Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name:________________________________________

1. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where $t$ is time in hours after 9:00 am and the rate $r(t)$ is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

10:00am $\Rightarrow$ 1 hr
1:00pm $\Rightarrow$ 4 hrs $\Big\} \Rightarrow \int_1^4 6t^{1/2}\,dt$

$$= 6 \cdot \frac{2}{3} t^{3/2} \Big]_1^4$$

$$= 4t^{3/2} \Big]_1^4$$

$$= 28$$

Answer:____________________

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Solve $\int_0^t 6t^{1/2}\,dt = 121$

$$4t^{3/2} = 121$$

$$t^{3/2} = \frac{121}{4}$$

$$t = \left(\frac{121}{4}\right)^{3/2}$$

Answer:____________________

2. Which derivative rule is undone by integration by substitution?

(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these

3. Which derivative rule is undone by integration by parts?

(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these

4. What would be the best substitution to make the solve the given integral?

$$\int e^{2x}\cos(e^{2x})\sin^3(e^{2x})\,dx$$

Check du is in the integral.

$u =$ $\sin(e^{2x})$

5. What would be the best substitution to make the solve the given integral?

$$\int \sec^2(5x)e^{\tan(5x))}\,dx$$

Check du is in the integral.

$u =$ $\tan(5x)$

6. Evaluate the definite integral.

$$\int_0^2 (5e^{2x} + 8)\, dx$$

$$\underbrace{\int_0^2 5e^{2x}dx}_{u\text{-sub}} + \int_0^2 8dx$$

$$\frac{5}{2}e^{2x}\Big]_0^2 + 8x\Big]_0^2$$

$$\frac{5}{2}(e^4 - e^0) + 8(2)$$

$$\frac{5}{2}e^4 - \frac{5}{2} + 16$$

$\int_0^2 (5e^{2x} + 8)\, dx =$ $\frac{5}{2}e^4 - \frac{27}{2}$

7. Evaluate the definite integral.

$$\int_0^{\pi/2} (x - 1)\sin(x)\, dx$$

$u = x-1$ $\quad dv = \sin(x)dx$ $\quad uv - \int v\,du$

$du = dx$ $\quad v = -\cos x$

$$= -(x-1)\cos x\Big]_0^{\pi/2} - \int_0^{\pi/2}(-\cos x)\,dx$$

$$= -(x-1)\cos x\Big]_0^{\pi/2} + \sin x\Big]_0^{\pi/2}$$

$$= \xrightarrow{0}\cancel{-\left(\frac{\pi}{2}-1\right)\cos\left(\frac{\pi}{2}\right)} - (-(0-1))\cos(0) + \xrightarrow{1}\cancel{\sin\left(\frac{\pi}{2}\right)} - \xrightarrow{0}\cancel{\sin(0)}$$

$$= 0 - 1 + 1$$

$\int_0^{\pi/2} (x - 1)\sin(x)\, dx =$ $0$

8. Evaluate the indefinite integral.

$$\int 9x^3 e^{-x^4}\, dx$$

$u = -x^4$

$du = -4x^3 dx$

$\frac{du}{-4x^3} = dx$

$\int 9x^3 e^u \frac{du}{-4x^3} = -\frac{9}{4}\int e^u du = -\frac{9}{4}e^u + C$

$= -\frac{9}{4}e^{-x^4} + C$

$\int 9x^3 e^{-x^4}\, dx =$ $-\frac{9}{4}e^{-x^4} + C$

9. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that $t$ hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$L'(t) = \sqrt{3t + 2} \quad \text{gallons per hour.}$$

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.

i.e. $\int_0^4 (3t+2)^{1/2} dt$

$u = 3t+2$

$du = 3dt$

$\frac{du}{3} = dt$

$\int u^{1/2} \frac{du}{3} = \frac{1}{3}\cdot\frac{2}{3}u^{3/2}$

$= \frac{2}{9}(3t+2)^{3/2}\Big]_0^4$

$\approx 11.0122$

Answer: 11.0122

10. Evaluate

$$\int 3x\ln(x^7)\,, dx$$

Rewrite $\int 3x \cdot 7\ln(x)dx = \int 21x\ln x dx$

$u = 21\ln x$ $\quad dv = x dx$ $\quad uv - \int v du$

$du = \frac{21}{x}dx$ $\quad v = \frac{x^2}{2}$

$= \frac{21x^2\ln x}{2} - \int \frac{x^2}{2}\cdot\frac{21}{x}dx$

$= \frac{21x^2\ln x}{2} - \frac{21}{2}\int x dx$

$= \frac{21x^2\ln x}{2} - \frac{21}{4}x^2 + C$

$$\int 3x\ln(x^7)\,, dx = \underline{\hspace{6cm}}$$

11. Evaluate

$$\int_1^e \frac{\ln(x^4)}{x}\,dx$$

Rewrite $\int_1^e \frac{4\ln x}{x}dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int \frac{4u}{x}\cdot x du = \int 4u du = \frac{4u^2}{2}$

$x du = dx$

$= 2(\ln x)^2\Big]_1^e = \underbrace{2(\ln e)^2}_{2} - \underbrace{2(\ln 1)^2}_{0}$

$$\int_1^e \frac{\ln(x^4)}{x}\,dx = \underline{\quad 2 \quad}$$

12. The population of pink elephants in Dumbo's dreams, in hundreds, $t$ years after the year 1980 is given by

$$P(t) = \frac{e^{5t}}{1+e^{5t}}$$

What is the average population during the decade between 1980 and 2000?

i.e. $\frac{1}{2000-1980}\int_0^{20} \frac{e^{5t}}{1+e^{5t}}\,dt$ $\quad u = 1+e^{5t}$ $\quad \frac{1}{20}\int \frac{e^{5t}}{u}\cdot\frac{du}{5e^{5t}}$

$du = 5e^{5t}\,dt$

$\frac{du}{5e^{5t}} = dt$

$= \frac{1}{100}\int \frac{du}{u} = \frac{1}{100}\ln|u| = \frac{1}{100}\ln|1+e^{5t}|\Big]_0^{20}$

$\approx 0.9931$

Answer: 0.9931 hundreds

$= 993$

13. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{3x+1}{x^2(x+1)^2(x^2+1)}$$

(A)

$$\frac{A}{x^2} + \frac{B}{(x+1)^2} + \frac{C}{x^2+1}$$

(B)

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{x^2+1}$$

(C)

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{Ex+F}{x^2+1}$$

(D)

$$\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{D}{x+1} + \frac{Ex+F}{(x+1)^2} + \frac{Gx+H}{x^2+1}$$

(E)

$$\frac{A}{x} + \frac{B}{(x+1)^2} + \frac{C}{x^2+1}$$

14. Determine the partial fraction decomposition of

$$\frac{7x^2+9}{x(x^2+3)}$$

$$\frac{A}{x}+\frac{Bx+C}{x^2+3}$$

$$=\frac{A(x^2+3)+x(Bx+C)}{x(x^2+3)}$$

$$=\frac{Ax^2+3A+Bx^2+Cx}{x(x^2+3)}$$

$$=\frac{(A+B)x^2+Cx+3A}{x(x^2+3)}$$

$$(A+B)x^2+Cx+3A=7x^2+0x+9$$

$$\begin{cases} A+B=7 \\ C=0 \\ 3A=9 \rightarrow A=2 \end{cases}$$

So $B=5$.

Answer: $\frac{2}{x}+\frac{5x}{x^2+3}$

15. Evaluate $\displaystyle\int \frac{5x^2+9}{x^2(x+3)}\,dx$

$$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}$$

$$=\frac{Ax(x+3)+B(x+3)+Cx^2}{x^2(x+3)}$$

$$=\frac{Ax^2+3Ax+Bx+3B+Cx^2}{x^2(x+3)}$$

$$=\frac{(A+C)x^2+(3A+B)x+3B}{x^2(x+3)}$$

$$(A+C)x^2+(3A+B)x+3B=5x^2+0x+9$$

$$\begin{cases} A+C=5 \\ 3A+B=0 \\ 3B=9 \rightarrow B=3 \end{cases}$$

$3A+B=0$
$3A+3=0$
$3A=-3$
$A=-1$

$-1+C=5$
$C=6$

$$\int -\frac{1}{x}\,dx+\int\frac{3}{x^2}\,dx+\int\frac{6}{x+3}\,dx$$

$\displaystyle\int \frac{5x^2+9}{x^2(x+3)}\,dx=$ $-\ln|x|-\frac{3}{x}+6\ln|x+3|+C$