}{du=-2t\,dt}$ $\quad \int \overset{2}{\cancel{-4t}}\, e^{u}\, \dfrac{du}{\cancel{-2t}}$

$\dfrac{du}{-2t} = dt$

$= \int 2e^{u}\,du = 2e^{u} + C$

$= 2e^{-t^2} + C$

② $S(0) = 8.2$ Find C.

$8.2 = 2e^{0} + C$

$8.2 = 2 + C$

$C = 6.2$

③ $S(t) = 2e^{-t^2} + 6.2$

$S(2) = 2e^{-4} + 6.2$

≈ 6.237

Answer: $\boxed{6.237}$

15. A biologist determines that, t hours after a bacterial colony was established, the population of bacteria in the colony is changing at a rate given by

$$P'(t) = \frac{5e^t}{1+e^t}$$

million bacteria per hour, $0 \leq t \leq 5$.

If the bacterial colony started with a population of 1 million, how many bacteria, in millions are present in the colony after the 5-hour experiment?

① $\int \frac{5e^t}{1+e^t}\,dt$ $\quad \frac{u=1+e^t}{du=e^t dt}$ $\quad \int \frac{5\cancel{e^t}}{u}\frac{du}{\cancel{e^t}} = \int \frac{5}{u}\,du$

$\frac{du}{e^t} = dt$

$= 5\ln|u| + c$

$= 5\ln|1+e^t| + c$

② $P(0)=1$ Find c.

$1 = 5\ln|1+e^0| + c$

$1 = 5\ln|1+1| + c$

$1 = 5\ln 2 + c$

$1 - 5\ln 2 = c$

③ $P(t) = 5\ln|1+e^t| + 1 - 5\ln 2$

$P(5) = 5\ln|1+e^5| + 1 - 5\ln 2$

≈ 22.57

Answer: $\boxed{22.57}$

16. Evaluate the indefinite integral

$$\int x(x^2+4)^3\,dx$$

$u = x^2+4$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$\int \cancel{x}\,u^3\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int u^3\,du = \frac{1}{2}\cdot\frac{u^4}{4} + C$

$= \frac{1}{8}(x^2+4)^4 + C$

$\int x(x^2+4)^3\,dx =$ $\boxed{\frac{1}{8}(x^2+4)^4 + C}$

17. Evaluate the definite integral.

$$\int_0^{\pi/4} 3\sin(2x)\,dx$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\int 3\sin(u)\,\frac{du}{2} = \frac{3}{2}\int \sin(u)\,du = -\frac{3}{2}\cos(u)$

$= -\frac{3}{2}\cos(2x)\Big]_0^{\pi/4}$

$= \cancel{-\frac{3}{2}\cos\left(\frac{2\pi}{4}\right)}^{\,0} - \left(-\frac{3}{2}\cos(0)\right)$

$= 3/2$

$\int_0^{\pi/4} 3\sin(2x)\,dx =$ $\boxed{3/2}$

18. Evaluate the indefinite integral.

$$\int (x+4)\sqrt{x^2+8x}\,dx$$

$u = x^2 + 8x$

$du = (2x+8)dx$

$du = 2(x+4)dx$

$\dfrac{du}{2(x+4)} = dx$

$\displaystyle \int \cancel{(x+4)}\sqrt{u}\,\frac{du}{2\cancel{(x+4)}}$

$\displaystyle = \frac{1}{2}\int u^{1/2}\,du$

$\displaystyle = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} u^{3/2} + C$

$\displaystyle = \frac{1}{3}(x^2+8x)^{3/2} + C$

$\int (x+4)\sqrt{x^2+8x}\,dx = \boxed{\frac{1}{3}(x^2+8x)^{3/2} + C}$

19. Evaluate the definite integral.

$$\int_0^9 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)}$$

$u = \sqrt{x} + 1$

$u = x^{1/2} + 1$

$du = \frac{1}{2}x^{-1/2}\,dx$

$du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$\displaystyle \int \frac{\cancel{2}\cancel{\sqrt{x}}\,du}{\cancel{2}\cancel{\sqrt{x}} \cdot u} = \int \frac{du}{u} = \ln|u|$

$\displaystyle = \ln|\sqrt{x}+1|\Big]_0^9$

$= \ln|\sqrt{9}+1| - \ln|\sqrt{0}+1|$

$= \ln(4)$

$\int_0^9 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)} = \boxed{\ln(4)}$

20. A tree is transplanted and after t years is growing at a rate

$$r'(t) = 1 + \frac{1}{(t+1)^2} \qquad \text{meters per year.}$$

After 2 years it has reached a height of 5 meters. How tall was the tree when it was originally transplanted? Round to one decimal place.

$$r(t) = \int \left(1 + \frac{1}{(t+1)^2}\right) dt$$

$$= \int \left(1 + (t+1)^{-2}\right) dt$$

$$= t + \frac{(t+1)^{-1}}{-1} + c$$

$$= t - \frac{1}{t+1} + C$$

Find c w/ $r(2) = 5$

$$5 = 2 - \frac{1}{2+1} + c$$

$$3 + \frac{1}{3} = c$$

$$\frac{10}{3} = c$$

So $r(t) = t - \frac{1}{t+1} + \frac{10}{3}$

$$r(0) = 0 - 1 + \frac{10}{3}$$

$$= 7/3 \approx 2.3$$

Height = $\boxed{2.3}$

21. The marginal revenue from the sale of x units of a particular product is estimated to be $R'(x) = 50 + 350xe^{-x^2}$ dollars per unit, and where $R(x)$ is revenue in dollars. What revenue should be expected from the sale of 100 units? Assume that $R(0) = 0$.

$$R(x) = \int 50 + 350xe^{-x^2}\,dx$$

$$= \int 50\,dx + \underbrace{\int 350xe^{-x^2}\,dx}$$

$$u = -x^2$$
$$du = -2x\,dx$$
$$\frac{du}{-2x} = dx$$

$$= \int 50\,dx + \int 350\cancel{x}e^{u}\,\frac{du}{-2\cancel{x}}$$

$$= \int 50\,dx - 175\int e^{u}\,du$$

$$= 50x - 175e^{u} + C$$

$$= 50x - 175e^{-x^2} + C$$

$$R(0) = 0$$
$$0 = 0 - 175 + C$$
$$C = 175$$

$$R(x) = 50x - 175e^{-x^2} + 175$$

$$R(100) \approx 5175$$

$R(100) =$ $\boxed{5175}$

22. Evaluate the indefinite integral

$$\int \frac{\ln(7x)}{x}\,dx$$

$$\frac{u=\ln(7x)}{du=\frac{1}{7x}\cdot 7\,dx}$$

$$du=\frac{1}{x}\,dx$$

$$\int u\,du = \frac{u^2}{2} = \frac{(\ln(7x))^2}{2} + C$$

$$\int \frac{\ln(7x)}{x}\,dx = \boxed{\frac{(\ln(7x))^2}{2} + C}$$

23. Evaluate

$$\int_1^e \frac{\ln(x^4)}{x}\,dx$$

Rewrite $\int_1^e \frac{4\ln x}{x}\,dx$ $\quad \frac{u=\ln x}{du=\frac{1}{x}dx}$ $\quad \int 4u\,du = \frac{4u^2}{2} = 2u^2 = 2(\ln x)^2\Big]_1^e$

$$= \underbrace{2(\ln e)^2}_{2} - \underbrace{2(\ln 1)^2}_{0}$$

$$= 2$$

$$\int_1^e \frac{\ln(x^4)}{x}\,dx = \boxed{2}$$

24. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx$$

$\dfrac{u = x-1}{du = dx}$ $\dfrac{dv = \sin(x)\,dx}{v = -\cos(x)}$ $uv - \int v\,du = -(x-1)\cos x\Big]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)\,dx$

$= -(x-1)\cos x\Big]_0^{\pi/2} + \sin(x)\Big]_0^{\pi/2}$

$= \cancel{-\left(\frac{\pi}{2}-1\right)\cos\left(\frac{\pi}{2}\right)} \to 0 - \left[-(0-1)\cos(0)\right]$

$+ \sin\left(\frac{\pi}{2}\right) \cancel{- \sin(0)} \to 0$

$= -1 + 1 = 0$

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx = \boxed{0}$$

25. Evaluate

$$\int 3x\ln(x^7)\,dx$$

Rewrite $\int 3x(7\ln(x))\,dx = \int 21x\ln x\,dx$

$\dfrac{u = 21\ln(x)}{du = \frac{21}{x}\,dx}$ $\dfrac{dv = x\,dx}{v = \frac{x^2}{2}}$ $uv - \int v\,du$

$= \dfrac{21x^2\ln x}{2} - \int \dfrac{x^2}{2}\cdot\dfrac{21}{x}\,dx$

$= \dfrac{21x^2\ln x}{2} - \int \dfrac{21}{2}x\,dx$

$= \dfrac{21x^2\ln x}{2} - \dfrac{21}{2}\cdot\dfrac{x^2}{2} + C$

$= \dfrac{21x^2\ln x}{2} - \dfrac{21x^2}{4} + C$

$$\int 3x\ln(x^7)\,dx = \boxed{\frac{21x^2\ln x}{2} - \frac{21x^2}{4} + C}$$

26. Evaluate

$$\int x^3 \ln(2x)\, dx$$

$u = \ln(2x)$ $\quad dv = x^3 dx$ $\quad uv - \int v\,du = \frac{x^4 \ln(2x)}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$

$du = \frac{1}{2x} \cdot 2dx$ $\quad v = \frac{x^4}{4}$

$du = \frac{1}{x} dx$

$= \frac{x^4 \ln(2x)}{4} - \frac{1}{4} \int x^3 dx$

$= \frac{x^4 \ln(2x)}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C$

$$\int x^3 \ln(2x)\, dx = \boxed{\frac{x^4}{4} \ln(2x) - \frac{x^4}{16} + C}$$

27. Evaluate the definite integral.

$$\int_0^3 5xe^{3x}\, dx$$

$u = 5x$ $\quad dv = e^{3x} dx$ $\quad uv - \int v\,du$

$du = 5dx$ $\quad v = \frac{1}{3} e^{3x}$

$= \frac{5x}{3} e^{3x} - \int \frac{5}{3} e^{3x} dx$

$= \left(\frac{5x}{3} e^{3x} - \frac{5}{3} \cdot \frac{e^{3x}}{3}\right)\Big]_0^3$

$= \frac{15}{3} e^9 - \frac{5}{9} e^9 - \left[0 - \frac{5}{9}\right]$

$= \frac{40}{9} e^9 + \frac{5}{9}$

$$\int_0^3 5xe^{3x}\, dx = \boxed{\frac{40}{9} e^9 + \frac{5}{9}}$$

28. The population of pink elephants in Dumbo's dreams, in hundreds, t years after the year 1980 is given by

$$P(t) = \frac{e^{5t}}{1+e^{5t}}$$

What is the average population during the decade between 1980 and 2000?

i.e. $\frac{1}{2000-1980}\int_0^{20} \frac{e^{5t}}{1+e^{5t}}\,dt$ $\quad u = 1+e^{5t}$, $du = 5e^{5t}dt$, $\frac{du}{5e^{5t}} = dt$ $\quad \frac{1}{20}\int \frac{e^{5t}}{u}\cdot\frac{du}{5e^{5t}}$

$= \frac{1}{100}\int \frac{du}{u} \quad = \frac{1}{100}\ln|u|$

$= \frac{1}{100}\ln|1+e^{5t}|\Big]_0^{20}$

≈ 0.9931

Answer: $\boxed{0.9931 \text{ hundreds or } 993}$

29. Evaluate the indefinite integral.

$$\int 20x\sin(2x)\,dx$$

$\dfrac{u = 20x}{du = 20\,dx}$ $\quad \dfrac{dv = \sin(2x)\,dx}{v = -\frac{\cos(2x)}{2}}$ $\quad uv - \int v\,du$

$= -\frac{20}{2}x\cos(2x) + \int \frac{20}{2}(+\cos(2x))\,dx$

$= -10x\cos(2x) + 10\int \cos(2x)\,dx$

$= -10x\cos(2x) + 10\,\frac{\sin(2x)}{2} + C$

$\boxed{-10x\cos(2x) + 5\sin(2x) + C}$

$\int 20x\sin(2x)\,dx =$ ____________________

30. The velocity of a cyclist during an hour-long race is given by the function

$$v(t) = 166te^{-2.2t} \text{ mi/hr}, \qquad 0 \le t \le 1$$

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?

① $\int 166te^{-2.2t}\,dt$

$u = 166t \qquad dv = e^{-2.2t}\,dt \qquad uv - \int v\,du$

$du = 166\,dt \qquad v = \dfrac{e^{-2.2t}}{-2.2}$

$$= \frac{166te^{-2.2t}}{-2.2} + \int \frac{e^{-2.2t}}{+2.2} \cdot 166\,dt$$

$$= -\frac{166te^{-2.2t}}{2.2} + \frac{166}{2.2} \cdot \frac{e^{-2.2t}}{-2.2} + C$$

$$= \frac{-166te^{-2.2t}}{2.2} - \frac{166e^{-2.2t}}{(2.2)^2} + C$$

② $s(0) = 0$. Find C.

$$0 = 0 - \frac{166}{(2.2)^2} + C \longrightarrow C = \frac{166}{(2.2)^2}$$

③ $$s(t) = \frac{-166te^{-2.2t}}{2.2} - \frac{166e^{-2.2t}}{(2.2)^2} + \frac{166}{(2.2)^2}$$

$$s(1) = -\frac{166}{2.2}e^{-2.2} - \frac{166}{(2.2)^2}e^{-2.2} + \frac{166}{(2.2)^2}$$

≈ 22.137

Answer: $\boxed{22.137}$

31. After t days, the growth of a plant is measured by the function $2000te^{-20t}$ inches per day. What is the change in the height of the plant (in inches) after the first 14 days?

$$\int_0^{14} 2000te^{-20t}\,dt$$

$$\frac{u = 2000t}{du = 2000\,dt} \qquad \frac{dv = e^{-20t}\,dt}{v = \frac{e^{-20t}}{-20}} \qquad uv - \int v\,du$$

$$= 2000t\left(\frac{e^{-20t}}{-20}\right) + \int\left(\frac{e^{-20t}}{+20}\right)2000\,dt$$

$$= -100\,te^{-20t} + 100\int e^{-20t}\,dt$$

$$= -100\,te^{-20t} + 100\left(\frac{e^{-20t}}{-20}\right)$$

$$= \left(-100\,te^{-20t} - 5\,e^{-20t}\right)\Big]_0^{14}$$

$$= 5$$

Answer: $\boxed{5}$

$u-5=2t$

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32. Evaluate the indefinite integral.

$$\int 4t\sqrt{2t+5}\,dt$$

$u=2t+5$

$du=2dt$

$\frac{du}{2}=dt$

$$\int 4t\,u^{1/2}\,\frac{du}{2}=\int 2t\,u^{1/2}\,du=\int (u-5)u^{1/2}\,du$$

$$=\int u^{3/2}-5u^{1/2}\,du$$

$$=\frac{2}{5}u^{5/2}-5\cdot\frac{2}{3}u^{3/2}+C$$

$$=\frac{2}{5}(2t+5)^{5/2}-\frac{10}{3}(2t+5)^{3/2}+C$$

$$\boxed{\frac{2}{5}(2t+5)^{5/2}-\frac{10}{3}(2t+5)^{3/2}+C}$$

$\int 6t\sqrt{2t+5}\,dt =$ ____________________

33. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x)=\frac{3x+1}{x^2(x+1)^2(x^2+1)}$$

(A) $\frac{A}{x^2}+\frac{B}{(x+1)^2}+\frac{C}{x^2+1}$

(B) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}+\frac{D}{(x+1)^2}+\frac{E}{x^2+1}$

$\boxed{\text{(C) } \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}+\frac{D}{(x+1)^2}+\frac{Ex+F}{x^2+1}}$

(D) $\frac{A}{x}+\frac{Bx+C}{x^2}+\frac{D}{x+1}+\frac{Ex+F}{(x+1)^2}+\frac{Gx+H}{x^2+1}$

(E) $\frac{A}{x}+\frac{B}{(x+1)^2}+\frac{C}{x^2+1}$

34. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{7x-5}{x^2(x^2+9)}$$

(A) $\frac{A}{x} + \frac{B}{x} + \frac{Cx+D}{x^2+9}$

$\boxed{\text{(B) } \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}}$

(C) $\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{Dx+E}{x^2+9}$

(D) $\frac{Ax+B}{x^2} + \frac{Cx+D}{x^2+9}$

(E) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3} + \frac{D}{x-3}$

(F) $\frac{Ax+B}{x^2} + \frac{C}{x+3} + \frac{D}{x-3}$

35. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)}$$

$\boxed{\text{(A) } \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+4}}$

(B) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{D}{x^2+4}$

(C) $\frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{D}{x-2} + \frac{E}{x^2+4}$

(D) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} + \frac{Dx}{x^2+4}$

(E) $\frac{A}{x-1} + \frac{Bx}{(x-1)^2} + \frac{C}{x-2} + \frac{Dx+E}{x^2+4}$

36. Determine the partial fraction decomposition of

$$\frac{7x^2+9}{x(x^2+3)}$$

$$\frac{A}{x}+\frac{Bx+C}{x^2+3}=\frac{A(x^2+3)+x(Bx+C)}{x(x^2+3)}$$

$$=\frac{Ax^2+3A+Bx^2+Cx}{x(x^2+3)}$$

$$=\frac{(A+B)x^2+Cx+3A}{x(x^2+3)}$$

$$(A+B)x^2+Cx+3A=7x^2+0x+9$$

$$\begin{cases} A+B=7 \\ C=0 \\ 3A=9 \rightarrow A=3 \end{cases}$$

So $B=4$

Answer: $\boxed{\frac{3}{x}+\frac{4x}{x^2+3}}$

37. Determine the partial fraction decomposition of

$$\frac{4x-11}{x^2-7x+10}$$

Factor $x^2-7x+10=(x-2)(x-5)$

$$\frac{4x-11}{(x-2)(x-5)}=\frac{A}{x-2}+\frac{B}{x-5}$$

$$=\frac{A(x-5)+B(x-2)}{(x-2)(x-5)}$$

$$=\frac{(A+B)x+(-5A-2B)}{(x-2)(x-5)}$$

So $4x-11=(A+B)x+(-5A-2B)$

$$\begin{cases} 4=A+B & \text{(i)} \\ -11=-5A-2B & \text{(ii)} \end{cases}$$

Multiply (i) by 5 and add (i) + (ii).

$$\begin{array}{rl} & 20=\cancel{5A}+5B \\ + & -11=\cancel{-5A}-2B \\ \hline & 9=3B \end{array}$$

$B=3$

Plug $B=3$ into (i)

$4=A+B$

$4=A+3$

$A=1$

Answer: $\boxed{\dfrac{1}{x-2}+\dfrac{3}{x-5}}$

38. Evaluate $\int \frac{5x^2+9}{x^2(x+3)}\,dx$

$$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}=\frac{Ax(x+3)+B(x+3)+Cx^2}{x^2(x+3)}$$

$$=\frac{Ax^2+3Ax+Bx+3B+Cx^2}{x^2(x+3)}$$

$$=\frac{(A+C)x^2+(3A+B)x+3B}{x^2(x+3)}$$

$$(A+C)x^2+(3A+B)x+3B=5x^2+0x+9$$

$$\begin{cases} A+C=5 \\ 3A+B=0 \\ 3B=9 \rightarrow B=3 \end{cases}$$

$3A+B=0$
$3A+3=0$
$3A=-3$
$A=-1$

$A+C=5$
$-1+C=5$
$C=6$

$$\int -\frac{1}{x}\,dx+\int\frac{3}{x^2}\,dx+\int\frac{6}{x+3}\,dx = -\ln|x|-\frac{3}{x}+6\ln|x+3|+c$$

$$\int \frac{5x^2+9}{x^2(x+3)}\,dx = \boxed{-\ln|x|-\frac{3}{x}+6\ln|x+3|+c}$$

39. Evaluate $\displaystyle\int \frac{x^2+2}{x^3+3x^2+2x}\,dx$

Factor $x^3+3x^2+2x = x(x^2+3x+2) = x(x+1)(x+2)$

So $\dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x+2} = \dfrac{A(x+1)(x+2)+Bx(x+2)+Cx(x+1)}{x(x+1)(x+2)}$

$$= \frac{A(x^2+3x+2)+B(x^2+2x)+C(x^2+x)}{x(x+1)(x+2)}$$

$$= \frac{(A+B+C)x^2+(3A+2B+C)x+2A}{x(x+1)(x+2)}$$

So $x^2+2 = (A+B+C)x^2+(3A+2B+C)x+2A$

$1\cdot x^2+0x+2 = (A+B+C)x^2+(3A+2B+C)x+2A$

$$\begin{cases} 1 = A+B+C & \text{(i)} \\ 0 = 3A+2B+C & \text{(ii)} \\ 2 = 2A & \text{(iii)} \end{cases}$$

Solve (iii).

$2 = 2A$

$A = 1$

Plug $A=1$ into (i) and (ii).

$$\begin{cases} 1 = 1+B+C & \text{(i)} \\ 0 = 3+2B+C & \text{(ii)} \end{cases}$$

Subtract the eqns.

$$\begin{array}{r} 1 = 1+B+\cancel{C} \\ -(0 = 3+2B+\cancel{C}) \\ \hline 1 = -2-B \\ +2 \quad +2 \\ \hline 3 = -B \end{array}$$

$B = -3$

Plug $B=-3$ into (i).

$1 = 1+B+C$

$1 = 1-3+C$

$1 = -2+C$

$3 = C$

Plug $A=1$, $B=-3$, $C=3$ into decomposition.

$\dfrac{1}{x}+\dfrac{-3}{x+1}+\dfrac{3}{x+2}$

So $\displaystyle\int \frac{1}{x}\,dx+\int\frac{-3}{x+1}\,dx+\int\frac{3}{x+2}\,dx$

$$\int \frac{x^2+2}{x^3+3x^2+2x}\,dx = \boxed{\ln|x| - 3\ln|x+1| + 3\ln|x+2| + C}$$

40. Evaluate $\displaystyle\int \frac{9x^2-4x+5}{(x-1)(x^2+1)}\,dx$

	Bx	C
x	Bx^2	Cx
-1	$-Bx$	$-C$

So $\displaystyle\frac{A}{x-1}+\frac{Bx+C}{x^2+1}=\frac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$

$$=\frac{Ax^2+A+Bx^2-Bx+Cx-C}{(x-1)(x^2+1)}$$

$$=\frac{(A+B)x^2+(C-B)x+(A-C)}{(x-1)(x^2+1)}$$

So $\begin{cases} A+B=9 & \text{(i)} \\ C-B=-4 & \text{(ii)} \\ A-C=5 & \text{(iii)} \end{cases}$

Add (i) and (ii)

$$\begin{array}{rl} & A+\cancel{B} = 9 \\ + & -\cancel{B}+C=-4 \\ \hline & A+C=5 \quad \text{(iv)} \end{array}$$

Add (iii) and (iv)

$$\begin{array}{rl} & A-\cancel{C}=5 \\ + & A+\cancel{C}=5 \\ \hline & 2A=10 \\ & A=5 \end{array}$$

Plug $A=5$ into (i)

$A+B=9$

$5+B=9$

$B=4$

Plug $A=5$ into (iii)

$A-C=5$

$5-C=5$

$C=0$

So $\displaystyle\frac{5}{x-1}+\frac{4x}{x^2+1}$

$$\int\frac{5}{x-1}\,dx+\underbrace{\int\frac{4x}{x^2+1}\,dx}$$

$$\frac{u=x^2+1}{du=2x\,dx}$$

$$=5\ln|x-1|+2\ln|x^2+1|+C$$

$\displaystyle\int\frac{x^2+2}{x^3+3x^2+2x}\,dx=$ $\boxed{5\ln|x-1|+2\ln|x^2+1|+C}$

41. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1-\cos x}\,dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$

(B) It is improper because of a discontinuity at $x = \pi/4$

(C) It is improper because of a discontinuity at $x = \pi/3$

(D) It is improper because of a discontinuity at $x = 0$

(E) It is improper because of a discontinuity at $x = \pi/2$

(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$1 - \cos x = 0$

$1 = \cos x$

$x = 0, \pi, 2\pi$

42. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \tan(x)\,dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$

(B) It is improper because of a discontinuity at $x = \pi/4$

(C) It is improper because of a discontinuity at $x = \pi/3$

(D) It is improper because of a discontinuity at $x = 0$

(E) It is improper because of a discontinuity at $x = \pi/2$

(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$\tan x = \frac{\sin x}{\cos x}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

43. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \cos(x)\,dx$$

$\rightarrow$ cos(x) is defined everywhere.

(A) It is improper because of a discontinuity at $x = \pi/6$

(B) It is improper because of a discontinuity at $x = \pi/4$

(C) It is improper because of a discontinuity at $x = \pi/3$

(D) It is improper because of a discontinuity at $x = 0$

(E) It is improper because of a discontinuity at $x = \pi/2$

(F) It is proper since it is defined on the interval $[0, \pi/2]$.

Bonus do this question w/ all trig functions

44. Evaluate the following integral;

$$\int_1^\infty \frac{5}{\sqrt{x}}dx$$

$$\int_1^\infty \frac{5}{\sqrt{x}}dx = \lim_{N\to\infty}\int_1^N 5x^{-1/2}dx = \lim_{N\to\infty}\left(5\cdot 2x^{1/2}\right)\Big]_1^N$$

$$= \lim_{N\to\infty}\left(10(N)^{1/2} - 10\right) = \infty$$

$$\int_1^\infty \frac{5}{\sqrt{x}}dx = \boxed{\infty}$$

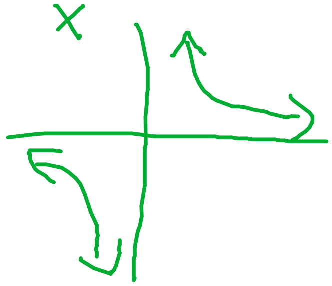
45. Evaluate the following integral;

$$\int_1^\infty \frac{3}{x^2}dx$$

$$\int_1^\infty \frac{3}{x^2}dx = \lim_{N\to\infty}\int_1^N 3x^{-2}dx = \lim_{N\to\infty}\left(\frac{3x^{-1}}{-1}\right]_1^N$$

$$= \lim_{N\to\infty}\left(-\frac{3}{x}\right)\Big]_1^N = \lim_{N\to\infty}\left(-\frac{3}{x} + \frac{3}{1}\right)$$

$y = -\frac{1}{x}$



$$\int_1^\infty \frac{3}{x^2}dx = \boxed{3}$$

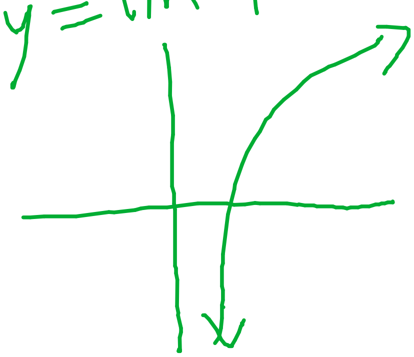
46. Evaluate the following integral;

$$\int_1^\infty \frac{10}{x}dx$$

$$\int_1^\infty \frac{10}{x}dx = \lim_{N\to\infty}\int_1^N \frac{10}{x}dx = \lim_{N\to\infty}\left(10\ln|x|\right)\Big]_1^N$$

$$= \lim_{N\to\infty}\left(10\ln|N| - 0\right)$$

$y = \ln|x|$



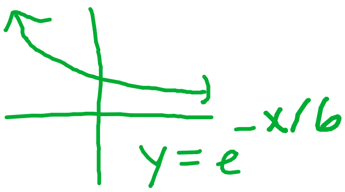
$$\int_1^\infty \frac{10}{x}dx = \boxed{\infty}$$

47. Evaluate the following integral;

$$\int_0^\infty e^{-x/6}dx$$

$$= \lim_{N\to\infty}\int_0^N e^{-x/6}dx = \lim_{N\to\infty}\left(-6e^{-x/6}\right)\Big]_0^N$$

$$= \lim_{N\to\infty}\left(-6e^{-N/6}+6\right) = 6$$



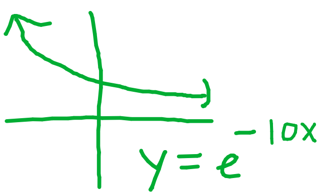
$$\int_0^\infty e^{-x/6}dx = \boxed{6}$$

48. Evaluate the following integral;

$$\int_0^\infty \frac{7}{e^{10x}}dx$$

$$\int_0^\infty 7e^{-10x}dx = \lim_{N\to\infty}\int_0^N 7e^{-10x}dx = \lim_{N\to\infty}\left(7\frac{e^{-10x}}{-10}\right)\Big]_0^N$$

$$= \lim_{N\to\infty}\left(\frac{7e^{-10N}}{-10}+\frac{7}{10}\right) = 0+\frac{7}{10}$$



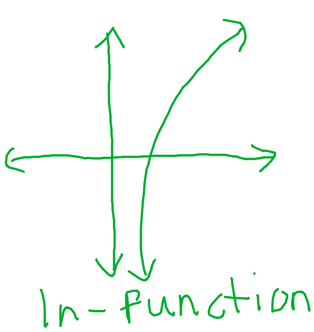
$$\int_0^\infty \frac{7}{e^{10x}}dx = \boxed{7/10}$$

49. Evaluate the definite integral

$$\int_2^\infty \frac{dx}{5x+2}$$

$$\lim_{N\to\infty}\int_2^N \frac{dx}{5x+2} \quad \begin{array}{l} u=5x+2 \\ du=5dx \\ \frac{du}{5}=dx \end{array} \quad \lim_{N\to\infty}\int \frac{1}{5}\frac{1}{u}du = \lim_{N\to\infty}\frac{1}{5}\ln|u| = \lim_{N\to\infty}\frac{1}{5}\ln|5x+2|\Big]_2^N$$

$$= \lim_{N\to\infty}\left(\frac{1}{5}\ln|5N+2| - \frac{1}{5}\ln|12|\right) = \infty$$



$$\int_2^\infty \frac{dx}{5x+2} = \boxed{\infty}$$

$P(t) = 3000e^{-0.080t}$

50. The rate at which a factory is dumping pollution into a river at any time t is given by $P(t) = P_0 e^{-kt}$, where P_0 is the rate at which the pollution is initially released into the river. If $P_0 = 3000$ and $k = 0.080$, find the total amount of pollution that will be released into the river into the indefinite future.

$$\int_0^\infty P(t)\,dt = \int_0^\infty 3000e^{-0.080t}\,dt = \lim_{N\to\infty}\int_0^N 3000e^{-0.080t}\,dt$$

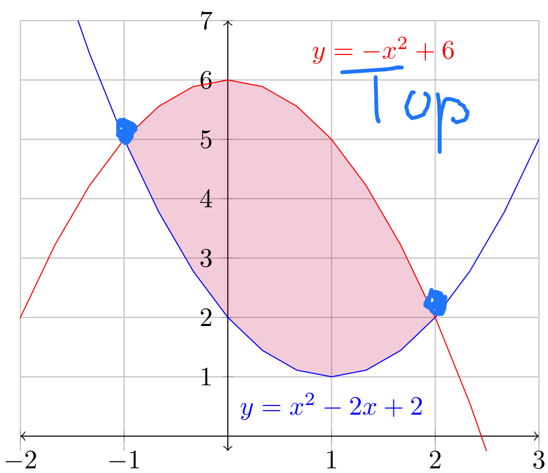
$$= \lim_{N\to\infty} \frac{3000}{-0.080} e^{-0.080t}\Big]_0^N$$

$$= \lim_{N\to\infty}\left(-37500e^{-0.080N} + 37500\right) = \boxed{37500}$$

Answer: ______

51. Set up the integral that computes the **AREA** shown to the right with respect to x.

DON'T COMPUTE IT!!!

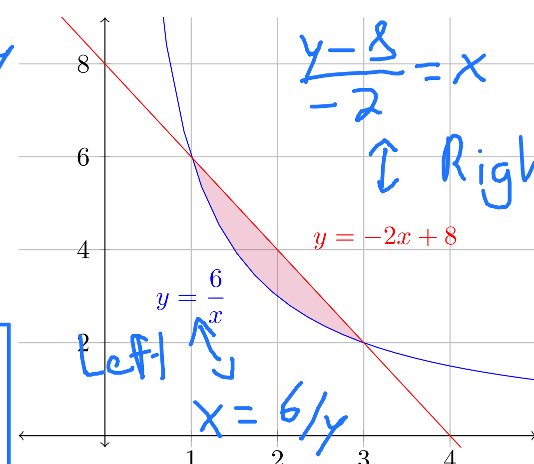


$$\boxed{\int_{-1}^{2} (-x^2+6) - (x^2 - 2x + 2)\,dx}$$

Area = ______

52. Set up the integral that computes the **AREA** shown to the right with respect to y. → dy

DON'T COMPUTE IT!!!



$$\boxed{\int_{2}^{6} \left(\frac{y-8}{-2}\right) - \frac{6}{y}\,dy}$$

Area = ______

53. Set up the integral that computes the **AREA** with respect to x of the region bounded by → dx problem

$$y = \frac{2}{x} \quad \text{and} \quad y = -x + 3$$

Bounds:

$\frac{2}{x} = -x+3$

$2 = -x^2 + 3x$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

Test Pt: $x = 1.5$

$y = \frac{2}{x} \Rightarrow y = \frac{2}{1.5} = \frac{4}{3} \approx 1.33 \rightarrow$ Bottom

$y = -x+3 \Rightarrow y = -1.5 + 3 = 1.5 \rightarrow$ Top

Area = $\boxed{\int_1^2 \left(-x + 3 - \frac{2}{x}\right) dx}$

54. Find the area of the region bounded by $y = 6x - x^2$ and $y = 2x^2$.

Bounds:

$6x - x^2 = 2x^2$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$x = 0, 2$

Test Pt: $x = 1$

$y = 6x - x^2 \Rightarrow y = 5 \rightarrow$ Top

$y = 2x^2 \Rightarrow y = 2 \rightarrow$ Bottom

$A = \int_0^2 \left[(6x - x^2) - 2x^2\right] dx$

$= \int_0^2 (6x - 3x^2)\, dx$

$= (3x^2 - x^3)\Big]_0^2 = 4$

Area = $\boxed{4}$

55. Find the area of the region bounded by $y = 2x - x^2$ and $y = x^2$.

Bounds:

$2x - x^2 = x^2$

$2x - 2x^2 = 0$

$2x(x-1) = 0$

$x = 0, 1$

$A = \int_0^1 (2x - x^2) - x^2 \, dx$

$= \int_0^1 2x - 2x^2 \, dx$

$= \left(\frac{2x^2}{2} - \frac{2x^3}{3}\right)\Big]_0^1 = \frac{1}{3}$

Test Pt: $x = 1/2$

$y = 2x - x^2 \longrightarrow y(\frac{1}{2}) = 3/4 \rightarrow$ Top

$y = x^2 \longrightarrow y(\frac{1}{2}) = 1/4 \rightarrow$ Bottom

Area = $\boxed{1/3}$

56. Calculate the **AREA** of the region bounded by the following curves.

$$x = 100 - y^2 \quad \text{and} \quad x = 2y^2 - 8$$

Bounds:

$100 - y^2 = 2y^2 - 8$

$108 = 3y^2$

$36 = y^2$

$y = \pm 6$

$A = \int_{-6}^{6} (100 - y^2) - (2y^2 - 8) \, dy$

$= \int_{-6}^{6} (108 - 3y^2) \, dy$

$= (108y - y^3)\Big]_{-6}^{6}$

$= 864$

Test Pt: $y = 0$

$x = 100 - y^2 \rightarrow x = 0 \rightarrow$ Right

$x = 2y^2 - 8 \rightarrow x = -8 \rightarrow$ Left

Area = $\boxed{864}$

57. Calculate the **AREA** of the region bounded by the following curves.

$$y = x^3 \quad \text{and} \quad y = x^2$$

Bounds:

$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0, 1$

Test Pt: $x = \frac{1}{2}$

$y = x^3 \rightarrow y = \frac{1}{8} \rightarrow$ Bottom

$y = x^2 \rightarrow y = \frac{1}{4} \rightarrow$ Top

$A = \int_0^1 (x^2 - x^3)\,dx$

$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big]_0^1$

$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

Area = $\boxed{1/12}$

58. After t hours studying, one student is working $Q_1(t) = 25 + 9t - t^2$ problems per hour, and a second student is working on $Q_2(t) = 5 - t + t^2$ problems per hour. How many more problems will the first student have done than the second student after 10 hours?

$\int_0^{10} Q_1(t) - Q_2(t)\,dt$

$= \int_0^{10} (25 + 9t - t^2) - (5 - t + t^2)\,dt$

$= \int_0^{10} (20 + 10t - 2t^2)\,dt$

$= \left(20t + 5t^2 - \frac{2}{3}t^3\right)\Big]_0^{10}$

$= \frac{100}{3}$

Answer: $\boxed{100/3}$

59. The birthrate of a particular population is modeled by $B(t) = 1000e^{0.036t}$ people per year, and the death rate is modeled by $D(t) = 725e^{0.019t}$ people per year. How much will the population increase in the span of 10 years? $(0 \leq t \leq 20)$ Round to the nearest whole number.

$$\int_0^{10} B(t) - D(t)\,dt = \int_0^{10} 1000e^{0.036t} - 725e^{0.019t}\,dt$$

$$= \left(\frac{1000}{0.036}e^{0.036t} - \frac{725}{0.019}e^{0.019t}\right)\Bigg]_0^{10}$$

$$\approx 4052$$

Answer: $\boxed{4052}$

60. Let R be the region shown below. Set up the integral that computes the **VOLUME** as R is rotated around the x-axis.

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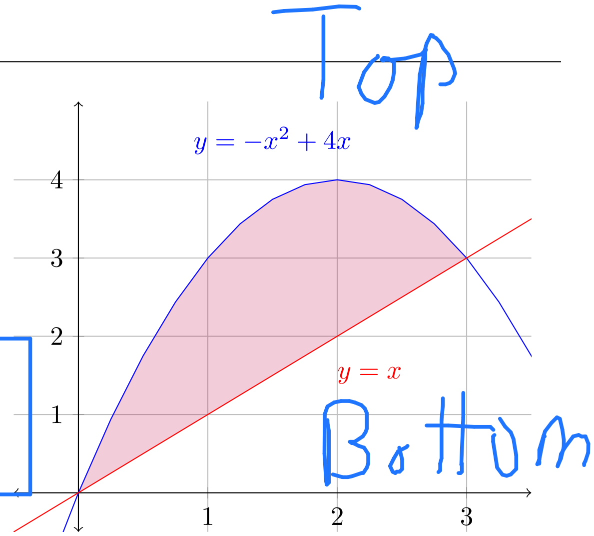


$$\boxed{\pi \int_0^3 \left[(-x^2+4x)^2 - (x)^2\right] dx}$$

Volume = ______________________

61. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{16 - x}, \quad y = 0 \quad \text{and} \quad x = 0$$

about the y-axis $\Rightarrow$ dy problem

Disk

$y = \sqrt{16-x}$

$y^2 = 16 - x$

$x = 16 - y^2$

Bounds: Given $y=0$

Plug $x=0$ into $y=\sqrt{16-x}$

$y = \sqrt{16-x}$

$y = \sqrt{16}$

$y = 4$

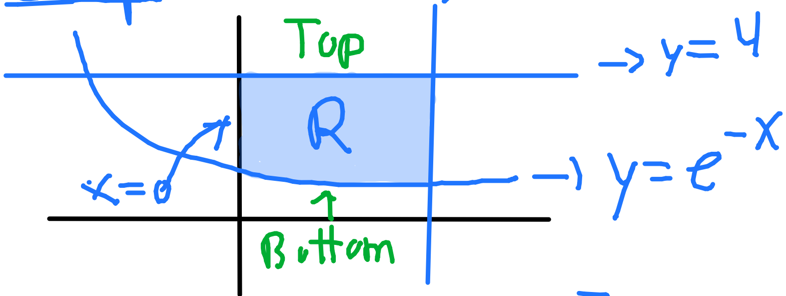
Volume = $\boxed{\pi \int_0^4 (16-y^2)^2\, dy}$

62. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = e^{-x}, \quad y = 4 \quad x = 0 \quad \text{and} \quad x = 10$$

about the x-axis $\rightarrow dx$

Graph:

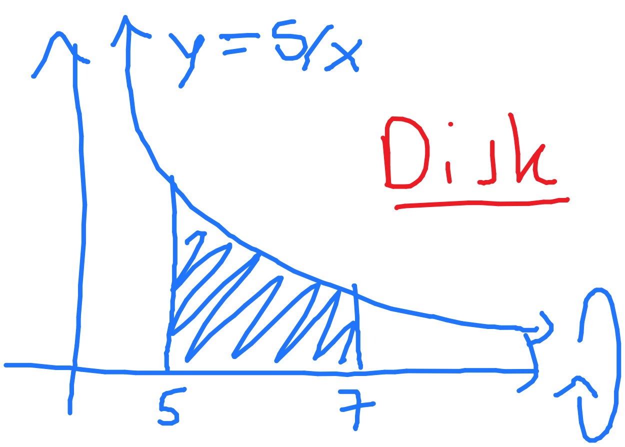


$$V = \pi \int_0^2 \left[4^2 - (e^{-x})^2 \right] dx$$

$$\boxed{\pi \int_0^2 (16 - e^{-2x})\, dx}$$

Volume = ______________________

63. Find the volume of the solid that results by revolving the region enclosed by the curves $y = \dfrac{5}{x}$, $y = 0$, $x = 5$, and $x = 7$ about the x-axis. $\Rightarrow dx$



Disk

$$V = \pi \int_5^7 \left(\frac{5}{x}\right)^2 dx$$

$$= \pi \int_5^7 \frac{25}{x^2}\, dx$$

$$= 25\pi \int_5^7 x^{-2}\, dx$$

$$= 25\pi \left(-\frac{1}{x}\right)\Big]_5^7$$

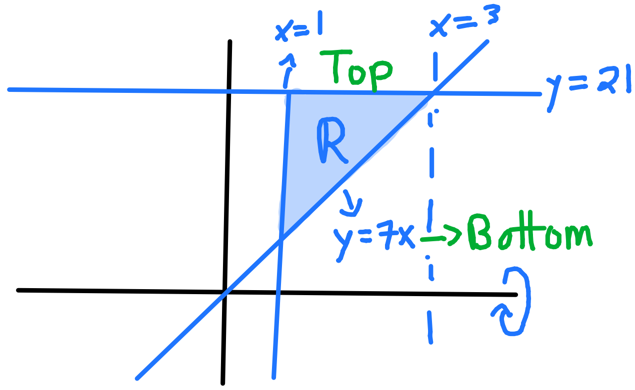
$$= \boxed{\frac{10\pi}{7}}$$

Volume = ______________________

64. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 21 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis $\longrightarrow dx$



Washer

$$V = \pi \int_1^3 \left[21^2 - (7x)^2\right] dx$$

$$= \pi \int_1^3 (441 - 49x^2)\, dx$$

$$= \pi \left(441x - \frac{49x^3}{3}\right)\Bigg]_1^3$$

$$= \frac{1274}{3}\pi$$

Volume $=$ $\boxed{\dfrac{1274\pi}{3}}$

65. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 0 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis $\longrightarrow dx$

Disk

$$V = \pi \int_1^3 (7x)^2\, dx$$

$$= \pi \int_1^3 49x^2\, dx$$

$$= \pi \left(\frac{49x^3}{3}\right)\Bigg]_1^3$$

$$= \frac{49\pi}{3}(3^3 - 1)$$

Volume $=$ $\boxed{\dfrac{1274}{3}\pi}$

66. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y = \sqrt{x}$$

about the y-axis

Graph:

Bounds:

$\sqrt{y} = y^2$

$y = y^4$

$0 = y^4 - y$

$0 = y(y^3 - 1)$

$y = 0, 1$

But y-axis $\Rightarrow dy$

Right $\to y = x^2 \to x = \sqrt{y}$

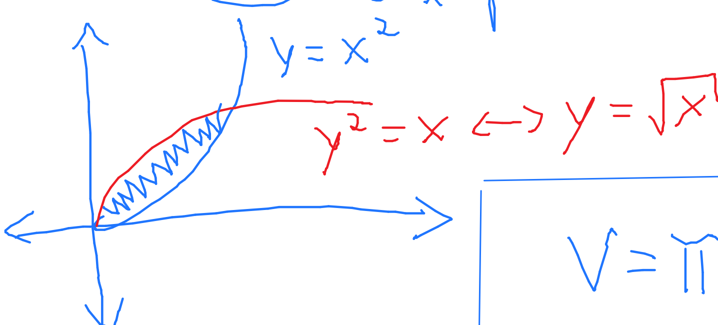
Left $\to y = \sqrt{x} \to x = y^2$

Volume = $\boxed{\pi \int_0^1 \left[(\sqrt{y})^2 - (y^2)^2\right] dy}$

67. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y^2 = x$$

about the x-axis $\to dx$ problem



$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$

Bounds:

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

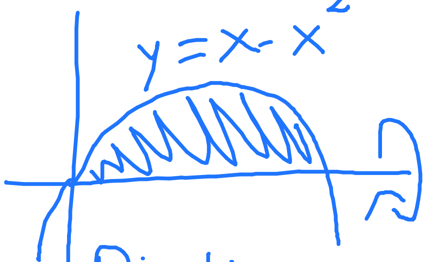
$x = 0, 1$

Volume = $\boxed{\pi \int_0^1 (x - x^4) \, dx}$

68. Find the **VOLUME** of the region bounded by

$$y = x - x^2, \quad \text{and} \quad y = 0$$

around the x-axis



Disk

Bounds:

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$

$$V = \pi \int_0^1 (x - x^2)^2\,dx$$

$$= \pi \int_0^1 (x^2 - 2x^3 + x^4)\,dx$$

$$= \pi \left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \Big]_0^1$$

$$= \frac{\pi}{30}$$

Volume $=$ $\boxed{\pi/30}$

69. Find the **VOLUME** of the solid generate by revolving the given region about the x-axis: $\rightarrow dx$

$$y = 8\sqrt{x}, \quad y = 0, \quad x = 3, \quad x = 6$$

$$V = \pi \int_3^6 (8\sqrt{x})^2\,dx$$

$$= \pi \int_3^6 64x\,dx$$

$$= \pi \, \frac{64x^2}{2} \Big]_3^6$$

$$= \pi \, 32x^2 \Big]_3^6$$

$$= 864\pi$$

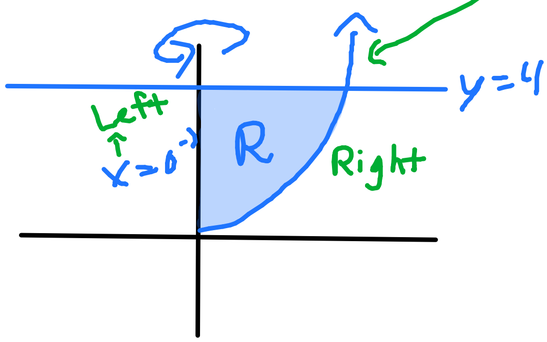
Volume $=$ $\boxed{864\pi}$

70. Find the **VOLUME** of the region bounded by

$$y = 4x^2, \quad x = 0, \quad y = 4$$

around the y-axis. $\to dy$

$x = \sqrt{\frac{y}{4}}$



Disk

$$V = \pi \int_0^4 \left(\sqrt{\frac{y}{4}}\right)^2 dy$$

$$= \pi \int_0^4 \frac{y}{4}\, dy$$

$$= \frac{\pi}{4} \cdot \frac{y^2}{2} \Big]_0^4$$

$$= \frac{\pi}{8} \cdot 16$$

$$= 2\pi$$

Volume $=$ $\boxed{2\pi}$

71. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x + 8, \quad \text{and} \quad y = (x-4)^2$$

about the x-axis

Bounds:

$x + 8 = (x-4)^2$

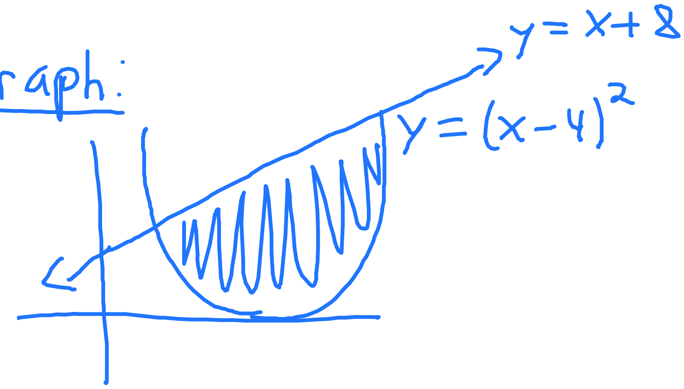
$x + 8 = x^2 - 8x + 16$

$0 = x^2 - 9x + 8$

$0 = (x-8)(x-1)$

$x = 1, 8$

Graph:



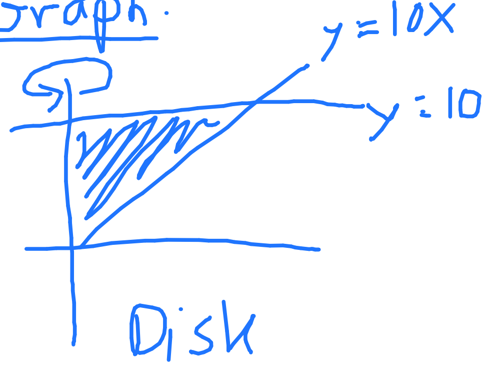
Volume $=$ $\boxed{\pi \int_1^8 \left[(x+8)^2 - (x-4)^4\right] dx}$

72. Find the **VOLUME** of the region bounded by

$$y = 10x, \quad x = 0, \quad y = 10$$

around the y-axis

Graph:



Disk

But y-axis $\Rightarrow$ dy problem

$y = 10x$

$\frac{y}{10} = x$

$$V = \pi \int_0^{10} \left(\frac{y}{10}\right)^2 dy$$

$$= \pi \int_0^{10} \frac{y^2}{100}\, dy$$

$$= \frac{\pi}{100}\left(\frac{y^3}{3}\right)\Big]_0^{10}$$

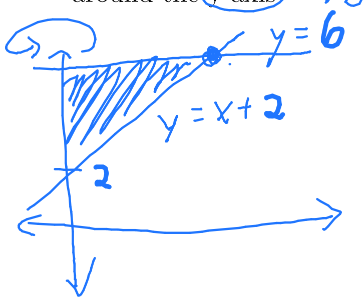
$$= \frac{10\pi}{3}$$

Volume $=$ $\boxed{10\pi/3}$

73. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 2, \quad x = 0, \quad y = 6$$ $\rightarrow x = y - 2$

around the y-axis $\rightarrow$ dy problem.



$$V = \pi \int_2^6 (y-2)^2\, dy$$

$$= \pi \int_2^6 (y^2 - 4y + 4)\, dy$$

$$= \pi\left(\frac{y^3}{3} - \frac{4y^2}{2} + 4y\right)\Big]_2^6$$

Volume $=$ $\boxed{64\pi/3}$

74. Find the **VOLUME** of the region bounded by

$$x + 3y = 9, \quad x = 0, \quad y = 0$$

around the y-axis

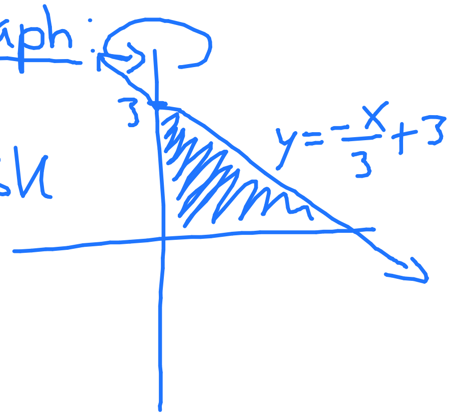
$x + 3y = 9$

$3y = -x + 9$

$y = \frac{-x}{3} + 3$

Graph:

Disk



But y-axis $\Rightarrow dy$

So $x + 3y = 9$

$x = 9 - 3y$

$$V = \pi \int_0^3 (9-3y)^2\,dy$$

$$= \pi \int_0^3 (81 - 54y + 9y^2)\,dy$$

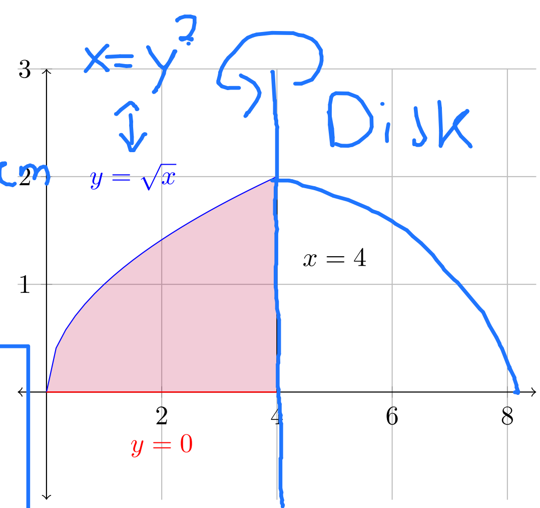
$$= \pi \left(81y - 27y^2 + 3y^3\right)\Big]_0^3$$

$$= 81\pi$$

Volume $=$ $\boxed{81\pi}$

75. Let R be the region shown to the right. Set up the integral that computes the **VOLUME** as R is rotated around the line $x = 4$. $\rightarrow$ dy problem

DON'T COMPUTE IT!!!



Volume $=$ $\boxed{\pi \int_0^2 (y^2-4)^2\,dy}$

76. **SET-UP using the washer method.** the **VOLUME** of the region bounded by

$$y = x^2, \quad y = 2x$$

around the x-axis $\to dx$

Note the bounds for all choices are the same.

(A) $\pi \int_0^2 (2x - x^2)^2 \, dx$

(B) $\boxed{\pi \int_0^2 (4x^2 - x^4) \, dx}$

(C) $\pi \int_0^2 (2x - x^2) \, dx$

(D) $\pi \int_0^2 (x^2 - 2x) \, dx$

(E) $\pi \int_0^2 (x^4 - 4x^2) \, dx$

(F) $2\pi \int_0^2 (x^3 - 2x^2) \, dx$

Test Pt: $x = 1$

$y = x^2 \to y = 1 \to$ Bottom

$y = 2x \to y = 2 \to$ Top

$V = \pi \int_0^2 (2x)^2 - (x^2)^2 \, dx$

$= \pi \int_0^2 4x^2 - x^4 \, dx$

77. Set up the integral needed to find the volume of the solid obtained when the region bounded by

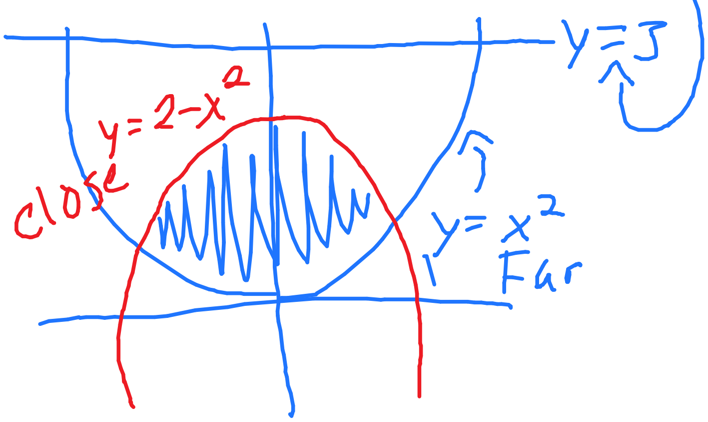
$$y = 2 - x^2 \quad \text{and} \quad y = x^2$$

is rotated about the line $y = 3$.

$y = 3 \Rightarrow dx$ problem

Graph

Washer



Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

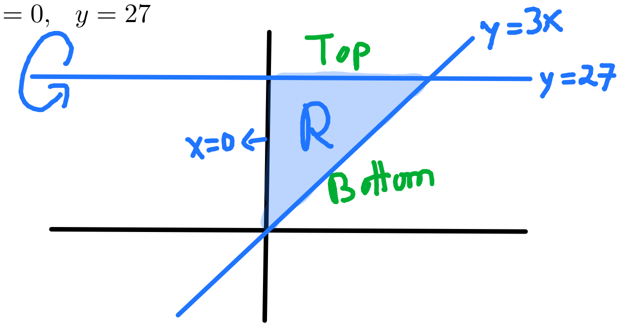
Volume $=$ $\boxed{\pi \int_{-1}^{1} (2 - x^2 - 3)^2 - (x^2 - 3)^2 \, dx}$

78. **SET-UP using the disk/washer method.** the **VOLUME** of the region bounded by

$$y = 3x, \quad x = 0, \quad y = 27$$

Disk

around the line $y = 27$ → dx



(A) $\pi \int_0^{27} (729 - 162x + 9x^2)\, dx$

(B) $\pi \int_0^{27} 9x^2\, dx$

(C) $\pi \int_0^{9} 9x^2\, dx$

(D) $\pi \int_0^{9} (9x^2 - 162x)\, dx$

(E) $\pi \int_0^{27} (729 - 9x^2)\, dx$

(F) $\boxed{\pi \int_0^{9} (729 - 162x + 9x^2)\, dx}$

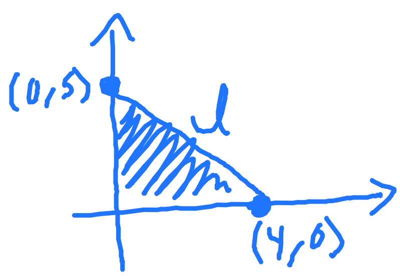
Bound: $3x = 27$
$x = 9$

$V = \pi \int_0^9 (3x - 27)^2\, dx$

$= \pi \int_0^9 (9x^2 - 162x + 729)\, dx$

dx

79. **SET-UP using the Shell method**, the integral that computes the **VOLUME** of the region in quadrant I enclosed by the region defined by a triangle with vertices at (0,0), (0,5), and (4,0) about the y-axis.



(A) $\pi \int_0^{5} \left(8x - \frac{5}{4}x^2\right) dx$

(B) $\pi \int_0^{5} \frac{5}{4}x^2\, dx$

(C) $\pi \int_0^{4} 4x^2\, dx$

(D) $\pi \int_0^{4} \left(8x - \frac{5}{4}x^2\right) dx$

(E) $\boxed{\pi \int_0^{4} \left(10x - \frac{5}{2}x^2\right) dx}$

(F) $\pi \int_0^{5} \left(10x - \frac{5}{2}x^2\right) dx$

$V = 2\pi \int_0^4 x \cdot l\, dx$

Find the eqn of the line, l.

$m = \frac{0-5}{4-0} = -\frac{5}{4}$

y-intercept is @ 5 b/c (0,5)

$l = -\frac{5}{4}x + 5$

$V = 2\pi \int_0^4 x\left(-\frac{5}{4}x + 5\right) dx$

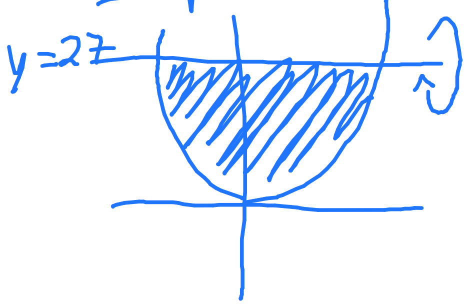
$= \pi \int_0^4 \left(10x - \frac{5}{2}x^2\right) dx$

80. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Graph Disk



$$V = \pi \int_0^3 (3x^2 - 27)^2 \, dx$$

$$= \pi \int_0^3 (9x^4 - 162x^2 + 729) \, dx$$

$$= \pi \left(\frac{9x^5}{5} - 54x^3 + 729x \right) \Big]_0^3$$

$$= 11664.4\pi$$

$y = 27 \Rightarrow dx$ problem

Bounds: Given $x = 0$

$27 = 3x^2$

$9 = x^2 \rightarrow x = 3$

$$\boxed{\frac{8322}{5}\pi}$$

Volume = ______________________

81. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

+ → dy

$$x = 2y - y^2, \quad \text{and} \quad x = 0$$

about the x-axis. → dy

Bounds: $0 = 2y - y^2$

$0 = y(2 - y)$

$y = 0, 2$

$$V = 2\pi \int_0^2 y(2y - y^2) \, dy$$

$$\boxed{2\pi \int_0^2 y(2y - y^2) \, dy}$$

Volume = ______________________

82. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 2 - x^2, \quad \text{and} \quad y = x^2$$

about the y-axis.

+ → dx

$$V = 2\pi \int_{-1}^{1} x(2 - x^2 - x^2)\,dx$$

Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

Test Pt: $x = 0$

$y = 2 - x^2 \rightarrow y = 2 \rightarrow$ Top

$y = x^2 \rightarrow y = 0 \rightarrow$ Bottom

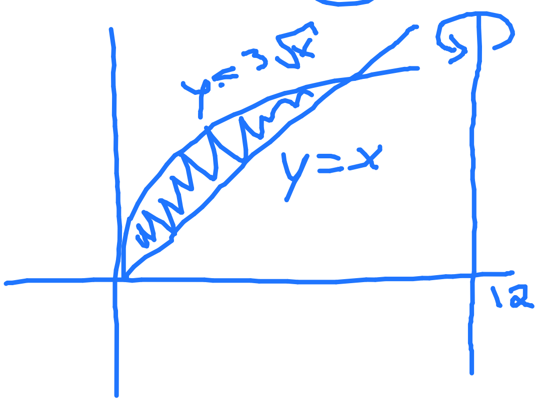
Volume $=$ $\boxed{2\pi \int_{-1}^{1} x(2 - 2x^2)\,dx}$

83. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 3\sqrt{x}, \quad \text{and} \quad y = x$$

about the $x = 12$.

+ → dx



Bounds

$3\sqrt{x} = x$

$9x = x^2$

$9x - x^2 = 0$

$x(9 - x) = 0$

$x = 0, 9$

★ Note $x = 12$ is on the right of our region.

★

$$\boxed{V = 2\pi \int_{0}^{9} (12 - x)(3\sqrt{x} - x)\,dx}$$

Volume $=$ ____________________

84. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = x, \quad \text{and } y = x^2$$

about the line $x = -2$. → $+$ → dx

Since $x=-2$ is on the left of our region

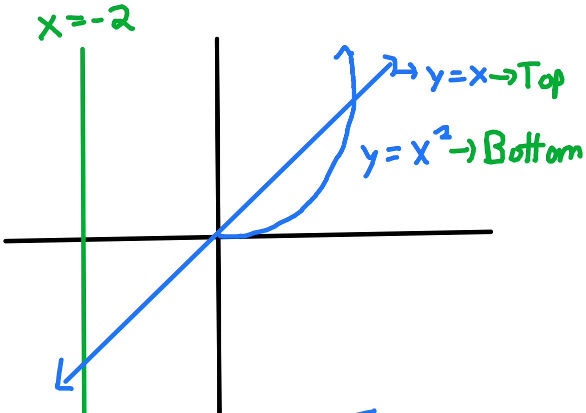
Bounds: $x = x^2$

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$

$$V = 2\pi \int_0^1 (x-(-2))\left[x - x^2\right]dx$$



Volume $=$ $\boxed{2\pi \int_0^1 (x+2)(x-x^2)dx}$

85. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 7x^2, \quad y = 0 \text{ and } \quad x = 2$$

about the line $x = 3$. → $+$ → dx

$$V = 2\pi \int_0^2 (\_\_\_\_)(7x^2)\,dx$$

Since $x=3$ is larger than the bounds,

$$V = 2\pi \int_0^2 (3-x)(7x^2)\,dx$$

Volume $=$ $\boxed{2\pi \int_0^2 (3-x)(7x^2)\,dx}$

86. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$x = y^2 + 1, \text{ and } \quad x = 2$$

$+$ $\to dy$

about the line $y = -2$. $\to dy$

Bounds: $y^2+1=2$

$y^2=1$

$y=\pm 1$

Since $y=-2$ is smaller than the bounds

$V = 2\pi \int_{-1}^{1} (y-(-2))(2-(y^2+1))\,dy$

Test Pt: $y=0$

$x=y^2+1 \to x=1 \to$ Left

$x=2 \to x=2 \to$ Right

Volume $=$ $\boxed{2\pi \int_{-1}^{1} (y+2)(2-(y^2+1))\,dy}$

87. The rate of change of the population $n(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate k in terms of minutes. Round to four decimal places.

Recall $N'=kN \to N=Ce^{kt}$

$N(0)=210$: $210=Ce^{k\cdot 0}$

$210=C \to N=210e^{kt}$

$N(5)=360$: $360=210e^{k\cdot 5}$

$\frac{12}{7}=e^{5k}$

$\ln(12/7)=5k$

$k=$ $\boxed{\frac{1}{5}\ln\left(\frac{12}{7}\right)}$

88. Let y denote the mass of a radioactive substance at time t. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time t in hours does half the original mass remain? Round your answer to 3 decimal places.

$$y' = -18y \Rightarrow y = Ce^{-18t}$$

$$y(0) = 20 \Rightarrow 20 = Ce^{-18(0)}$$

$$20 = C \quad \Rightarrow y = 20e^{-18t}$$

We want solve $\frac{1}{2}(20) = y(t)$ for t.

$$10 = 20e^{-18t}$$

$$1/2 = e^{-18t}$$

$$\ln(1/2) = -18t$$

$$\frac{\ln(1/2)}{-18} = t$$

t = $\boxed{0.039}$

89. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

Rewrite: $y\,dy = 3x^2\,dx$

$$\int y\,dy = \int 3x^2\,dx$$

$$\frac{y^2}{2} = x^3 + C$$

$$y^2 = 2x^3 + C$$

$$y = \pm\sqrt{2x^3 + C}$$

y = $\boxed{\pm\sqrt{2x^3 + C}}$

90. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

Rewrite $dy = 5y\,dx$

$$\frac{dy}{y} = 5dx$$

$$\int \frac{dy}{y} = \int 5dx$$

$$\ln|y| = 5x + C$$

$$|y| = e^{5x+C}$$

$$\pm y = e^{C} e^{5x}$$

$$y = \underbrace{\pm e^{C}}_{= C} e^{5x}$$

$$y = Ce^{5x}$$

or memorize

$$\frac{dy}{dx} = ky \Rightarrow y = Ce^{kx}$$

y = $\boxed{Ce^{5x}}$

91. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

Rewrite: $y\,dy = -x\,dx$

$$\int y\,dy = \int -x\,dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

$$y^2 = -x^2 + C$$

$$y = \pm\sqrt{C - x^2}$$

y = $\boxed{\pm\sqrt{C-x^2}}$

92. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

Note there are 2 ways to do this problem.
① Separation of Variables
② First-Order Linear Eqn

By method 1,

$$\frac{dy}{dt} = 15y$$

$$\frac{dy}{y} = 15\,dt$$

$$\int \frac{dy}{y} = \int 15\,dt$$

$$\ln|y| = 15t + C$$

$$y = e^{15t + C}$$

$$y = e^{C} e^{15t}$$

$$y = Ce^{15t}$$

$y =$ $\boxed{Ce^{15t}}$

93. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$$y\,dy = 3\,dx$$

$$\int y\,dy = \int 3\,dx$$

$$\frac{y^2}{2} = 3x + C$$

$$y^2 = 6x + 2C$$

$$y^2 = 6x + C$$

$$y = \pm\sqrt{6x + C}$$

$y =$ $\boxed{\pm\sqrt{6x+C}}$

94. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$$\frac{dy}{y} = 3x^2\,dx$$

$$\int \frac{dy}{y} = \int 3x^2\,dx$$

$$\ln|y| = x^3 + C$$

$$y = e^{x^3+C}$$

$$y = e^C e^{x^3}$$

$$y = Ce^{x^3}$$

$y =$ $\boxed{Ce^{x^3}}$

95. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$$dy = 8e^{-4t}e^{-y}\,dt$$

$$e^{y}\,dy = 8e^{-4t}\,dt$$

$$\int e^{y}\,dy = \int 8e^{-4t}\,dt$$

$$e^{y} = \frac{8}{-4}e^{-4t} + C$$

$$e^{y} = -2e^{-4t} + C$$

$$y = \ln(-2e^{-4t} + C)$$

$y =$ $\boxed{\ln(-2e^{-4t}+C)}$

96. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$2y\,dy = (3x+2)\,dx$

$\int 2y\,dy = \int (3x+2)\,dx$

$y^2 = \frac{3x^2}{2} + 2x + C$

So $y^2 = \frac{3x^2}{2} + 2x + 16$

$y = \pm\sqrt{\frac{3x^2}{2} + 2x + 16}$

when $y(0) = 4$

$4^2 = 0 + 0 + C$

$16 = C$

$y =$ $\boxed{\pm\sqrt{\frac{3x^2}{2} + 2x + 16}}$

97. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$\frac{dy}{y} = \frac{5}{6x+3}\,dx$

$\int \frac{dy}{y} = \int \frac{5}{6x+3}\,dx$

$\ln|y| = \frac{5}{6}\ln|6x+3| + C$

$y = \exp\left[\frac{5}{6}\ln|6x+3| + C\right]$

$y = e^C \exp\left[\ln|6x+3|^{5/6}\right]$

$y = C \cdot |6x+3|^{5/6}$

When $y(0) = 1$

$1 = C \cdot |6(0)+3|^{5/6}$

$1 = C \cdot 3^{5/6}$

$C = 3^{-5/6}$

$y =$ $\boxed{3^{-5/6} \cdot |6x+3|^{5/6}}$

98. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \quad \text{where} \quad y = 10 \quad \text{when} \quad x = 2$$

Find the value of the integration constant, C.

$$dy = 11x^2e^{-x^3}dx$$

$$\int dy = \underbrace{\int 11x^2e^{-x^3}dx}$$

$$u = -x^3$$

$$du = -3x^2dx$$

$$y = \int -\frac{11}{3}e^u du$$

$$y = -\frac{11}{3}e^{-x^3} + C$$

When $y=10$ and $x=2$

$$10 = -\frac{11}{3}e^{-2^3} + C$$

$$10 = -\frac{11}{3}e^{-8} + C$$

$$C = 10 + \frac{11}{3}e^{-8}$$

$C = \boxed{10 + \frac{11}{3}e^{-8}}$

99. Find the particular solution to the given differential equation if $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$$y^2dy = xdx$$

$$\int y^2dy = \int xdx$$

$$\frac{y^3}{3} = \frac{x^2}{2} + C$$

Find C w/ $y(2)=3$

$$\frac{3^3}{3} = \frac{2^2}{2} + C$$

$$9 = 2 + C$$

$$7 = C$$

$$\frac{y^3}{3} = \frac{x^2}{2} + 7$$

$$y^3 = \frac{3x^2}{2} + 21$$

$$y = \sqrt[3]{\frac{3x^2}{2} + 21}$$

$y = \boxed{\sqrt[3]{\frac{3x^2}{2} + 21}}$

100. Calculate the constant of integration, C, for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \qquad y(1) = 2$$

Rewrite $6y\,dy = 7x^3\,dx$

$\int 6y\,dy = \int 7x^3\,dx$

$3y^2 = \frac{7x^4}{4} + C$

Note we want C when $y(1)=2$

$3(2)^2 = \frac{7(1)^4}{4} + C$

$12 = \frac{7}{4} + C$

$C = 41/4$

$C=$ $\boxed{\frac{41}{4}}$

101. The volume of an object $V(t)$ in cubic millimeter at any time t in seconds changes according to the model

$$\frac{dV}{dt} = \cos\left(\frac{t}{10}\right),$$

where $V(0) = 5$. Find the volume of the object at $t = 3$ seconds. Round to 4 decimal places.

Rewrite $dV = \cos\left(\frac{t}{10}\right)dt$

$\int dV = \int \cos\left(\frac{t}{10}\right)dt$

$V = 10\sin\left(\frac{t}{10}\right) + C$

Find C w/ $V(0)=5$

$5 = 10\sin\left(\frac{0}{10}\right) + C$

$C = 5$

So $V = 10\sin\left(\frac{t}{10}\right) + 5$

$V(3) = 10\sin\left(\frac{3}{10}\right) + 5$

≈ 7.9552

$V(3) =$ $\boxed{7.9552}$

102. What is the **integrating factor** of the following differential equation?

$$\frac{2y' + \left(\frac{6}{x}\right) y}{2} = \frac{10\ln(x)}{2}$$

$$y' + \frac{3}{x} y = 5\ln x$$

$$P(x) = \frac{3}{x} \quad Q(x) = 5\ln x$$

$$u(x) = \exp\left[\int \frac{3}{x}\,dx\right]$$
$$= \exp[3\ln x]$$
$$= \exp[\ln x^3]$$
$$= x^3$$

$u(x) =$ $\boxed{x^3}$

103. What is the **integrating factor** of the following differential equation?

$$y' + \left(\frac{2x+3}{x}\right) y = 10\ln(x)$$

$$P(x) = \frac{2x+3}{x} \quad Q(x) = 10\ln(x)$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int \frac{2x+3}{x}\,dx\right]$$
$$= \exp\left[\int 2 + \frac{3}{x}\,dx\right]$$
$$= \exp[2x + 3\ln x]$$

$$= e^{2x+3\ln x}$$
$$= e^{2x} \cdot e^{3\ln x}$$
$$= e^{2x} \cdot e^{\ln x^3}$$
$$= x^3 e^{2x}$$

$u(x) =$ $\boxed{x^3 e^{2x}}$

104. What is the **integrating factor** of the following differential equation?

$$\frac{x^8 y' - 14x^7 y}{x^8} = \frac{32e^{7x}}{x^8}$$

$$y' + \underbrace{\left(\frac{-14}{x}\right)}_{P} y = \underbrace{\frac{32e^{7x}}{x^8}}_{Q}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int -\frac{14}{x}\,dx\right]$$
$$= \exp\left[-14 \ln x\right]$$
$$= \exp\left[\ln x^{-14}\right]$$
$$= x^{-14}$$
$$= \frac{1}{x^{14}}$$

$u(x) =$ $\boxed{1/x^{14}}$

105. What is the **integrating factor** of the following differential equation?

$$\frac{(x+1)\frac{dy}{dx} - 2(x^2+x)y}{(x+1)} = \frac{(x+1)e^{x^2}}{(x+1)}$$

$$\frac{dy}{dx} - \frac{2x(x+1)}{(x+1)} y = e^{x^2}$$

$$\frac{dy}{dx} + (-2x) \cdot y = e^{x^2}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int -2x\,dx\right]$$
$$= \exp\left[-x^2\right]$$

$u(x) =$ $\boxed{e^{-x^2}}$

106. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) = \exp\left[\int P(x)\,dx\right]$

$= \exp\left[\int \cot x\,dx\right]$

$= \exp\left[\int \frac{\cos x}{\sin x}\,dx\right]$

$u = \sin x$

$du = \cos x\,dx$

$= \exp\left[\int \frac{du}{u}\right]$

$= \exp[\ln u]$

$u(x) = \exp[\ln \sin x]$

$= \sin x$

$u(x) =$ $\boxed{\sin x}$

107. What is the **integrating factor** of the following differential equation?

$$y' + \tan(x) \cdot y = \sec(x)$$

$u(x) = \exp\left[\int P(x)\,dx\right]$

$= \exp\left[\int \tan x\,dx\right]$

$= \exp\left[\int \frac{\sin x}{\cos x}\,dx\right]$

$u = \cos x$

$du = -\sin x\,dx$

$= \exp\left[-\int \frac{du}{u}\right]$

$= \exp[-\ln u]$

$u(x) = \exp[-\ln(\cos x)]$

$= \exp[\ln(\cos x)^{-1}]$

$= (\cos x)^{-1} = \sec x$

$u(x) =$ $\boxed{\sec(x)}$

Note there are 2 ways to do this problem.
(1) Separation of Variables
(2) First-Order Linear Eqn

108. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + (4x - 1)y = 8x - 2$$

$P(x) = 4x - 1 \qquad Q(x) = 8x - 2$

$u(x) = \exp\left[\int (4x-1)\,dx\right]$

$= \exp\left[2x^2 - x\right]$

$= e^{2x^2 - x}$

$y\,u(x) = \int Q(x)u(x)\,dx + C$

$y e^{2x^2-x} = \underbrace{\int (8x-2)e^{2x^2-x}\,dx}_{\substack{u = 2x^2 - x \\ du = 4x-1\,dx}} + C$

$y e^{2x^2-x} = \int \frac{8x-2}{4x-1} e^u\,du + C$

$y e^{2x^2-x} = \int \frac{2\cancel{(4x-1)}}{\cancel{4x-1}} e^u\,du + C$

$y e^{2x^2-x} = \int 2e^u\,du + C$

$y e^{2x^2-x} = 2e^u + C$

$y e^{2x^2-x} = 2e^{2x^2-x} + C$

$y = \frac{2e^{2x^2-x} + C}{e^{2x^2-x}}$

$y = 2 + Ce^{-(2x^2-x)}$

$= 2 + Ce^{x-2x^2}$

$y =$ $\boxed{2 + Ce^{x-2x^2}}$

109. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + \frac{6y}{x} = x + 10$$

$P(x) = \frac{6}{x} \quad Q(x) = x + 10$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int \frac{6}{x}\,dx\right]$$
$$= \exp[6 \ln x]$$
$$= \exp[\ln(x^6)]$$
$$= x^6$$

$$y \cdot u(x) = \int Q(x)u(x)\,dx + C$$
$$y\,x^6 = \int (x+10)x^6\,dx + C$$
$$y\,x^6 = \int (x^7 + 10x^6)\,dx + C$$
$$y\,x^6 = \frac{x^8}{8} + \frac{10x^7}{7} + C$$
$$y = \frac{x^2}{8} + \frac{10x}{7} + \frac{C}{x^6}$$

$y = \boxed{\frac{x^2}{8} + \frac{10x}{7} + \frac{C}{x^6}}$

110. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2(y+4) \text{ and } y(0) = 3$$

$y' = 6x^2y + 24x^2$

$y' - 6x^2y = 24x^2$

$P(x) = -6x^2 \quad Q(x) = 24x^2$

$u(x) = \exp\left[\int -6x^2 dx\right]$

$= \exp\left[-2x^3\right]$

$= e^{-2x^3}$

$y \cdot u(x) = \int Q(x)u(x)\,dx + C$

$ye^{-2x^3} = \underbrace{\int 24x^2 e^{-2x^3} dx}_{\substack{u=-2x^3 \\ du=-6x^2dx}} + C$

$ye^{-2x^3} = \int -4e^u du + C$

$ye^{-2x^3} = -4e^u + C$

$ye^{-2x^3} = -4e^{-2x^3} + C$

$y = -4 + Ce^{2x^3}$

With $y(0) = 3$

$3 = -4 + Ce^{2\cdot 0^3}$

$3 = -4 + C$

$7 = C$

So $y = -4 + 7e^{2x^3}$

$y =$ $\boxed{-4+7e^{2x^3}}$

111. Solve the initial value problem.

$$x^4 y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$$\frac{x^4 y' + 4x^3 y}{x^4} = \frac{10x^9}{x^4}$$

$$y' + \frac{4}{x} \cdot y = 10x^5$$

$$P(x) = \frac{4}{x} \quad Q(x) = 10x^5$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$

$$= \exp\left[\int \frac{4}{x}\,dx\right]$$

$$= \exp[4 \ln x]$$

$$= \exp[\ln x^4]$$

$$= x^4$$

$$y \cdot u(x) = \int Q(x)u(x)\,dx + C$$

$$y \cdot x^4 = \int 10x^5 x^4\,dx + C$$

$$y \cdot x^4 = \int 10x^9\,dx + C$$

$$y \cdot x^4 = x^{10} + C$$

$$y = \frac{x^{10}}{x^4} + \frac{C}{x^4}$$

$$y = x^6 + \frac{C}{x^4}$$

$$23 = 1 + \frac{C}{1}$$

$$22 = C$$

$$y = x^6 + \frac{22}{x^4}$$

$y =$ $\boxed{x^6 + \frac{22}{x^4}}$

112. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \ldots$$

$$= 1 - \frac{6}{10} + \frac{36}{100} - \frac{216}{1000} + \ldots$$

$$= 1 - \frac{6}{10} + \left(\frac{6}{10}\right)^2 - \left(\frac{6}{10}\right)^3 + \ldots$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n$$

Answer: $\boxed{\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n}$

(b) Use the sum from (a) and compute the sum.

$$\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n = \frac{1}{1-(-6/10)} = \frac{1}{1+6/10} = \frac{1}{16/10} = \frac{10}{16} = \frac{5}{8}$$

Answer: $\boxed{5/8}$

113. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Note $r = 3/2$ and

$\left|\frac{3}{2}\right| < 1$ is false

So the sum diverges

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \boxed{\text{diverges}}$$

114. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$$= \frac{6}{1-(-1/9)}$$

$$= \frac{6}{1+1/9}$$

$$= \frac{6}{10/9}$$

$$= 6 \cdot \frac{9}{10}$$

$$= 3 \cdot \frac{9}{5} = \frac{27}{5}$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ $\boxed{27/5}$

115. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$$= \sum_{n=0}^{\infty} 7\left(\frac{1}{4}\right)^n$$

$$= \frac{7}{1-1/4}$$

$$= \frac{7}{3/4}$$

$$= 7 \cdot \frac{4}{3} = \frac{28}{3}$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ $\boxed{28/3}$

116. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$= \frac{5^3}{6} + \frac{5^4}{6^2} + \frac{5^5}{6^3} + \cdots$$

$$= \frac{5^3}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \cdots\right)$$

$$= \frac{125}{6} \sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n = \frac{125}{6} \cdot \frac{1}{1-5/6}$$

$$= \frac{125}{6} \cdot \frac{1}{1/6} = \frac{125}{6} \cdot \frac{6}{1} = 125$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \boxed{125}$$

117. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n}{3 \cdot 3^{2n}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3}\left(-\frac{2}{9}\right)^n$$

$$= \frac{1/3}{1-(-2/9)}$$

$$= \frac{1/3}{1+2/9}$$

$$= \frac{1/3}{11/9}$$

$$= \frac{1}{3} \cdot \frac{9}{11}$$

$$= 3/11$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \boxed{3/11}$$

118. Evaluate the sum of the following infinite series.

$$\sum_{n=0}^{\infty}(-1)^n \frac{5^{n+1}}{3^{2n}}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \, 5^n \cdot 5^1}{(3^2)^n}$$

$$= \sum_{n=0}^{\infty} 5\left(-\frac{5}{9}\right)^n$$

$$= \frac{5}{1-(-5/9)}$$

$$= \frac{5}{14/9}$$

$$= \frac{5}{1} \cdot \frac{9}{14}$$

$$= \frac{45}{14}$$

Answer: $\boxed{45/14}$

119. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \frac{4(3)^{n-1}}{5^n}$$

$$= \frac{4(3)^0}{5^1} + \frac{4(3)^1}{5^2} + \frac{4(3)^2}{5^3} + \frac{4(3)^3}{5^4} + \ldots$$

$$= \frac{4}{5}\left(1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \left(\frac{3}{5}\right)^3 + \ldots\right)$$

$$= \frac{4}{5} \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n$$

$$= \frac{4}{5} \cdot \frac{1}{1-3/5}$$

$$= \frac{4}{5} \cdot \frac{1}{2/5} = \frac{4}{5} \cdot \frac{5}{2} = 2$$

Answer: $\boxed{2}$

120. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty}\left(\frac{3^{n-1}}{4^n}+\frac{(-1)^{n+1}}{9^n}\right)$$

$$= \sum_{n=1}^{\infty}\left(\frac{3^{-1}}{1}\cdot\frac{3^n}{4^n}+\frac{(-1)^1}{1}\cdot\frac{(-1)^n}{9^n}\right)$$

$$= \sum_{n=1}^{\infty}\left(\frac{1}{3}\left(\frac{3}{4}\right)^n-\left(\frac{-1}{9}\right)^n\right)$$

$$= \frac{1}{3}\left(\frac{3}{4}\right)^1-\left(\frac{-1}{9}\right)^1$$

$$+\frac{1}{3}\left(\frac{3}{4}\right)^2-\left(\frac{-1}{9}\right)^2$$

$$+\frac{1}{3}\left(\frac{3}{4}\right)^3-\left(\frac{-1}{9}\right)^3$$

$$+\ldots$$

$$=\frac{1}{3}\left(\frac{3}{4}\right)\left[1+\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2+\ldots\right]$$

$$-\left(\frac{1}{-9}\right)\left[1+\left(-\frac{1}{9}\right)+\left(\frac{-1}{9}\right)^2\ldots\right]$$

$$=\frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n+\frac{1}{9}\sum_{n=0}^{\infty}\left(\frac{-1}{9}\right)^n$$

$$=\frac{1}{4}\cdot\frac{1}{1-3/4}+\frac{1}{9}\cdot\frac{1}{1-(-1/9)}$$

Answer: $\boxed{1.1}$

121. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty}3(-2x)^n$$

Remember

$$\sum_{n=0}^{\infty}\square^n=\frac{1}{1-\square} \quad \text{where } |\square|<1$$

$$|-2x|<1$$

$$|2x|<1$$

$$2|x|<1$$

$$|x|<1/2=R$$

$R = \boxed{1/2}$

122. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

Remember

$$\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square} \text{ where } |\square| < 1$$

$|7x^2| < 1$

$7|x^2| < 1$

$|x^2| < 1/7$

$-1/7 < x^2 < 1/7$

By algebra

$x^2 < 1/7$

$x < \pm\sqrt{1/7}$

$|x| < \sqrt{1/7}$

$R =$ $\boxed{\sqrt{1/7}}$

123. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$$\frac{3}{1+2x} = \frac{3}{1}\cdot\frac{1}{1+2x} = \frac{3}{1}\cdot\frac{1}{1-(-2x)}$$

$$\frac{1}{1-(-2x)} = \sum_{n=0}^{\infty}(-2x)^n \text{ where } |-2x| < 1$$

$$f(x) = \frac{3}{1-(-2x)} = 3\sum_{n=0}^{\infty}(-2x)^n \text{ where } 2|x| < 1$$

$$= \sum_{n=0}^{\infty} 3(-1)^n 2^n x^n \text{ where } |x| < 1/2$$

$\dfrac{3}{1+2x} =$ $\sum_{n=0}^{\infty} 3(-1)^n 2^n x^n$

$R =$ $1/2$

124. Express $f(x) = \dfrac{x}{4+3x^2}$ as a power series.

$$\frac{x}{4(1+3x^2/4)} = \frac{x}{4} \cdot \frac{1}{1-(-(3x^2/4))}$$

$$\frac{1}{1-(-3x^2/4)} = \sum_{n=0}^{\infty} \left(\frac{-3x^2}{4}\right)^n$$

$$f(x) = \frac{x}{4} \cdot \frac{1}{1-(-3x^2/4)} = \frac{x}{4} \sum_{n=0}^{\infty} \left(\frac{-3x^2}{4}\right)^n$$

$$f(x) = \frac{x}{4} \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n}}{4^n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n+1}}{4^{n+1}}$$

$\dfrac{x}{4+3x^2} =$ $\boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n+1}}{4^{n+1}}}$

125. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\, dx$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\sin(x^{3/2}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x^{3/2})^{2n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}$$

$$\int \sin(x^{3/2})\, dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}\, dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{3n+3/2}\, dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{3n+5/2}}{3n+5/2}$$

$$= \frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6 \cdot (3+5/2)} + \frac{x^{17/2}}{5!\,(6+5/2)}$$

$\int \sin(x^{3/2})\, dx =$ $\boxed{\frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6 \cdot (3+5/2)} + \frac{x^{17/5}}{5!\,(6+5/2)}}$

126. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int e^{-3x}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-3x} = \sum_{n=0}^{\infty} \frac{(-3x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!}$$

$$\int e^{-3x}\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \int x^n\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \cdot \frac{x^{n+1}}{(n+1)}$$

$$= \frac{(-1)^0 3^0}{0!} \cdot \frac{x^1}{1} + \frac{(-1)^1 3^1}{1!} \cdot \frac{x^2}{2} + \frac{(-1)^2 3^2}{2!} \cdot \frac{x^3}{3}$$

$\int e^{-3x}\,dx =$ $\boxed{x - \frac{3}{2}x^2 + \frac{3}{2}x^3}$

127. Evaluate the following indefinite integral as a series. [Leave your answer as a sum.]

$$\int 5e^{5x^3}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{5x^3} = \sum_{n=0}^{\infty} \frac{(5x^3)^n}{n!} = \sum_{n=0}^{\infty} \frac{5^n x^{3n}}{n!}$$

$$5e^{5x^3} = 5\sum_{n=0}^{\infty} \frac{5^n x^{3n}}{n!} = \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} x^{3n}$$

$$\int 5e^{5x^3}\,dx = \int \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} x^{3n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \int x^{3n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \cdot \frac{x^{3n+1}}{(3n+1)}$$

$\int 5e^{5x^3}\,dx =$ $\boxed{\sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \cdot \frac{x^{3n+1}}{(3n+1)}}$

128. Use the first three terms of the powers series representation of the $f(x) = \dfrac{3x}{10+2x}$ to estimate $f(0.5)$. Round to 4 decimal places.

$$\frac{3x}{10\left(1+\frac{2}{10}x\right)} = \frac{3x}{16} \cdot \frac{1}{1-\left(-\frac{2}{10}x\right)}$$

$$\frac{1}{1-\left(-\frac{2}{10}x\right)} = \sum_{n=0}^{\infty} \left(-\frac{2}{10}x\right)^n$$

$$f(x) = \frac{3x}{10} \cdot \frac{1}{1-\left(-\frac{2}{10}x\right)} = \frac{3x}{10} \sum_{n=0}^{\infty} \left(-\frac{2}{10}x\right)^n$$

$$f(x) = \frac{3x}{10} \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^n}{10^n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^n \cdot 3^1 x^{n+1}}{10^{n+1}}$$

$$f(0.5) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n \cdot 3 \cdot (0.5)^{n+1}}{10^{n+1}}$$

$$= \frac{3(0.5)}{10} - \frac{2 \cdot 3(0.5)^2}{10^2} + \frac{2^2 \cdot 3(0.5)^3}{10^3}$$

$$\approx 0.1365$$

$f(0.5) \approx$ $\boxed{0.1365}$

129. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5+x^6}\,dx$$

$$\frac{x}{5+x^6} = \frac{x}{5-(-x^6)} = \frac{x}{5[1-(-x^6/5)]} = \frac{x}{5} \cdot \frac{1}{1-(-x^6/5)}$$

$$\frac{1}{1-(-x^6/5)} = \sum_{n=0}^{\infty} \left(-\frac{x^6}{5}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{5^n}$$

$$\frac{x}{5} \cdot \frac{1}{1-(-x^6/5)} = \frac{x}{5} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{5^n} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{5^{n+1}}$$

$$\int_0^{0.24} \frac{x}{5+x^6}\,dx = \int_0^{0.24} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{5^{n+1}}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}} \int_0^{0.24} x^{6n+1}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}} \cdot \frac{x^{6n+2}}{(6n+2)}\Bigg]_0^{0.24}$$

$$= \left(\frac{1}{5} \cdot \frac{x^2}{2} - \frac{1}{5^2} \cdot \frac{x^8}{8} + \frac{1}{5^3} \cdot \frac{x^{14}}{14}\right)\Bigg]_0^{0.24}$$

$$\approx 0.00576$$

$\int_0^{0.24} \frac{x}{5+x^6}\,dx \approx$ $\boxed{0.00576}$

130. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\, dx$$

$$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = \sum_{n=0}^{\infty} (-x^4)^n = \sum_{n=0}^{\infty} (-1)^n x^{4n}$$

$$\int_0^{0.11} \frac{1}{1+x^4} dx = \int_0^{0.11} \sum_{n=0}^{\infty} (-1)^n x^{4n} dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \int_0^{0.11} x^{4n} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{4n+1}\Bigg]_0^{0.11}$$

$$= \left(x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13}\right)\Bigg]_0^{0.11}$$

$\int_0^{0.11} \frac{1}{1+x^4}\, dx \approx$ $\boxed{0.11000}$

131. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\, dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$$

$$\int_0^{0.23} e^{-x^2} dx = \int_0^{0.23} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^{0.23} x^{2n} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{2n+1}}{2n+1}\Bigg]_0^{0.23}$$

$$= \left(\frac{x}{0!} - \frac{x^3}{1!(3)} + \frac{x^5}{2!(5)}\right)\Bigg]_0^{0.23}$$

$$= \left(x - \frac{x^3}{3} + \frac{x^5}{10}\right)\Bigg]_0^{0.23}$$

$\int_0^{0.23} e^{-x^2}\, dx \approx$ $\boxed{0.226}$

132. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}(x^{1/2})^{2n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$f(x) = 4x\cos(\sqrt{x}) = 4x\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1}$$

$$\int_0^{0.45} 4x\cos(\sqrt{x})dx = \int_0^{0.45} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\int_0^{0.45} x^{n+1} dx$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\frac{x^{n+2}}{n+2}\Bigg]_0^{0.45}$$

$$= \left(\frac{4x^2}{0!(2)} - \frac{4x^3}{2!(3)} + \frac{4x^4}{4!(4)} - \frac{4x^5}{6!(5)}\right)\Bigg]_0^{0.45}$$

$$= \left(2x^2 - \frac{2x^3}{3} + \frac{x^4}{24} - \frac{x^5}{900}\right)\Bigg]_0^{0.45}$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ $\boxed{0.34593}$

133. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1 + x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

Note $1.56 = 1 + 0.56$

$$\ln(1+0.56) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}(0.56)^n = 0.56 - \frac{(0.56)^2}{2} + \frac{(0.56)^3}{3}$$

$\ln(1.56) \approx$ $\boxed{0.46174}$

134. Use the first 4 terms of the Macluarin series for $f(x) = \sin(x)$ to evaluate $\sin(0.75)$. Round to 5 decimal places.

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin(0.75) = \sum_{n=0}^{\infty} \frac{(-1)^n (0.75)^{2n+1}}{(2n+1)!} = \frac{0.75}{1!} - \frac{(0.75)^3}{3!} + \frac{(0.75)^5}{5!} - \frac{(0.75)^7}{7!}$$

$\sin(0.75) \approx$ $\boxed{0.74631}$

135. Given $f(x, y) = 3x^3y^2 - x^2y^{1/3}$, evaluate $f(3, -8)$.

$$f(3,-8) = 3(3)^3(-8)^2 - (3)^2(-8)^{1/3}$$

$f(3, -8) =$ $\boxed{5202}$

136. Find the domain of

$$f(x, y) = \frac{-5x}{\sqrt{x + 9y + 1}}$$

$$\frac{1}{\sqrt{?}} \rightarrow ? > 0$$

$$x + 9y + 1 > 0$$

Domain = $\{(x, y) \mid x + 9y + 1 > 0\}$

137. Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

$\sqrt{?} \to ? \geq 0$

$\sqrt{x+y-1} \to x+y-1 \geq 0$

$x+y \geq 1$

$\ln(?) \to ? > 0$

$\ln(y-11) \to y-11 > 0$

$y > 11$

$\frac{1}{?} \to ? \neq 0$

$\ln(y-11)-9 \neq 0$

$\ln(y-11) \neq 9$

$y-11 \neq e^9$

$y \neq e^9+11$

Domain = $\boxed{\{(x,y) \mid x+y \geq 1,\ y > 11,\ y \neq 11+e^9\}}$.

138. Find the domain of

$$f(x,y) = \frac{\ln(x^2-y+3)}{\sqrt{x-6}}$$

$\ln(?) \to ? > 0$

$\ln(x^2-y+3) \to x^2-y+3 > 0$

$x^2+3 > y$

$\frac{1}{\sqrt{?}} \to ? > 0$

$\frac{1}{\sqrt{x-6}} \to x-6 > 0$

$x > 6$

Domain = $\boxed{\{(x,y) \mid x > 6,\ x^2+3 > y\}}$

139. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \quad C = \ln(36)$$

$\ln(x^2+y^2)=\ln(36)$

$x^2+y^2=36$

$x^2+y^2=6^2$

(a) Parabola with vertices at $(0, 0)$

(b) Circle with center at $(0, \ln(36))$ and radius 6

(c) Parabola with vertices at $(0, \ln(36))$

(d) Circle with center at $(0, 0)$ and radius 6

(e) Increasing Logarithm Function

140. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

$\ln(y-e^{5x})=C$

$y-e^{5x}=e^C$

$y-e^{5x}=C$

$y=e^{5x}+C$

(a) Increasing exponential functions

(b) Rational Functions with x-axis symmetry

(c) Natural logarithm functions

(d) Decreasing exponential functions

(e) Rational Functions with y-axis symmetry

141. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

$\sqrt{x^2+y^2}=C$

$x^2+y^2=C^2$

(a) Lines

(b) Parabolas

(c) Circles

(d) Point at the origin

(e) Ellipses

(f) Hyperbolas

142. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

$\cos(y+4x^2)=C$

$y+4x^2=\cos^{-1}(C)$

$y+4x^2=C$

$y=-4x^2+C$

(a) Lines

(b) Parabolas

(c) Circles

(d) Point at the origin

(e) Ellipses

(f) Hyperbolas

143. For the following function $f(x, y)$, evaluate $f_y(-2, -3)$.

$$f(x, y) = 8x^4y^5 + 3x^3 - 12y^2$$

$$f_y(x,y) = \frac{d}{dy}\left(8x^4y^5 + 3x^3 - 12y^2\right)$$

$$= 8x^4\frac{d}{dy}(y^5) + 3x^3\frac{d}{dy}(1) - \frac{d}{dy}(12y^2)$$

$$= (8x^4)(5y^4) + (3x^3)(0) - 24y$$

$$= 40x^4y^4 - 24y$$

$$f_y(-2,-3) = 40(-2)^4(-3)^4 - 24(-3)$$

$$= 51912$$

$f_y(-2, -3) =$ $\boxed{51912}$

144. Compute $f_x(6, 5)$ when

$$f(x, y) = \frac{(6x - 6y)^2}{\sqrt{y^2 - 1}}$$

$$f_x(x,y) = \frac{d}{dx}\left(\frac{(6x-6y)^2}{\sqrt{y^2-1}}\right)$$

$$= \frac{1}{\sqrt{y^2-1}}\frac{d}{dx}\left((6x-6y)^2\right)$$

$$= \frac{1}{\sqrt{y^2-1}} \cdot 2(6x-6y)\frac{d}{dx}(6x+6y)$$

$$= \frac{1}{\sqrt{y^2-1}} \cdot 2(6x-6y) \cdot 6$$

$$= \frac{72x-72y}{\sqrt{y^2-1}}$$

$f_x(6, 5)=$ $\boxed{72/\sqrt{24}}$

145. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$$f(x,y) = 3x^2 \cdot \frac{y^3}{(y-1)^2}$$

$$f_x(x,y) = \frac{d}{dx}\left(3x^2 \cdot \frac{y^3}{(y-1)^2}\right) = \frac{y^3}{(y-1)^2} \cdot \frac{d}{dx}\left(3x^2\right) = \frac{y^3}{(y-1)^2} \cdot 6x$$

$$f_y(x,y) = \frac{d}{dy}\left(3x^2 \cdot \frac{y^3}{(y-1)^2}\right) = 3x^2 \frac{d}{dy}\left(\frac{y^3}{(y-1)^2}\right) = 3x^2\left(\frac{3y^2(y-1)^2 - y^3 \cdot 2(y-1)}{(y-1)^4}\right)$$

$$= 3x^2\left(\frac{\cancel{(y-1)}\left[3y^2(y-1) - 2y^3\right]}{(y-1)^{\cancel{4}\,3}}\right) = \frac{3x^2(3y^3 - 3y^2 - 2y^3)}{(y-1)^3}$$

$$= \frac{3x^2(y^3 - 3y^2)}{(y-1)^3}$$

$f_x(x,y) =$ $\boxed{6xy^3/(y-1)^2}$

$f_y(x,y) =$ $\boxed{\dfrac{3x^2(y^3-3y^2)}{(y-1)^3}}$

146. Find the first order partial derivatives of

$$f(x,y) = x\sin(xy)$$

$$f_x(x,y) = \frac{d}{dx}(x\sin(xy)) = \frac{d}{dx}(x)\sin(xy) + x\frac{d}{dx}(\sin(xy))$$

$$= \sin(xy) + x\cos(xy)\frac{d}{dx}(xy)$$

$$= \sin(xy) + x \cdot y\cos(xy)$$

$$f_y(x,y) = \frac{d}{dy}(x\sin(xy)) = x\frac{d}{dy}(\sin(xy))$$

$$= x\cos(xy)\frac{d}{dy}(xy)$$

$$= x^2\cos(xy)$$

$f_x(x,y) =$ $\boxed{\sin(xy) + xy\cos(xy)}$

$f_y(x,y) =$ $\boxed{x^2\cos(xy)}$

147. Find the first order partial derivatives of $f(x, y) = (xy - 1)^2$

$$f_x(x,y) = \frac{d}{dx}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dx}(xy-1)$$
$$= 2(xy-1)\,y$$
$$= 2xy^2 - 2y$$

$$f_y(x,y) = \frac{d}{dy}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dy}(xy-1)$$
$$= 2(xy-1)\,x$$
$$= 2x^2y - 2x$$

$f_x(x, y) =$ $\boxed{2xy^2 - 2y}$

$f_y(x, y) =$ $\boxed{2x^2y - 2x}$

148. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$$f_x(x,y) = \frac{d}{dx}(x)\,e^{x^2+xy+y^2} + x\frac{d}{dx}\left(e^{x^2+xy+y^2}\right)$$
$$= e^{x^2+xy+y^2} + x\left(e^{x^2+xy+y^2}\right)(2x+y)$$
$$= (1+2x^2+xy)e^{x^2+xy+y^2}$$

$$f_y(x,y) = x\frac{d}{dy}\left(e^{x^2+xy+y^2}\right) = x\left(e^{x^2+xy+y^2}\right)(x+2y)$$
$$= (x^2+2xy)e^{x^2+xy+y^2}$$

$f_x(x, y) =$ $\boxed{(1+2x^2+xy)e^{x^2+xy+y^2}}$

$f_y(x, y) =$ $\boxed{(x^2+2xy)e^{x^2+xy+y^2}}$

149. Find the first order partial derivatives of $f(x, y) = -7\tan(x^7y^8)$

$$f_x(x,y) = -7\frac{d}{dx}(\tan(x^7y^8)) = -7\sec^2(x^7y^8)\frac{d}{dx}(x^7y^8)$$
$$= -7\cdot 7x^6y^8\sec^2(x^7y^8) = -49x^6y^8\sec^2(x^7y^8)$$

$$f_y(x,y) = -7\frac{d}{dy}(\tan(x^7y^8)) = -7\sec^2(x^7y^8)\frac{d}{dy}(x^7y^8)$$
$$= -7\cdot 8x^7y^7\sec^2(x^7y^8)$$
$$= -56x^7y^7\sec^2(x^7y^8)$$

$f_x(x, y) =$ $-49x^6y^8\sec^2(x^7y^8)$

$f_y(x, y) =$ $-56x^7y^7\sec^2(x^7y^8)$

150. Find the first order partial derivatives of $f(x, y) = y\cos(x^2y)$

$$f_x(x,y) = y\frac{d}{dx}(\cos(x^2y)) = y(-\sin(x^2y))\frac{d}{dx}(x^2y) = -y\sin(x^2y)[2xy]$$
$$= -2xy^2\sin(x^2y)$$

$$f_y(x,y) = \frac{d}{dy}(y)\cos(x^2y) + y\frac{d}{dy}(\cos(x^2y))$$
$$= \cos(x^2y) + y(-\sin(x^2y))\frac{d}{dy}(x^2y)$$
$$= \cos(x^2y) - y\sin(x^2y)[x^2]$$
$$= \cos(x^2y) - x^2y\sin(x^2y)$$

$f_x(x, y) =$ $-2xy^2\sin(x^2y)$

$f_y(x, y) =$ $\cos(x^2y) - x^2y\sin(x^2y)$

151. Find the first order partial derivatives of $f(x,y) = xe^{xy}$

$$f_x = \frac{\partial}{\partial x}(xe^{xy}) = \frac{\partial}{\partial x}(x)e^{xy} + x\frac{\partial}{\partial x}(e^{xy})$$
$$= e^{xy} + xe^{xy}\frac{\partial}{\partial x}(xy)$$
$$= e^{xy} + xe^{xy}(y)$$
$$= e^{xy}(1+xy)$$

$$f_y = \frac{\partial}{\partial y}(xe^{xy}) = x\frac{\partial}{\partial y}(e^{xy})$$
$$= xe^{xy}\frac{\partial}{\partial y}(xy)$$
$$= xe^{xy}\cdot x$$
$$= x^2e^{xy}$$

$f_x(x,y) = \boxed{e^{xy}(1+xy)}$

$f_y(x,y) = \boxed{x^2e^{xy}}$

152. Given the function $f(x,y) = x^3y^2 - 3x + 5y - 5x^2y^3$, compute $f_{xx}(x,y)$

$$f_x = \frac{\partial}{\partial x}(x^3y^2 - 3x + 5y - 5x^2y^3)$$
$$= 3x^2y^2 - 3 + 0 - 10xy^3$$

$$f_{xx} = \frac{\partial}{\partial x}(3x^2y^2 - 3 - 10xy^3)$$
$$= 6xy^2 + 0 - 10y^3$$

$f_{xx}(x,y) = \boxed{6xy^2 - 10y^3}$

153. Given the function $f(x, y) = 4x^5 \tan(3y)$, compute $f_{xy}(2, \pi/3)$

$$f_x(x,y) = \frac{d}{dx}\left(4x^5 \tan(3y)\right) = \tan(3y) \cdot \frac{d}{dx}(4x^5)$$
$$= \tan(3y) \cdot (20x^4)$$

$$f_{xy}(x,y) = \frac{d}{dy}(f_x(x,y)) = \frac{d}{dy}(\tan(3y) \cdot (20x^4)) = 20x^4 \frac{d}{dy}(\tan(3y))$$
$$= 20x^4 \cdot \sec^2(3y) \cdot 3$$
$$= 60x^4 \sec^2(3y)$$

$$f_{xy}(2, \pi/3) = 60(2)^4 \sec^2(3\pi/3)$$
$$= 60(16)\sec^2(\pi)$$
$$= 960$$

$f_{xy}(2, \pi/3) =$ $\boxed{960}$

154. Given the function $f(x, y) = x^3 \sin(y)$, compute $f_{xy}(2, 0)$

$$f_x = \frac{\partial}{\partial x}\left(x^3 \sin(y)\right) = \sin(y) \frac{\partial}{\partial x}(x^3) = 3x^2 \sin(y)$$

$$f_{xy} = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}\left(3x^2 \sin(y)\right) = 3x^2 \frac{\partial}{\partial y}(\sin(y))$$

$$= 3x^2 \cos(y)$$

$$f_{xy}(2,0) = 3(2)^2 \cos(0) = 12$$

$f_{xy}(2, 0) =$ $\boxed{12}$

155. Find the second order partial derivatives of

$$f(x, y) = x^2 y \ln(7x)$$

$f(x,y) = (x^2 \ln(7x))\, y$

$f_x(x,y) = \frac{d}{dx}\left((x^2 \ln(7x)) \cdot y\right) = y \frac{d}{dx}\left(x^2 \ln(7x)\right)$

$= y\left(2x\ln(7x) + x^2 \frac{1}{7x} \cdot 7\right) = y(2x\ln(7x) + x)$

$f_{xx}(x,y) = \frac{d}{dx}\left(y(2x\ln(7x) + x)\right) = y \frac{d}{dx}(2x\ln(7x) + x)$

$= y\left(2\ln(7x) + 2x \cdot \frac{1}{7x} \cdot 7 + 1\right) = y(2\ln(7x) + 2 + 1)$

$= y(2\ln(7x) + 3)$

$f_{xy}(x,y) = \frac{d}{dy}\left(y(2x\ln(7x) + x)\right) = (2x\ln(7x) + x)\frac{d}{dy}(y)$

$= 2x\ln(7x) + x$

$f_y(x,y) = \frac{d}{dy}\left((x^2 \ln(7x)) \cdot y\right) = (x^2 \ln(7x)) \frac{d}{dy}(y) = x^2 \ln(7x)$

$f_{yy}(x,y) = \frac{d}{dy}(x^2 \ln(7x)) = 0$

$f_{xx}(x, y) =$ $(2\ln(7x) + 3)y$

$f_{xy}(x, y) =$ $2x\ln(7x) + x$

$f_{yy}(x, y) =$ 0

156. A function $f(x, y)$ has 2 critical points. The partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 8x - 16y \quad \text{and} \quad f_y(x, y) = 8y^2 - 16x$$

One of the critical points is $(0, 0)$. Find the second critical point of $f(x, y)$.

$$\begin{cases} 8x - 16y = 0 & (1) \\ 8y^2 - 16x = 0 & (2) \end{cases}$$

Solve (1) for x.

$8x = 16y$

$x = 2y$

Plug $x = 2y$ into (2).

$8y^2 - 16(2y) = 0$

$8y^2 - 32y = 0$

$8y(y-4) = 0$

$y = 0, 4$

Plug $y = 0, 4$ into $x = 2y$.

$y = 0 \to x = 0 \to (0,0)$

$y = 4 \to x = 8 \to (8,4)$

$(a, b) =$ $\boxed{(8,4)}$

157. Find the discriminant of

$$f(x, y) = e^x \sin(y)$$

Simplify your answer. Note: $\sin^2(y) + \cos^2(y) = 1$.

$f_x(x,y) = e^x \sin(y)$

$f_{xx}(x,y) = e^x \sin(y)$

$f_{xy}(x,y) = e^x \cos(y)$

$f_y(x,y) = e^x \cos(y)$

$f_{yy}(x,y) = -e^x \sin(y)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= (e^x \sin(y))(-e^x \sin(y)) - (e^x \cos(y))^2$

$= -e^{2x} \sin^2(y) - e^{2x} \cos^2(y)$

$= -e^{2x}(\sin^2(y) + \cos^2(y))$

$= -e^{2x}(1)$

$D(x, y) =$ $\boxed{-e^{2x}}$

158. Using the information in the table below, classify the critical points for the function $g(x, y)$.

(a,b)	$g_{xx}(a,b)$	$g_{yy}(a,b)$	$g_{xy}(a,b)$
$(4,5)$	0	4	-2
$(5,-10)$	5	-10	6
$(10,10)$	-4	-6	-4
$(7,9)$	5	7	4
$(4,8)$	2	2	2

- (4,5) is saddle pt
- (5,-10) is saddle pt
- (10,10) is relative max
- (7,9) is relative min
- (4,8) is inconclusive

$D(4,5) = (0)(4) - (-2)^2 = -4 < 0$
$\rightarrow$ saddle pt

$D(5,-10) = (5)(-10) - 6^2 = -86 < 0$
$\rightarrow$ saddle pt

$D(10,10) = (-4)(-6) - (-4)^2 = 8 > 0$
$\rightarrow$ relative
$g_{xx} = -4 < 0 \rightarrow$ max

$D(7,9) = (5)(7) - (4)^2 = 19 > 0$
$\rightarrow$ relative
$g_{xx} = 5 > 0 \rightarrow$ min

$D(4,8) = (2)(2) - 2^2 = 0 \rightarrow$ Inconclusive

159. Given the information below, which critical point(s) (a, b) would be classified as a relative maximum?

(a,b)	$f_{xx}(a,b)$	$f_{yy}(a,b)$	$f_{xy}(a,b)$
$(7,8)$	-5	-5	10
$(-8,-1)$	-4	-7	-2
$(1,7)$	-10	-1	6

$D(7,8) = (-5)(-5) - 10^2 < 0 \rightarrow$ saddle pt

$D(-8,-1) = (-4)(-7) - (-2)^2 > 0 \rightarrow$ relative extrema

$f_{xx}(-8,-1) < 0 \rightarrow$ relative min

$D(1,7) = (-10)(-1) - 6^2 < 0 \Rightarrow$ saddle pt

Answer: $(-8,-1)$

160. Classify the critical points of the function $f(x, y)$ given the partial derivatives:

$$f_x(x, y) = x - y \qquad f_y(x, y) = y^3 - x$$

(a) Two saddle points and one local minimum

(b) Two saddle points and one local maximum

(c) One saddle point, one local maximum, and one local minimum

(d) Three saddle points

(e) Two local minimums and one saddle point

$f_x = 0$, $x - y = 0$, $x = y$

$f_y = 0$, $y^3 - x = 0$, $y^3 = x$

$\begin{cases} x = y \\ y^3 = x \end{cases} \Rightarrow y = y^3$, $y - y^3 = 0$, $y(1 - y^2) = 0$, $y = 0, \pm 1$

$f_x = x - y$, $f_{xx} = 1$, $f_{xy} = -1$

$f_y = y^3 - x$, $f_{yy} = 3y^2$

$D = f_{xx} f_{yy} - (f_{xy})^2 = (1)(3y^2) - (-1)^2 = 3y^2 - 1$

Note we don't need to find the x-values b/c D which we found on the left only has y's.

When $y = 0$, $D = -1 < 0 \rightarrow$ saddle

When $y = -1$, $D = 2 > 0 \rightarrow$ rel extrema

When $y = +1$, $D = 2 > 0 \rightarrow$ rel extrema

Check $f_{xx} = 1 > 0 \rightarrow$ rel mins @ $y = \pm 1$

Answer:

161. The critical points for a function $f(x, y)$ are (0,0) and (8,4). Given that the partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 3x - 6y \qquad f_y(x, y) = 3y^2 - 6x$$

Classify each critical point as a maximum, minimum, or saddle point.

	$f_{xx} = 3$	$f_{yy} = 6y$	$f_{xy} = -6$	D
(0,0)	3	0	−6	−36
(8,4)	3	24	−6	36

$D(0,0) < 0 \rightarrow$ saddle pt

$D(8,4) > 0$ and $f_{xx}(8,4) > 0$ → rel min

(0,0) is saddle pt

(8,4) is rel min

162. Find all local maximum and minimum points of

$$f(x,y) = 4x^2 - xy + 8y^2 - 46x - 26y + 11$$

$\begin{cases} f_x = 8x - y - 46 = 0 & ① \\ f_y = -x + 16y - 26 = 0 & ② \end{cases}$

Multiply ② by 8. Then add

$8\cancel{x} - y + 46 = 0$

$-\cancel{8}x + 128y - 208 = 0$

$127y - 162 = 0$

$y = \frac{162}{127}$

Plug $y = \frac{162}{127}$ into ①

$8x - \frac{162}{127} - 46 = 0$

$x = \frac{1501}{254}$

$\left.\begin{array}{l} f_{xx} = 8 \\ f_{xy} = -1 \\ f_{yy} = 16 \end{array}\right\} \to D = f_{xx}f_{yy} - (f_{xy})^2$

$= 8(16) - (-1)^2 > 0$

and $f_{xx} = 8 > 0$

For all pts. So we have only rel min

$\underbrace{\text{Critical Pt}}$

$\left(\frac{1501}{254}, \frac{162}{127}\right)$

Local max at None

Local min at $\left(\frac{1501}{254}, \frac{162}{127}\right)$

163. Fleet feet stores two most sold running shoes brands are Aesics and Brookes. The total venue from selling x pairs of Aesics and y pairs of Brookes is given by

$$R(x,y) = -10x^2 - 16y^2 - 4xy + 84 + 204y$$

where x and y are in **thousands of units**. Determine the number of Brookes shoes to be sold to maximize the revenue.

First find the critical pts.

$\begin{cases} R_x = -20x - 4y = 0 & ① \\ R_y = -32y - 4x + 204 = 0 & ② \end{cases}$

Divide ① and ② by -4.

$\begin{cases} 5x + y = 0 & ① \\ x + 8y - 51 = 0 & ② \end{cases}$

$\Rightarrow \begin{cases} 5x + y = 0 & ① \\ x + 8y = 51 & ② \end{cases}$

Multiply ② by 5.

$\Rightarrow \begin{cases} 5x + y = 0 & ① \\ 5x + 40y = 255 & ② \end{cases}$

Subtract ① and ②

$-39y = -255$

$y \approx 6.5$

$\Rightarrow y = 7$

The # of Brookes shoes sold is $\boxed{7000}$

164. Find the point(s) (x, y) where the function $f(x, y) = 3x^2 + 4xy + 6x - 15$ attains maximal value, subject to the constraint $x + y = 10$.

$f = 3x^2 + 4xy + 6x - 15$

$f_x = 6x + 4y + 6$

$f_y = 4x$

$g = x + y = 10$

$g_x = 1$

$g_y = 1$

System
$$\begin{cases} 6x + 4y + 6 = \lambda & (1) \\ 4x = \lambda & (2) \\ x + y = 10 & (3) \end{cases}$$

Set (1) = (2)

$6x + 4y + 6 = 4x$

$2x + 4y + 6 = 0$

$2x = -4y - 6$

$x = -2y - 3$

Plug $x = -2y - 3$ into (3)

$x + y = 10$

$-2y - 3 + y = 10$

$-y - 3 = 10$

$-y = 13$

$y = -13$

Plug $y = -13$ into $x = -2y - 3$.

$x = -2(-13) - 3$

$= 26 - 3$

$= 23$

(x,y) = $\boxed{(23, -13)}$

165. Find the maximum of the function using LaGrange Multipliers of the function $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$.

$f = x^2 + 2y^2$

$f_x = 2x$

$f_y = 4y$

$g = x^2 + y^2 = 1$

$g_x = 2x$

$g_y = 2y$

System:
$$\begin{cases} 2x = 2x\lambda & (1) \\ 4y = 2y\lambda & (2) \\ x^2 + y^2 = 1 & (3) \end{cases}$$

Solve (1).

$2x = 2x\lambda$

$2x - 2x\lambda = 0$

$2x(1 - \lambda) = 0$

$x = 0, \lambda = 1$

Plug $x = 0$ into (3)

$0^2 + y^2 = 1$

$y = \pm 1$

Pts: $(0, 1), (0, -1)$

Plug $\lambda = 1$ into (2)

$4y = 2y$

only true when $y = 0$

Plug $y = 0$ into (3)

$x^2 + 0^2 = 1$

$x = \pm 1$

Pts: $(1, 0), (-1, 0)$

Now plug the pts into $f(x, y) = x^2 + 2y^2$

$\left.\begin{array}{l} f(0,1) = 2 \\ f(0,-1) = 2 \end{array}\right\} \to$ max

$\left.\begin{array}{l} f(1,0) = 1 \\ f(-1,0) = 1 \end{array}\right\} \to$ Min

Maximum Value = $\boxed{2}$

166. Find the minimum value of the function $f(x,y) = 2x^2y - 3y^2$ subject to the constraint $x^2 + 2y = 1$.

$f = 2x^2y - 3y^2$ $\quad g = x^2 + 2y = 1$

$f_x = 4xy$ $\quad g_x = 2x$

$f_y = 2x^2 - 6y$ $\quad g_y = 2$

System
$$\begin{cases} 4xy = 2x\lambda & (1) \\ 2x^2 - 6y = 2\lambda & (2) \\ x^2 + 2y = 1 & (3) \end{cases}$$

Solve (1)

$4xy - 2x\lambda = 0$

$2x(2y - \lambda) = 0$

$x = 0, \ \lambda = 2y$

Plug $x=0$ into (3)

$0^2 + 2y = 1$

$y = 1/2$

Pts: $(0, 1/2)$

Plug $\lambda = 2y$ into (2)

$2x^2 - 6y = 2(2y)$

$2x^2 - 6y = 4y$

$2x^2 = 10y$

$x^2 = 5y$

Plug $x^2 = 5y$ into (3)

$5y + 2y = 1$

$7y = 1$

$y = 1/7$

Plug $y = 1/7$ into $x^2 = 5y$

$x^2 = \frac{5}{7}$

$x = \pm\sqrt{\frac{5}{7}}$

Pts: $\left(\sqrt{\frac{5}{7}}, \frac{1}{7}\right), \left(-\sqrt{\frac{5}{7}}, \frac{1}{7}\right)$

Test for Min

$f(0, 1/2) = -3/4$

$f\left(\pm\sqrt{\frac{5}{7}}, \frac{1}{7}\right) = \frac{1}{7}$

Minimum Value = $\boxed{-3/4}$

167. Locate and classify the points that maximize and minimize the function $f(x,y) = 5x^2 + 10y$ subject to the constraint $5x^2 + 5y^2 = 5$.

$f = 5x^2 + 10y$ $\quad g = 5x^2 + 5y^2 = 5$

$f_x = 10x$ $\quad g_x = 10x$

$f_y = 10$ $\quad g_y = 10y$

System:
$$\begin{cases} 10x = 10x\lambda & (1) \\ 10 = 10y\lambda & (2) \\ 5x^2 + 5y^2 = 5 & (3) \end{cases}$$

Solve (1)

$10x - 10x\lambda = 0$

$10x(1 - \lambda) = 0$

$x = 0, \ \lambda = 1$

Plug $x = 0$ into (3)

$5y^2 = 5$

$y^2 = 1$

$y = \pm 1$

Pts: $(0,1)(0,-1)$

Plug $\lambda = 1$ into (2)

$10 = 10y$

$y = 1$

Plug $y = 1$ into (3)

$5x^2 + 5 = 5$

$5x^2 = 0$

$x = 0$

Pt: $(0,1)$ again

Test w/ $f(x,y)$

$f(0,-1) = -10$

$f(0,1) = 10$

Minimum Value occurs at -10

Maximum Value occurs at 10

9
11

168. Find the maximum value of the function $f(x, y) = 8x - 11y^2$ subject to the constraint $x^2 + 11y^2 = 25$.

$f_x = 8 \quad g_x = 2x$

$f_y = -22y \quad g_y = 22y$

$$\begin{cases} 8 = 2x\lambda & (1) \\ -22y = 22y\lambda & (2) \\ x^2 + 11y^2 = 25 & (3) \end{cases}$$

Solve (1)

$-22y = 22y\lambda$

$0 = 22y\lambda + 22y$

$0 = 22y(\lambda + 1)$

$y = 0, \lambda = -1$

Plug $y=0$ into (3)

$x^2 + 0 = 25$

$x = \pm 5$

Critical Pt: $(5,0), (-5,0)$

Plug $\lambda = -1$ into (1)

$8 = -2x$

$x = -4$

Plug $x = -4$ into (3)

$16 + 11y^2 = 25$

$11y^2 = 9$

$y^2 = \frac{9}{11}$

$y = \pm\sqrt{\frac{9}{11}}$

Critical Pt: $\left(-4, \sqrt{\frac{9}{11}}\right), \left(-4, -\sqrt{\frac{9}{11}}\right)$

$f(5,0) = 40 \rightarrow$ max

$f(-5,0) = -40$

$f\left(-4, \sqrt{\frac{9}{11}}\right) = -49$

$f\left(-4, -\sqrt{\frac{9}{11}}\right) = -49$

Max value is $\boxed{40}$

169. A factory can produce a chocolate bar with a weight of $W(x, y) = \frac{xy}{100}$ with the weight W in ounces and x and y are the percentages of cocoa and sugar respectively. The percentage of cocoa and sugar are constrained to $2x + y = 75$. What is the weight, in ounces, of the largest chocolate bar that can be produced? Round to 2 decimal places.

$W(x,y) = xy/100 \quad g(x,y) = 2x + y = 75$

$W_x = y/100 \quad g_x = 2$

$W_y = x/100 \quad g_y = 1$

$$\begin{cases} \frac{y}{100} = 2\lambda & (1) \\ \frac{x}{100} = \lambda & (2) \\ 2x + y = 75 & (3) \end{cases}$$

Plug (2) into (1).

$\frac{y}{100} = \frac{2x}{100}$

$y = 2x$

Plug $y = 2x$ into (3)

$2x + 2x = 75$

$4x = 75$

$x = 18.75$

Plug $x = 18.75$ into

$y = 2x$

$y = 37.5$

$W(18.75, 37.5) = 7.03$

Weight of Largest Chocolate Bar = $\boxed{7.03}$

170. We are baking a tasty treat where customer satisfaction is given by $S(x, y) = 6x^{3/2}y$. Here, x and y are the amount of sugar and spice respectively. If the sugar and spice we use must satisfy $9x + y = 4$, what is the maximum customer satisfaction we can achieve? (Note: the function is defined only for $x \geq 0$ and $y \geq 0$.) Round your answer to 2 decimal places.

$S = 6x^{3/2}y$ $\quad g = 9x + y = 4$

$S_x = 9x^{1/2}y$ $\quad g_x = 9$

$S_y = 6x^{3/2}$ $\quad g_y = 1$

System: $\begin{cases} 9x^{1/2}y = 9\lambda & ① \\ 6x^{3/2} = \lambda & ② \\ 9x^2 + y^2 = 4 & ③ \end{cases}$

Plug ② in ①

$\cancel{9}x^{1/2}y = \cancel{9}(6x^{3/2})$

$x^{1/2}y = 6x^{3/2}$

$x^{1/2}y - 6x^{3/2} = 0$

$x^{1/2}(y - 6x) = 0$

$x = 0, \quad y = 6x$

Plug $x=0$ into ③

$0 + y = 4$

Pt: $(0,4)$

Plug $y=6x$ into

$9x + 6x = 4$

$15x = 4$

$x = \frac{4}{15}$

Plug $x = \frac{4}{15}$ into $y = 6x$

$y = \frac{8}{5}$

Pt: $\left(\frac{4}{15}, \frac{8}{5}\right)$

Test for max

$S(0,4) = 0$

$S\left(\frac{4}{15}, \frac{8}{5}\right) \approx 1.32$ ← max

Maximum Value = $\boxed{1.32}$

171. A customer has \$280 to spend on two items, Item A, which costs \$2 per unit, and Item B, which costs \$5 per unit. If the enjoyment of each item by the customer is given by $f(A, B) = 100AB^3$, how many of each unit should be purchase to maximize the enjoyment of the customer?

$f = 100AB^3$

$g = 2A + 5B = 280$

$f_A = 100B^3$ $\quad g_A = 2$

$f_B = 300AB^2$ $\quad g_B = 5$

$\begin{cases} 100B^3 = 2\lambda & ① \\ 300AB^2 = 5\lambda & ② \\ 2A + 5B = 280 & ③ \end{cases}$

Simplify ① and ②

$\begin{cases} 50B^3 = \lambda & ① \\ 60AB^2 = \lambda & ② \\ 2A + 5B = 280 & ③ \end{cases}$

Set ① = ②

$50B^3 = 60AB^2$

$50B^3 - 60AB^2 = 0$

$10B^2(5B - 6A) = 0$

$B = 0, \quad B = \frac{6A}{5}$

Plug $B=0$ into ③

$2A + 0 = 280$

$A = 140$

Plug $B = \frac{6A}{5}$ into ③

$2A + 5\left(\frac{6A}{5}\right) = 280$

$2A + 6A = 280$

$8A = 280$

$A = 35$

So $B = \frac{6}{5} \cdot 35 = 42$

$f(140, 0) = 0$

$f(35, 42) = 259308000$

Units of A: $\boxed{35}$

Units of B: $\boxed{42}$

172. Evaluate the following double integral.

$$\int_0^2 \int_0^3 (x+y)\, dy\, dx$$

$$= \int_0^2 \left(xy + \frac{y^2}{2} \right) \Big]_0^3 dx$$

$$= \int_0^2 \left(3x + \frac{9}{2} \right) dx$$

$$= \left(\frac{3x^2}{2} + \frac{9}{2} x \right) \Big]_0^2$$

$$= 15$$

$$\int_0^2 \int_0^3 (x+y)\, dy\, dx = \boxed{15}$$

173. Evaluate the double integral

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x)\, dy\, dx$$

$$\int_0^{\pi/3} \sec^2(x) \left(\int_0^2 25y^4\, dy \right) dx$$

$$= \int_0^{\pi/3} \sec^2(x) \left(5y^5 \Big]_0^2 \right) dx$$

$$= \int_0^{\pi/3} \sec^2(x)\, (20)\, dx$$

$$= 20 \int_0^{\pi/3} \sec^2(x)\, dx$$

$$= 20 \tan x \Big]_0^{\pi/3}$$

$$= 20\sqrt{3}$$

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x)\, dy\, dx = \boxed{20\sqrt{3}}$$

174. Evaluate the double integral

$$\int_0^{\pi/2}\int_0^1 12x^3\sin(y)\,dx\,dy$$

$= \int_0^{\pi/2} \sin(y)\left(\int_0^1 12x^3\,dx\right)dy$

$= \int_0^{\pi/2} \sin(y)\left(3x^4\Big]_0^1\right)dy$

$= \int_0^{\pi/2} \sin(y)\,(3)\,dy$

$= 3\int_0^{\pi/2} \sin(y)\,dy$

$= -3\cos(y)\Big]_0^{\pi/2}$

$= -3\cos\left(\frac{\pi}{2}\right) - \left(-3\cos(0)\right)$

$= 0 - (-3)$

$= 3$

$$\int_0^1\int_0^{\pi/2} 12x^3\sin(y)\,dx\,dy = \boxed{3}$$

175. Evaluate the double integral

$$\int_0^{\pi/2}\int_0^1 16y^3\cos(x)\,dy\,dx$$

$= \int_{x=0}^{x=\pi/2}\left[\int_{y=0}^{y=1} 16y^3\cos(x)\,dy\right]dx$

$= \int_{x=0}^{x=\pi/2} 16\cos(x)\left[\int_{y=0}^{y=1} y^3\,dy\right]dx$

$= \int_{x=0}^{x=\pi/2} 16\cos(x)\left(\frac{y^4}{4}\right)\Big]_{y=0}^{y=1}dx$

$= \int_{x=0}^{x=\pi/2} \frac{16}{4}\cos(x)\,dx$

$= 4\sin(x)\Big]_{x=0}^{x=\pi/2}$

$= 4$

$$\int_0^1\int_0^{\pi/2} 16y^3\cos(x)\,dy\,dx = \boxed{4}$$

176. Evaluate the double integral

$$\int_0^4 \int_2^y (y+x)\,dx\,dy$$

$$\int_{y=0}^{y=4}\int_{x=2}^{x=y} (y+x)\,dx\,dy$$

$$= \int_{y=0}^{y=4}\left(\left(xy+\frac{x^2}{2}\right)\Big]_{x=2}^{x=y}\right)dy$$

$$= \int_{y=0}^{y=4}\left(y^2+\frac{y^2}{2}-(2y+2)\right)dy$$

$$= \int_{y=0}^{y=4}\left(\frac{3}{2}y^2-2y-2\right)dy$$

$$= \left(\frac{3}{2}\cdot\frac{y^3}{3}-\frac{2y^2}{2}-2y\right)\Big]_{y=0}^{y=4}$$

$$= \left(\frac{y^3}{2}-y^2-2y\right)\Big]_{y=0}^{y=4}$$

$$\int_0^4 \int_2^y (y+x)\,dx\,dy = \boxed{8}$$

$$= 8$$

177. Evaluate the double integral

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\,dy\,dx$$

$$= \int_{x=1}^{x=2}\int_{y=1}^{y=x^2} xy^{-2}\,dy\,dx$$

$$= \int_{x=1}^{x=2} x\left(\int_{y=1}^{y=x^2} y^{-2}\,dy\right)dx$$

$$= \int_{x=1}^{x=2} x\left(-y^{-1}\Big]_{y=1}^{y=x^2}\right)dx$$

$$= \int_{x=1}^{x=2} x\left(-\frac{1}{y}\Big]_{y=1}^{y=x^2}\right)dx$$

$$= \int_{x=1}^{x=2} x\left(-\frac{1}{x^2}+1\right)dx$$

$$= \int_1^2\left(x-\frac{1}{x}\right)dx$$

$$= \left(\frac{x^2}{2}-\ln(x)\right)\Big]_1^2$$

$$= (2-\ln(2))-\left(\frac{1}{2}-0\right)$$

$$= \frac{3}{2}-\ln(2)$$

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\,dy\,dx = \boxed{\frac{3}{2}-\ln(2)}$$

178. Compute the following definite integral.

$$\int_0^7 \int_1^x 36x \, dy \, dx$$

$= \int_0^7 36x\left(\int_1^x dy\right)dx$

$= \int_0^7 36x \left.(y)\right]_1^x dx$

$= \int_0^7 36x[x-1]\,dx$

$= \int_0^7 (36x^2-36x)\,dx$

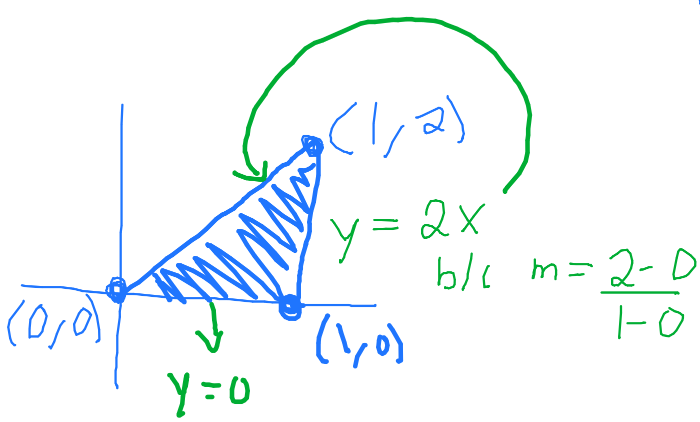
$= \left.\left(\frac{36x^3}{3} - \frac{36x^2}{2}\right)\right]_0^7$

$= \left.\left(12x^3 - 18x^2\right)\right]_0^7$

$= 3234$

$$\int_0^7 \int_1^x 36x \, dy \, dx = \boxed{3234}$$

179. Find the bounds for the integral $\iint_R f(x,y)\,dA$ where R is a triangle with vertices (0,0), (1,0), and (1,2).



Hence $\int_0^1 \int_0^{2x} f(x,y) \, dy\,dx$

Answer: $\boxed{\int_0^1 \int_0^{2x} f(x,y) \, dy\,dx}$

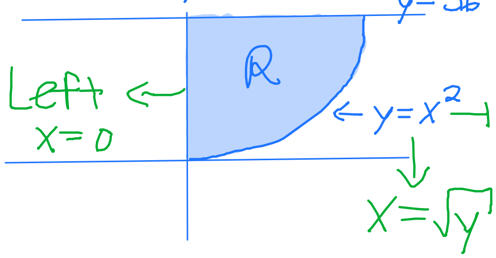
180. Switch the order of integration on the follow integral

$$\int_0^6 \int_{x^2}^{36} f(x,y)\,dy\,dx$$

The bounds tell me

$0 \leq x \leq 6$

$x^2 \leq y \leq 36$



So $0 \leq x \leq \sqrt{y}$

what does y range from?

$0 \leq y \leq 36$

Answer: $\boxed{\int_0^{36} \int_0^{\sqrt{y}} f(x,y)\,dx\,dy}$

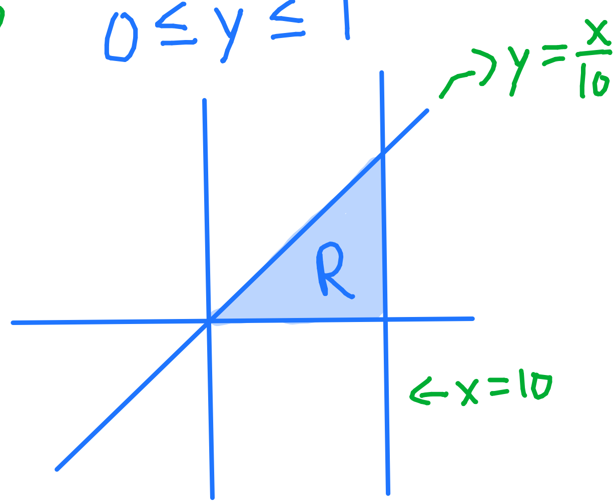
181. Switch the order of integration on the follow integral

$$\int_0^1 \int_{10y}^{10} f(x,y)\,dx\,dy$$

The bounds tell me

$y = \frac{x}{10} \leftarrow 10y \leq x \leq 10$

$0 \leq y \leq 1$



Answer: $\boxed{\int_0^1 \int_0^{x/10} f(x,y)\,dy\,dx}$

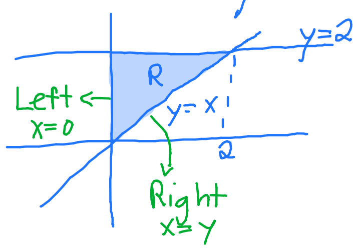
182. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2}\,dy\,dx$$

(Hint: Change the order of integration)

Bounds: $0 \le x \le 2$

$x \le y \le 2$



So $0 \le y \le 2$

$0 \le x \le y$

$$\int_0^2 \int_x^2 4e^{y^2}\,dy\,dx$$

$$= \int_{y=0}^{y=2} \int_{x=0}^{x=y} 4e^{y^2}\,dx\,dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2}\left(\int_{x=0}^{x=y} dx\right)dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2}\left(x\Big]_{x=0}^{x=y}\right)dy$$

$$= \int_{y=0}^{y=2} 4ye^{y^2}\,dy$$

$$\underset{du=2y\,dy}{\overset{u=y^2}{=}} \int 2e^u\,du$$

$$= 2e^u.$$

$$= 2e^{y^2}\Big]_{y=0}^{y=2}$$

$$= 2e^4 - 2$$

$$\int_0^2 \int_x^2 4e^{y^2}\,dy\,dx = \boxed{2e^4-2}$$

183. Evaluate the double integral

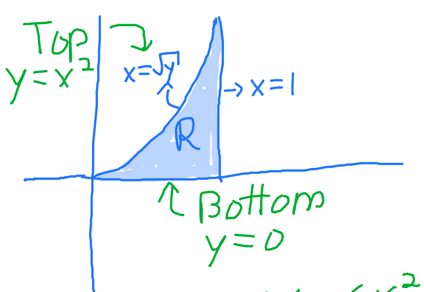
$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\,dx\,dy$$

Round your answer to 2 decimal places.

(Hint: Change the order of integration)

Bounds: $0 \le y \le 1$

$\sqrt{y} \le x \le 1$



New Bounds: $0 \le y \le x^2$

$0 \le x \le 1$

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\,dx\,dy$$

$$= \int_{x=0}^{x=1} \int_{y=0}^{y=x^2} \sin(x^3)\,dy\,dx$$

$$= \int_{x=0}^{x=1} \sin(x^3)\left(\int_{y=0}^{y=x^2} dy\right)dx$$

$$= \int_{x=0}^{x=1} \sin(x^3)\left(y\Big]_{y=0}^{y=x^2}\right)dx$$

$$= \int_{x=0}^{x=1} \sin(x^3)\cdot x^2\,dx$$

$$\underset{du=3x^2dx}{\overset{u=x^3}{=}} \int \frac{1}{3}\sin(u)\,du$$

$$= -\frac{1}{3}\cos(u)$$

$$= -\frac{1}{3}\cos(x^3)\Big]_{x=0}^{x=1}$$

$$\approx 0.15$$

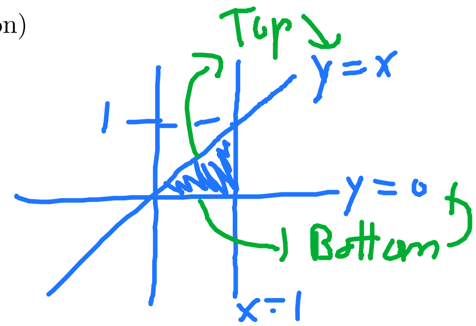
$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\,dx\,dy = \boxed{0.15}$$

184. Evaluate the double integral

$$\int_0^1 \int_y^1 2e^{x^2}\,dx\,dy = \int_{y=0}^{y=1} \int_{x=y}^{x=1} 2e^{x^2}\,dx\,dy$$

(Hint: Change the order of integration)

Draw the region
$y=0,\ y=1$
$x=y,\ x=1$



So our new bounds are

$$= \int_{x=0}^{x=1} \left[\int_{y=0}^{y=x} 2e^{x^2}\,dy \right] dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} \left[\int_{y=0}^{y=x} dy \right] dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} (y) \Big]_{y=0}^{y=x}\, dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} \cdot x\,dx$$

$$\frac{u = x^2}{du = 2x\,dx} \qquad \int \cancel{2} e^u \cancel{x}\, \frac{du}{\cancel{2x}} = \int e^u = e^u$$

$$\frac{du}{2x} = dx \qquad = e^{x^2} \Big]_{x=0}^{x=1}$$

$$\int_0^1 \int_y^1 2e^{x^2}\,dx\,dy = \underline{\boxed{e-1}}$$