MA 16020 Exam 2 Study Guide: Cal 2

An improper integral is when
(1) we have $\pm\infty$ in the bounds, or
(2) we have a discontinuity within the bounds
↓
Check if the integrand is undefined and check if that value is in the interval.

When computing them, rewrite with a limit

ex. $\int_0^{\infty} e^{-x}\,dx = \lim_{N\to\infty} \int_0^{N} e^{-x}\,dx$

To review limit check MA 16020 Exam 2 Study Guide: Cal 1.

Area Between Two Curves

The area between two curves can be described two ways:

$A = \int_a^b (\text{Top} - \text{Bottom})\,dx$ → You want $y = \text{something}_x$ for Top and Bottom

or $A = \int_c^d (\text{Right} - \text{Left})\,dy$ → You want $x = \text{something}_y$ for Right and Left

Volume of Solids of Revolution

Read the problem to see if a particular method is asked for. Plus try to draw the regions.

When the region "hugs" the line of rotation ⇒ Disk

- x-axis ⇒ dx problem ⇒ $V=\int_a^b \pi(f(x))^2\,dx$
- y-axis ⇒ dy problem ⇒ $V=\int_c^d \pi(g(y))^2\,dy$
- the line $y=\#$ ⇒ dx problem ⇒ $V=\int_a^b \pi(f(x)-\#)^2\,dx$
- the line $x=\#$ ⇒ dy problem ⇒ $V=\int_c^d \pi(g(y)-\#)^2\,dy$

When there is a "gap" between the region and the line of rotation ⇒ Washer

- x-axis ⇒ dx problem ⇒ $V=\int_a^b \pi(R^2-r^2)\,dx$
- y-axis ⇒ dy problem ⇒ $V=\int_c^d \pi(R^2-r^2)\,dy$
- the line $y=\#$ ⇒ dx problem ⇒ $V=\int_a^b \pi\left[(R-\#)^2-(r-\#)^2\right]dx$
- the line $x=\#$ ⇒ dy problem ⇒ $V=\int_c^d \pi\left[(R-\#)^2-(r-\#)^2\right]dy$

Where R is the farthest from the line of rotation and r is the closest to the line of rotation

---

But if you find solving for x or y, in either method, is hard ⇒ Shell

- x-axis ⇒ dy problem ⇒ $V=\int_c^d 2\pi y(\text{Right}-\text{Left})\,dy$
- y-axis ⇒ dx problem ⇒ $V=\int_a^b 2\pi x(\text{Top}-\text{Bottom})\,dx$

Growth & Decay Differential Equations

- Proportional to population $\Rightarrow y' = \frac{dy}{dt} = ky$

$$\Rightarrow y = Ce^{kt}$$

- Half-life Problems $\Rightarrow y = Ce^{kt}$ with

$$k = \frac{\ln(1/2)}{\text{half-life}} = \frac{-\ln(2)}{\text{half-life}}$$

Separation of Variables

Solve differential equations of the type

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

The idea is to try to get terms w/ y on one-side and x-terms on the other. Then integrate and solve for y.