MATH 16020: Applied Calculus 2 Practice Exam 3 Wed., Apr. 5, 2023

Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:________________________________

1. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

$y =$ ________________________________

2. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$y =$ ________________________________

3. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$y =$ ______________________________

4. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$y =$ ______________________________

5. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$y =$ ____________________

6. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$y =$ ____________________

7. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2 e^{-x^3} \quad \text{where} \quad y = 10 \quad \text{when} \quad x = 2$$

Find the value of the integration constant, C.

$C =$ ______________________

8. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right) y = 10 \ln(x)$$

$u(x) =$ ______________________

9. What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = (x+1)e^{x^2}$$

$u(x) =$ ____________________

10. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) =$ ____________________

11. Solve the initial value problem.

$$x^4 y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$y =$ ______________________________

12. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \dots$$

Answer:______________________________

(b) Use the sum from (a) and compute the sum.

Answer:______________________________

13. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \underline{\hspace{8cm}}$$

14. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ ______________________________

15. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ ______________________________

16. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \underline{\hspace{8cm}}$$

17. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \underline{\hspace{8cm}}$$

18. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

$R =$ ______________________________

19. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

$R =$ ______________________________

20. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$\dfrac{3}{1+2x} =$ ______________________________

$R =$ ______________________________

21. Express $f(x) = \dfrac{5x}{3+2x^2}$ as a power series and determine it's radius of converge.

$\dfrac{5x}{3+2x^2} =$ ______________________________

$R =$ ______________________________

22. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$\int \sin(x^{3/2})\,dx =$ ________________________

23. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx$$

$\int_0^{0.11} \frac{1}{1+x^4}\,dx \approx$ ________________________

24. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\, dx$$

$\int_0^{0.23} e^{-x^2}\, dx \approx$ ______________________

25. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\, dx$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\, dx \approx$ ______________________

26. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$\ln(1.56) \approx$ ______________________

27. Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

Domain = ______________________

28. Find the domain of

$$f(x,y) = \frac{\ln(x^2-y+3)}{\sqrt{x-6}}$$

Domain = ______________________

29. Describe the indicated level curves $f(x,y) = C$

$$f(x,y) = \ln(x^2 + y^2) \;\; C = \ln(36)$$

(a) Parabola with vertices at $(0,0)$
(b) Circle with center at $(0, \ln(36))$ and radius 6
(c) Parabola with vertices at $(0, \ln(36))$
(d) Circle with center at $(0,0)$ and radius 6
(e) Increasing Logarithm Function

30. What do the level curves for the following function look like?

$$f(x,y) = \ln(y - e^{5x})$$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

31. What do the level curves for the following function look like?

$$f(x,y) = \sqrt{y + 4x^2}$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

32. Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y^2-1}}$$

$f_x(6,5)$= ______________________

33. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$f_x(x,y) =$ ______________________

$f_y(x,y) =$ ______________________

34. Find the first order partial derivatives of

$$f(x, y) = (xy - 1)^2$$

$f_x(x, y) =$ ______________________________

$f_y(x, y) =$ ______________________________

35. Given the function $f(x, y) = 4x^5 \tan(3y)$, compute $f_{xy}(2, \pi/3)$

$f_{xy}(2, \pi/3) =$ ______________________________

36. Find the second order partial derivatives of

$$f(x, y) = x^2 y \ln(7x)$$

$f_{xx}(x, y) =$ ______________________________

$f_{xy}(x, y) =$ ______________________________

$f_{yy}(x, y) =$ ______________________________