Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: Solutions

1. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

Note there are 2 ways to do this problem.
① Separation of Variables
② First-Order Linear Eqn

By method 1,

$\frac{dy}{dt} = 15y$

$\frac{dy}{y} = 15\,dt$

$\int \frac{dy}{y} = \int 15\,dt$

$\ln|y| = 15t + C$

$y = e^{15t+C}$

$y = e^{C}e^{15t}$

$y = Ce^{15t}$

$y = \boxed{Ce^{15t}}$

2. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$y\,dy = 3\,dx$

$\int y\,dy = \int 3\,dx$

$\frac{y^2}{2} = 3x + C$

$y^2 = 6x + 2C$

$y^2 = 6x + C$

$y = \pm\sqrt{6x+C}$

$y = \boxed{\pm\sqrt{6x+C}}$

3. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$$\frac{dy}{y} = 3x^2\,dx$$

$$\int \frac{dy}{y} = \int 3x^2\,dx$$

$$\ln|y| = x^3 + C$$

$$y = e^{x^3 + C}$$

$$y = e^{C}e^{x^3}$$

$$y = Ce^{x^3}$$

$y =$ $\boxed{Ce^{x^3}}$

4. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$$dy = 8e^{-4t}e^{-y}\,dt$$

$$e^{y}\,dy = 8e^{-4t}\,dt$$

$$\int e^{y}\,dy = \int 8e^{-4t}\,dt$$

$$e^{y} = \frac{8}{-4}e^{-4t} + C$$

$$e^{y} = -2e^{-4t} + C$$

$$y = \ln(-2e^{-4t} + C)$$

$y =$ $\boxed{\ln(-2e^{-4t} + C)}$

5. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$2y\,dy = (3x+2)\,dx$

$\int 2y\,dy = \int (3x+2)\,dx$

$y^2 = \frac{3x^2}{2} + 2x + C$

So $y^2 = \frac{3x^2}{2} + 2x + 16$

$y = \pm\sqrt{\frac{3x^2}{2} + 2x + 16}$

when $y(0) = 4$

$4^2 = 0 + 0 + C$

$16 = C$

$y =$ $\boxed{\pm\sqrt{\frac{3x^2}{2} + 2x + 16}}$

6. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$\frac{dy}{y} = \frac{5}{6x+3}\,dx$

$\int \frac{dy}{y} = \int \frac{5}{6x+3}\,dx$

$\ln|y| = \frac{5}{6}\ln|6x+3| + C$

$y = \exp\left[\frac{5}{6}\ln|6x+3| + C\right]$

$y = e^C \exp\left[\ln|6x+3|^{5/6}\right]$

$y = C \cdot |6x+3|^{5/6}$

When $y(0) = 1$

$1 = C \cdot |6(0)+3|^{5/6}$

$1 = C \cdot 3^{5/6}$

$C = 3^{-5/6}$

$y =$ $\boxed{3^{-5/6} \cdot |6x+3|^{5/6}}$

7. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \quad \text{where} \;\; y = 10 \;\; \text{when} \;\; x = 2$$

Find the value of the integration constant, C.

$dy = 11x^2e^{-x^3}dx$

$\int dy = \underbrace{\int 11x^2e^{-x^3}dx}$

$u = -x^3$

$du = -3x^2dx$

$y = \int -\frac{11}{3}e^u du$

$y = -\frac{11}{3}e^{-x^3} + C$

When $y = 10$ and $x = 2$

$10 = -\frac{11}{3}e^{-2^3} + C$

$10 = -\frac{11}{3}e^{-8} + C$

$C = 10 + \frac{11}{3}e^{-8}$

$C =$ $\boxed{10 + \frac{11}{3}e^{-8}}$

8. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right)y = 10\ln(x)$$

$\frac{2y' + \left(\frac{6}{x}\right)y}{2} = \frac{10\ln(x)}{2}$

$y' + \frac{3}{x}y = 5\ln x$

$P(x) = \frac{3}{x}$ $\quad Q(x) = 5\ln x$

$u(x) = \exp\left[\int \frac{3}{x}dx\right]$

$= \exp[3\ln x]$

$= \exp[\ln x^3]$

$= x^3$

$u(x) =$ $\boxed{x^3}$

9. What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = (x+1)e^{x^2}$$

$$\underbrace{(x+1)\frac{dy}{dx} - 2(x^2+x)y}_{(x+1)} = \underbrace{(x+1)e^{x^2}}_{(x+1)}$$

$$\frac{dy}{dx} - \frac{2x(x+1)}{(x+1)}y = e^{x^2}$$

$$\frac{dy}{dx} + (-2x)\cdot y = e^{x^2}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$

$$= \exp\left[\int -2x\,dx\right]$$

$$= \exp\left[-x^2\right]$$

$u(x) =$ $\boxed{e^{-x^2}}$

10. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x)\cdot y = \sin^2(x)$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$

$$= \exp\left[\int \cot x\,dx\right]$$

$$= \exp\left[\int \frac{\cos x}{\sin x}\,dx\right]$$

$u = \sin x$

$du = \cos x\,dx$

$$= \exp\left[\int \frac{du}{u}\right]$$

$$= \exp\left[\ln u\right]$$

$$u(x) = \exp\left[\ln \sin x\right]$$

$$= \sin x$$

$u(x) =$ $\boxed{\sin x}$

11. Solve the initial value problem.

$$x^4y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$$\frac{x^4y' + 4x^3y}{x^4} = \frac{10x^9}{x^4}$$

$$y' + \frac{4}{x} \cdot y = 10x^5$$

$$P(x) = \frac{4}{x} \quad Q(x) = 10x^5$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int \frac{4}{x}\,dx\right]$$
$$= \exp[4\ln x]$$
$$= \exp[\ln x^4]$$
$$= x^4$$

$$y \cdot u(x) = \int Q(x)u(x)\,dx + C$$
$$y \cdot x^4 = \int 10x^5x^4\,dx + c$$
$$y \cdot x^4 = \int 10x^9\,dx + C$$
$$y \cdot x^4 = x^{10} + C$$
$$y = \frac{x^{10}}{x^4} + \frac{C}{x^4}$$
$$y = x^6 + \frac{C}{x^4}$$

$$23 = 1 + \frac{C}{1}$$
$$22 = C$$
$$y = x^6 + \frac{22}{x^4}$$

$y =$ $\boxed{x^6 + \frac{22}{x^4}}$

12. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \ldots$$

$$= 1 - \frac{6}{10} + \frac{36}{100} - \frac{216}{1000} + \ldots$$

$$= 1 - \frac{6}{10} + \left(\frac{6}{10}\right)^2 - \left(\frac{6}{10}\right)^3 + \ldots$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n$$

Answer: $\boxed{\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n}$

(b) Use the sum from (a) and compute the sum.

$$\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n = \frac{1}{1-(-6/10)} = \frac{1}{1+6/10} = \frac{1}{16/10} = \frac{10}{16} = \frac{5}{8}$$

Answer: $\boxed{5/8}$

13. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Note $r = 3/2$ and

$\left|\frac{3}{2}\right| < 1$ is false

So the sum diverges

$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n =$ $\boxed{\text{diverges}}$

14. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$$= \frac{6}{1-(-1/9)}$$

$$= \frac{6}{1+1/9}$$

$$= \frac{6}{10/9}$$

$$= 6 \cdot \frac{9}{10}$$

$$= 3 \cdot \frac{9}{5} = \frac{27}{5}$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ $\boxed{27/5}$

15. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$$= \sum_{n=0}^{\infty} 7\left(\frac{1}{4}\right)^n$$

$$= \frac{7}{1-1/4}$$

$$= \frac{7}{3/4}$$

$$= 7 \cdot \frac{4}{3} = \frac{28}{3}$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ $\boxed{28/3}$

16. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$= \frac{5^3}{6} + \frac{5^4}{6^2} + \frac{5^5}{6^3} + \cdots$$

$$= \frac{5^3}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \cdots\right)$$

$$= \frac{125}{6} \sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n = \frac{125}{6} \cdot \frac{1}{1-5/6}$$

$$= \frac{125}{6} \cdot \frac{1}{1/6} = \frac{125}{6} \cdot \frac{6}{1} = 125$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \boxed{125}$$

17. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n}{3 \cdot 3^{2n}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3} \left(-\frac{2}{9}\right)^n$$

$$= \frac{1/3}{1-(-2/9)}$$

$$= \frac{1/3}{1+2/9}$$

$$= \frac{1/3}{11/9}$$

$$= \frac{1}{3} \cdot \frac{9}{11}$$

$$= 3/11$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \boxed{3/11}$$

18. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

Remember

$$\sum_{n=0}^{\infty} \Box^n = \frac{1}{1-\Box} \quad \text{where } |\Box| < 1$$

$|-2x| < 1$

$|2x| < 1$

$2|x| < 1$

$|x| < 1/2 = R$

$R =$ $\boxed{1/2}$

19. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

Remember

$$\sum_{n=0}^{\infty} \Box^n = \frac{1}{1-\Box} \quad \text{where } |\Box| < 1$$

$|7x^2| < 1$

$7|x^2| < 1$

$|x^2| < 1/7$

$-1/7 < x^2 < 1/7$

By algebra

$x^2 < 1/7$

$x < \pm\sqrt{1/7}$

$|x| < \sqrt{1/7}$

$R =$ $\boxed{\sqrt{1/7}}$

20. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$$\frac{3}{1+2x} = \frac{3}{1} \cdot \frac{1}{1+2x} = \frac{3}{1} \cdot \frac{1}{1-(-2x)}$$

$$\frac{1}{1-(-2x)} = \sum_{n=0}^{\infty} (-2x)^n \text{ where } |-2x| < 1$$

$$f(x) = \frac{3}{1-(-2x)} = 3\sum_{n=0}^{\infty} (-2x)^n \text{ where } 2|x|<1$$

$$= \sum_{n=0}^{\infty} 3(-1)^n 2^n x^n \text{ where } |x| < 1/2$$

$$\frac{3}{1+2x} = \boxed{\sum_{n=0}^{\infty} 3(-1)^n 2^n x^n}$$

$$R = \boxed{1/2}$$

21. Express $f(x) = \dfrac{5x}{3+2x^2}$ as a power series and determine it's radius of converge.

$$\frac{5x}{3(1+2x^2/3)} = \frac{5x}{3} \cdot \frac{1}{1-(-(2x^2/3))}$$

$$\frac{1}{1-(-2x^2/3)} = \sum_{n=0}^{\infty} \left(-\frac{2x^2}{3}\right)^n \text{ where } \left|-\frac{2x^2}{3}\right| < 1$$

$$\frac{2}{3}|x^2| < 1$$

$$|x^2| < \frac{3}{2}$$

$$-\frac{3}{2} < x^2 < \frac{3}{2}$$

By algebra

$$x^2 < \frac{3}{2}$$

$$-\sqrt{\frac{3}{2}} < x < \sqrt{\frac{3}{2}}$$

$$|x| < \sqrt{\frac{3}{2}}$$

$$f(x) = \frac{5x}{3} \cdot \frac{1}{1-(-2x^2/3)} = \frac{5x}{3} \sum_{n=0}^{\infty} \left(\frac{-2x^2}{3}\right)^n$$

$$f(x) = \frac{5x}{3} \sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{2n}}{3^n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 2^n \cdot 5 \cdot x^{2n+1}}{3^{n+1}}$$

$$\frac{5x}{3+2x^2} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 2^n \cdot 5 \cdot x^{2n+1}}{3^{n+1}}}$$

$$R = \boxed{\sqrt{3/2}}$$

22. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\sin(x^{3/2}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x^{3/2})^{2n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}$$

$$\int \sin(x^{3/2})\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{3n+3/2}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{3n+5/2}}{3n+5/2}$$

$$= \frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6 \cdot (3+5/2)} + \frac{x^{17/5}}{5!\,(6+5/2)}$$

$\int \sin(x^{3/2})\,dx = \boxed{\dfrac{x^{5/2}}{5/2} - \dfrac{x^{11/2}}{6 \cdot (3+5/2)} + \dfrac{x^{17/5}}{5!\,(6+5/2)}}$

23. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx$$

$$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = \sum_{n=0}^{\infty} (-x^4)^n = \sum_{n=0}^{\infty} (-1)^n x^{4n}$$

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx = \int_0^{0.11} \sum_{n=0}^{\infty} (-1)^n x^{4n}\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \int_0^{0.11} x^{4n}\,dx$$

$$= \sum_{n=0}^{\infty} \left. \frac{(-1)^n x^{4n+1}}{4n+1} \right]_0^{0.11}$$

$$= \left. \left(x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13}\right) \right]_0^{0.11}$$

$\int_0^{0.11} \frac{1}{1+x^4}\,dx \approx \boxed{0.11000}$

24. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$$

$$\int_0^{0.23} e^{-x^2}dx = \int_0^{0.23} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^{0.23} x^{2n} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{2n+1}}{2n+1}\Bigg]_0^{0.23}$$

$$= \left(\frac{x}{0!} - \frac{x^3}{1!(3)} + \frac{x^5}{2!(5)}\right)\Bigg]_0^{0.23}$$

$$= \left(x - \frac{x^3}{3} + \frac{x^5}{10}\right)\Bigg]_0^{0.23}$$

$\int_0^{0.23} e^{-x^2}\,dx \approx$ $\boxed{0.226}$

25. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x^{1/2})^{2n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$f(x) = 4x\cos(\sqrt{x}) = 4x\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1}$$

$$\int_0^{0.45} 4x\cos(\sqrt{x})dx = \int_0^{0.45} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\int_0^{0.45} x^{n+1} dx$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\frac{x^{n+2}}{n+2}\Bigg]_0^{0.45}$$

$$= \left(\frac{4x^2}{0!(2)} - \frac{4x^4}{2!(4)} + \frac{4x^6}{4!(6)} - \frac{4x^8}{6!(8)}\right)\Bigg]_0^{0.45}$$

$$= \left(2x^2 - \frac{x^4}{2} + \frac{x^6}{36} - \frac{x^8}{1440}\right)\Bigg]_0^{0.45}$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ $\boxed{0.34847}$

26. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

Note $1.56 = 1 + 0.56$

$$\ln(1+0.56) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (0.56)^n$$

$$= 0.56 - \frac{(0.56)^2}{2} + \frac{(0.56)^3}{3}$$

$\ln(1.56) \approx$ $\boxed{0.46174}$

27. Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11) - 9}$$

$\sqrt{?} \to ? \geq 0$

$\sqrt{x+y-1} \to x+y-1 \geq 0$

$x+y \geq 1$

$\ln(?) \to ? > 0$

$\ln(y-11) \to y-11 > 0$

$y > 11$

$\frac{1}{?} \to ? \neq 0$

$\ln(y-11) - 9 \neq 0$

$\ln(y-11) \neq 9$

$y - 11 \neq e^9$

$y \neq e^9 + 11$

Domain = $\boxed{\{(x,y) \mid x+y \geq 1,\ y > 11,\ y \neq 11 + e^9\}}$

28. Find the domain of

$$f(x,y) = \frac{\ln(x^2 - y + 3)}{\sqrt{x-6}}$$

$\ln(?) \to ? > 0$

$\ln(x^2 - y + 3) \to x^2 - y + 3 > 0$

$x^2 + 3 > y$

$\frac{1}{\sqrt{?}} \to ? > 0$

$\frac{1}{\sqrt{x-6}} \to x - 6 > 0$

$x > 6$

Domain = $\boxed{\{(x,y) \mid x > 6,\ x^2 + 3 > y\}}$

29. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \quad C = \ln(36)$$

$\ln(x^2+y^2)=\ln(36)$

$x^2+y^2=36$

$x^2+y^2=6^2$

(a) Parabola with vertices at $(0, 0)$
(b) Circle with center at $(0, \ln(36))$ and radius 6
(c) Parabola with vertices at $(0, \ln(36))$
(d) Circle with center at $(0, 0)$ and radius 6
(e) Increasing Logarithm Function

30. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

$\ln(y-e^{5x})=C$

$y-e^{5x}=e^C$

$y-e^{5x}=C$

$y=e^{5x}+C$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

31. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

$\sqrt{y+4x^2}=C$

$y+4x^2=C^2$

$y+4x^2=C$

$y=-4x^2+C$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

32. Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y^2-1}}$$

$$f_x(x,y) = \frac{d}{dx}\left(\frac{(6x+6y)^2}{\sqrt{y^2-1}}\right)$$

$$= \frac{1}{\sqrt{y^2-1}}\frac{d}{dx}\left((6x+6y)^2\right)$$

$$= \frac{1}{\sqrt{y^2-1}}\cdot 2(6x+6y)\frac{d}{dx}(6x+6y)$$

$$= \frac{1}{\sqrt{y^2-1}}\cdot 2(6x+6y)\cdot 6$$

$$= \frac{72x+72y}{\sqrt{y^2-1}}$$

$f_x(6,5)=$ $\boxed{792/\sqrt{24}}$

33. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$$f(x,y) = 3x^2\cdot\frac{y^3}{(y-1)^2}$$

$$f_x(x,y) = \frac{d}{dx}\left(3x^2\cdot\frac{y^3}{(y-1)^2}\right) = \frac{y^3}{(y-1)^2}\cdot\frac{d}{dx}\left(3x^2\right) = \frac{y^3}{(y-1)^2}\cdot 6x$$

$$f_y(x,y) = \frac{d}{dy}\left(3x^2\cdot\frac{y^3}{(y-1)^2}\right) = 3x^2\frac{d}{dy}\left(\frac{y^3}{(y-1)^2}\right) = 3x^2\left(\frac{3y^2(y-1)^2-y^3\cdot 2(y-1)}{(y-1)^4}\right)$$

$$= 3x^2\left(\frac{\cancel{(y-1)}\left[3y^2(y-1)-2y^3\right]}{(y-1)^{\cancel{4}\,3}}\right) = \frac{3x^2(3y^3-3y^2-2y^3)}{(y-1)^3}$$

$$= \frac{3x^2(y^3-3y^2)}{(y-1)^3}$$

$f_x(x,y) =$ $6xy^3/(y-1)^2$

$f_y(x,y) =$ $\dfrac{3x^2(y^3-3y^2)}{(y-1)^3}$

34. Find the first order partial derivatives of

$$f(x,y) = (xy-1)^2$$

$$f_x(x,y) = \frac{d}{dx}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dx}(xy-1)$$
$$= 2(xy-1)\,y$$
$$= 2xy^2 - 2y$$

$$f_y(x,y) = \frac{d}{dy}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dy}(xy-1)$$
$$= 2(xy-1)\,x$$
$$= 2x^2y - 2x$$

$f_x(x,y) =$ $\boxed{2xy^2 - 2y}$

$f_y(x,y) =$ $\boxed{2x^2y - 2x}$

35. Given the function $f(x,y) = 4x^5 \tan(3y)$, compute $f_{xy}(2, \pi/3)$

$$f_x(x,y) = \frac{d}{dx}\left(4x^5 \tan(3y)\right) = \tan(3y)\cdot\frac{d}{dx}(4x^5)$$
$$= \tan(3y)\cdot(20x^4)$$

$$f_{xy}(x,y) = \frac{d}{dy}(f_x(x,y)) = \frac{d}{dy}\left(\tan(3y)\cdot(20x^4)\right) = 20x^4\frac{d}{dy}(\tan(3y))$$
$$= 20x^4\cdot\sec^2(3y)\cdot 3$$
$$= 60x^4\sec^2(3y)$$

$$f_{xy}(2, \pi/3) = 60(2)^4\sec^2(3\pi/3)$$
$$= 60(16)\sec^2(\pi)$$
$$= 960$$

$f_{xy}(2, \pi/3) =$ $\boxed{960}$

36. Find the second order partial derivatives of

$$f(x,y) = x^2 y \ln(7x)$$

$$f(x,y) = (x^2 \ln(7x))\, y$$

$$f_x(x,y) = \frac{d}{dx}\left((x^2 \ln(7x)) \cdot y\right) = y \frac{d}{dx}\left(x^2 \ln(7x)\right)$$

$$= y\left(2x\ln(7x) + x^2 \frac{1}{7x} \cdot 7\right) = y(2x\ln(7x) + x)$$

$$f_{xx}(x,y) = \frac{d}{dx}\left(y(2x\ln(7x) + x)\right) = y \frac{d}{dx}(2x\ln(7x) + x)$$

$$= y\left(2\ln(7x) + 2x \cdot \frac{1}{7x} \cdot 7 + 1\right) = y(2\ln(7x) + 2 + 1)$$

$$= y(2\ln(7x) + 3)$$

$$f_{xy}(x,y) = \frac{d}{dy}\left(y(2x\ln(7x) + x)\right) = (2x\ln(7x) + x)\frac{d}{dy}(y)$$

$$= 2x\ln(7x) + x$$

$$f_y(x,y) = \frac{d}{dy}\left((x^2\ln(7x)) \cdot y\right) = (x^2 \ln(7x))\frac{d}{dy}(y) = x^2 \ln(7x)$$

$$f_{yy}(x,y) = \frac{d}{dy}(x^2 \ln(7x)) = 0$$

$f_{xx}(x,y) =$ $(2\ln(7x) + 3)y$

$f_{xy}(x,y) =$ $2x\ln(7x) + x$

$f_{yy}(x,y) =$ 0