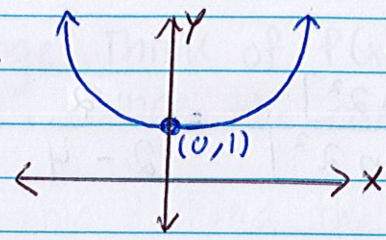
Lesson 26: Intro to Functions of Several Variables

Single Variable Functions

- Can be written as $\boxed{y=f(x)}$
  i.e. y is a function of x
- Takes as an input a # and produces as an output a #
- Graph: a curve in the xy-plane
  i.e. 2-D

ex. $f(x)=x^2+1$



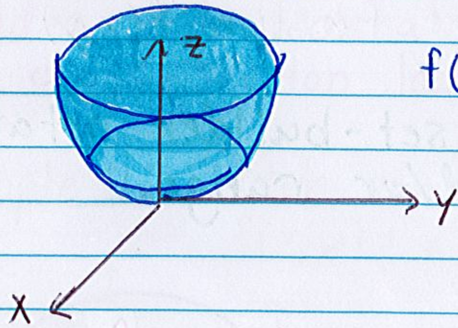
Multivariable Functions

Definition: A multivariable function has 2 or more variables as inputs.

In this class, we are limitting ourselves to just 2 variables. So

- Can be written as $\boxed{z=f(x,y)}$
  i.e. z is a function of x and y
- Takes as an input a pair of # or a point and produces as an output a #
- Graph: A Surface in xyz-space
  i.e. 3-D

ex. $f(x,y)=x^2+y^2$



Recall the idea of cross sections and how stacking multiple 2-D objects became one 3-D object. Well this is a bunch of circles with increasing radii stacked on top of another.

Example 1: Compute the indicated functional value

ⓐ $f(x,y) = \dfrac{3x+2y}{2x+3y}$; $f(-4,6)$

$$f(-4,6) = \frac{3(-4)+2(6)}{2(-4)+3(6)} = \frac{-12+12}{-8+18} = \frac{0}{10} = 0$$

ⓑ $f(x,y) = \dfrac{e^{xy}}{2-e^{xy}}$; $f(\ln 2, 2)$ and $f(1, \ln 1)$

$$f(\ln 2, 2) = \frac{e^{2\ln 2}}{2-e^{2\ln 2}} = \frac{\exp[\ln 2^2]}{2-\exp[\ln 2^2]} = \frac{4}{2-4} = \frac{4}{-2} = -2$$

$$f(1, \ln 1) = \frac{e^{\ln 1}}{2-e^{\ln 1}} = \frac{1}{2-1} = \frac{1}{1} = 1$$

Domain and Range for Multivariable Functions

Just as for functions of one variables, we can find the domain and range of functions of two variable in a similar fashion. If you need a review of domain and range of one variable functions, look at the Algebra Review pdf for Lesson 18.

The main difference for two variable functions is

Domain: All points (x,y) in the xy-plane for which $f(x,y)$ is defined

Range: All values that the function $f(x,y)$ produces

Notation: We will be using set-builder notation for denoting the domain and/or range.

i.e. $\{x \mid x \geq 4\}$

Example 2: Describe the domain and range of the function
$$f(x,y) = \sqrt{x^2 - y}$$

Domain: Recall domain of $\sqrt{?}$ is $? \geq 0$. So
$$x^2 - y \geq 0$$
$$x^2 \geq y$$

Hence the domain is $\{(x,y) \mid x^2 \geq y\}$. Note this domain consists parabolas.

Range: Think of $f(x,y) = z$. Recall that the range of $\sqrt{}$ is ≥ 0. Hence the range is $\{z \mid z \geq 0\}$

Example 3: Find the domain of $f(x,y) = \dfrac{\sqrt{x-18}}{\ln(y-9) - 3}$

To find the domain, we need to play with 3 portions of the function.

- $\sqrt{x-18} \Rightarrow x - 18 \geq 0$, so $x \geq 18$
- $\ln(y-9) \Rightarrow y - 9 > 0$, so $y > 9$
- $\ln(y-9) - 3 \neq 0$
  - $\ln(y-9) \neq 3$
  - $y - 9 \neq e^3$
  - $y \neq e^3 + 9$

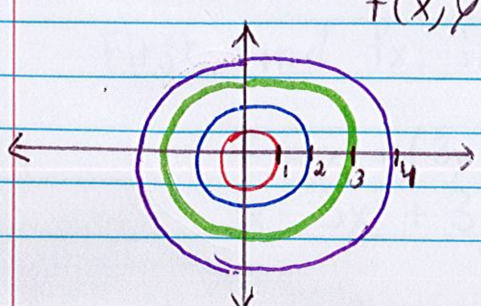
Henee the domain is $\{(x,y) \mid x \geq 18, y > 9, y \neq e^3 + 9\}$

Level Curves

Definition: The level curves of a function of two variables are the curves with equations $f(x,y) = C$ where C is a constant (in the range of f).

The idea is to look at various values of z and see what each cross-section looks like.

Example 4: Sketch the indicated level curves
$$f(x,y) = x^2 + y^2, \quad C = 1, 4, 9, 16$$



$x^2 + y^2 = 1 = 1^2$

$x^2 + y^2 = 4 = 2^2$

$x^2 + y^2 = 9 = 3^2$

$x^2 + y^2 = 16 = 4^2$

Example 6: What do the level curves for $f(x,y)=11\sqrt{y+6x^2}$ look like?

Let $f(x,y)=C$. So $C=11\sqrt{y+6x^2}$. Now solve for y.

$$\frac{C}{11}=\sqrt{y+6x^2}$$
$$C=\sqrt{y+6x^2}$$
$$C^2=y+6x^2$$
$$C=y+6x^2$$
$$y=-6x^2+C$$

So the level curves are in the shapes of parabolas.

Example 5: What does the level curve for $f(x,y)=\ln(x^2+y^2)$ at $C=\ln(36)$ look like?

Let $f(x,y)=C$. So

$$\ln(x^2+y^2)=\ln(36)$$
$$x^2+y^2=36$$
$$x^2+y^2=6^2$$

So the level curve is a circle with center at the origin and with a radius of 6.