Lesson 8: Integration by Parts II

Recall from Last time, we found the formula for Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

We were also given an acronym to help in choosing what u should be.

L - Logarithmic
A - Algebraic (like polynomials)
T - Trigonometric
E - Exponential

Example 1: Evaluate

ⓐ $\int (t-3)e^t\,dt$ $\quad \dfrac{u=t-3 \qquad dv=e^t\,dt}{du=dt \qquad v=e^t}$ $\quad uv-\int v\,du$

$$= (t-3)e^t - \int e^t\,dt = (t-3)e^t - e^t + C$$

$$= (t-3-1)e^t + C = (t-4)e^t + C$$

ⓑ $\int \ln x\,dx$ $\quad \dfrac{u=\ln x \qquad dv=dx}{du=1/x\,dx \qquad v=x}$ $\quad uv-\int v\,du$

$$= x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - \int dx$$

$$= x\ln x - x + C$$

Example 2: Evaluate

ⓐ $\int x^2\sin x\,dx$ $\quad \dfrac{u=x^2 \qquad dv=\sin x\,dx}{du=2x\,dx \qquad v=-\cos x}$ $\quad uv-\int v\,du$

$$= -x^2\cos x - \int(-\cos x)2x\,dx$$

$$= -x^2\cos x + \underbrace{\int 2x\cos x\,dx}$$

Note to integrate this we need to do another integration by parts

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$$\int 2x\cos x\,dx \quad \frac{u=2x}{du=2dx} \quad \frac{dv=\cos x\,dx}{v=\sin x} \quad uv-\int v\,du$$

$$= 2x\sin x - \int 2\sin x\,dx = 2x\sin x - (-2\cos x)$$

$$= 2x\sin x + 2\cos x$$

$$= x^2\cos x + 2x\sin x + 2\cos x + C$$

(b) $\int 12x(\ln(4x))^2\,dx$

$u=(\ln(4x))^2$ $\quad dv=12x\,dx$

$du = 2(\ln(4x)) \cdot \frac{1}{4x} \cdot 4\,dx$ $\quad v=\frac{12x^2}{2}$

$du = \frac{2(\ln(4x))}{x}dx$ $\quad v=6x^2$

$$= 6x^2(\ln(4x))^2 - \int 6x^2 \cdot \frac{2(\ln(4x))}{x}dx$$

$$= 6x^2(\ln(4x))^2 - \underbrace{\int 12x\ln(4x)\,dx}$$

Note to integrate this we need to do another integration by parts

$\int 12x\ln(4x)\,dx$ $\quad u=\ln(4x)$ $\quad dv=12x\,dx$

$du=\frac{1}{4x}\cdot 4dx$ $\quad v=\frac{12x^2}{2}$

$du = \frac{1}{x}dx$ $\quad v=6x^2$

$$= 6x^2\ln(4x) - \int 6x^2 \cdot \frac{1}{x}dx$$

$$= 6x^2\ln(4x) - \int 6x\,dx$$

$$= 6x^2\ln(4x) - \frac{6x^2}{2} = 6x^2\ln(4x) - 3x^2$$

$$= 6x^2(\ln(4x))^2 - (6x^2\ln(4x) - 3x^2) + C$$

$$= 6x^2(\ln(4x))^2 - 6x^2\ln(4x) + 6x^2 + C$$

Example 3: The velocity of a car over the time period $0 \le t \le 3$ is given by the function

$$v(t) = 50te^{-t/4} \text{ miles per hour}$$

where t is time in hours. What was the distance the car traveled in the first 30 minutes?

Note t is in hours. So 30 mins $\Rightarrow$ 0.5 hrs

We need to compute

$$\int_0^{0.5} 50te^{-t/4}\,dt \qquad \frac{u = 50t}{du = 50\,dt} \quad \frac{dv = e^{-t/4}\,dt}{v = -4e^{-t/4}} \qquad uv - \int v\,du$$

$$= 50t(-4e^{-t/4})\Big]_0^{0.5} - \int_0^{0.5} -4e^{-t/4}(50)\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200\int_0^{0.5} e^{-t/4}\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200(-4)e^{-t/4}\Big]_0^{0.5}$$

$$= -200(0.5)e^{-0.5/4} - (-200(0)e^{-0/4}) - 800e^{-0.5/4}$$

$$- (-800e^{-0/4})$$

$$= -100e^{-1/8} - 0 - 800e^{-1/8} + 800$$

$$= -900e^{-1/8} + 800$$

$$\approx 5.75$$

Example 4: A model for the ability of a child to memorize information, measured on a scale from 1 to 100, is given by

$$M(t) = 1 + 3.4t\ln(t)$$

when $2 \le t \le 8$ where t is the child's age in years. Find the child's average memorization ability between ages 3 and 6.

$$M_{AVE}(t) = \frac{1}{6-3}\int_3^6 (1 + 3.4t\ln(t))\,dt$$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\int_3^6 t\ln(t)\,dt$$

$u = \ln(t)$ $\quad dv = t\,dt$

$du = \frac{1}{t}\,dt$ $\quad v = \frac{t^2}{2}$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\left(\frac{t^2}{2}\ln(t)\Big]_3^6 - \int_3^6 \frac{t^2}{2}\cdot\frac{1}{t}\,dt\right)$$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\left(\frac{t^2}{2}\ln(t)\Big]_3^6 - \int_3^6 \frac{1}{2}t\,dt\right)$$

$$= \frac{1}{3}t\Big]_3^6 + \frac{3.4}{6}t^2\ln(t)\Big]_3^6 - \frac{3.4}{6}\cdot\frac{t^2}{2}\Big]_3^6$$

$$= \frac{1}{3}(6) - \frac{1}{3}(3) + \frac{3.4}{6}(6)^2\ln(6) - \frac{3.4}{6}(3)^2\ln(3)$$

$$- \frac{3.4}{12}(6)^2 - \left(-\frac{3.4}{12}(3)^2\right)$$

$$\approx 26.299$$