Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:______________________________

1. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where $t$ is time in hours after 9:00 am and the rate $r(t)$ is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

Answer:______________________________

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Answer:______________________________

2. Which derivative rule is undone by integration by substitution?

(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these

3. Which derivative rule is undone by integration by parts?

(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these

4. What would be the best substitution to make the solve the given integral?

$$\int e^{2x} \cos(e^{2x}) \sin^3(e^{2x})\, dx$$

$u =$ ____________________

5. What would be the best substitution to make the solve the given integral?

$$\int \sec^2(5x) e^{\tan(5x)}\, dx$$

$u =$ ____________________

6. Find the area under the curve $y = 14e^{7x}$ for $0 \leq x \leq 4$.

Area = ______________________

7. Evaluate the definite integral.

$$\int_0^2 (5e^{2x} + 8)\, dx$$

$\int_0^2 (5e^{2x} + 8)\, dx =$ ______________________

8. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx$$

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx = \underline{\hspace{6cm}}$$

9. Evaluate the indefinite integral.

$$\int 9x^3 e^{-x^4}\,dx$$

$$\int 9x^3 e^{-x^4}\,dx = \underline{\hspace{8cm}}$$

10. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that $t$ hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$L'(t) = \sqrt{3t+2} \quad \text{gallows per hour.}$$

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.

Answer:____________________

11. Evaluate

$$\int 3x \ln(x^7)\, dx$$

$\int 3x \ln(x^7)\, dx$=____________________

12. Evaluate the indefinite integral

$$\int \frac{\ln(5x)}{x}\,dx$$

$$\int \frac{\ln(5x)}{x}\,dx = \underline{\hspace{6cm}}$$

13. Evaluate

$$\int_1^e \frac{\ln(x^4)}{x}\,dx$$

$$\int_1^e \frac{\ln(x^4)}{x}\,dx = \underline{\hspace{6cm}}$$

14. The population of pink elephants in Dumbo's dreams, in hundreds, $t$ years after the year 1980 is given by

$$P(t) = \frac{e^{5t}}{1 + e^{5t}}$$

What is the average population during the decade between 1980 and 2000?

Answer:____________________________

15. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{3x + 1}{x^2(x+1)^2(x^2+1)}$$

(A)

$$\frac{A}{x^2} + \frac{B}{(x+1)^2} + \frac{C}{x^2+1}$$

(B)

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{x^2+1}$$

(C)

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{Ex+F}{x^2+1}$$

(D)

$$\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{D}{x+1} + \frac{Ex+F}{(x+1)^2} + \frac{Gx+H}{x^2+1}$$

(E)

$$\frac{A}{x} + \frac{B}{(x+1)^2} + \frac{C}{x^2+1}$$

16. Determine the partial fraction decomposition of

$$\frac{7x^2+9}{x(x^2+3)}$$

Answer:______________________________

17. Evaluate $\int \frac{5x^2+9}{x^2(x+3)}\,dx$

$\int \frac{5x^2+9}{x^2(x+3)}\,dx=$______________________________

18. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1 - \cos x}\, dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

19. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \tan(x)\, dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

20. Evaluate the following integral;

$$\int_0^{\infty} e^{-3x} dx$$

$$\int_0^{\infty} e^{-3x} dx \quad = \quad \underline{\hspace{6cm}}$$

21. Evaluate the following integral;

$$\int_1^{\infty} \frac{5}{\sqrt{x}} dx$$

$$\int_1^{\infty} \frac{5}{\sqrt{x}} dx \quad = \quad \underline{\hspace{6cm}}$$

22. Evaluate the following integral;

$$\int_1^{\infty} \frac{3}{x^2} dx$$

$$\int_1^{\infty} \frac{3}{x^2} dx \quad = \quad \underline{\hspace{6cm}}$$

23. Evaluate the following integral;

$$\int_1^{\infty} \frac{10}{x} dx$$

$$\int_1^{\infty} \frac{10}{x} dx \quad = \quad \underline{\hspace{6cm}}$$

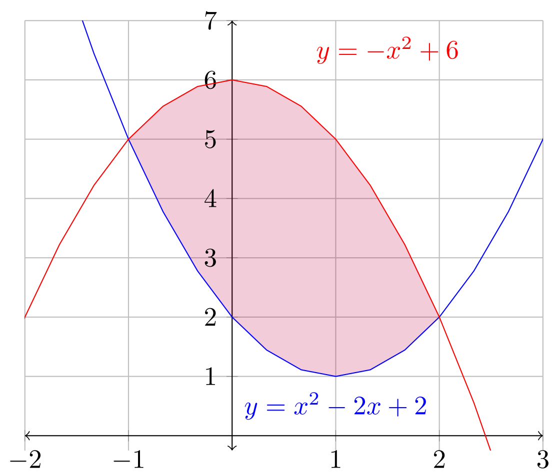
24. Evaluate the definite integral

$$\int_2^\infty \frac{dx}{5x+2}$$

$$\int_2^\infty \frac{dx}{5x+2} = \underline{\hspace{5cm}}$$

25. Set up the integral that computes the **AREA** shown to the right with respect to $x$.

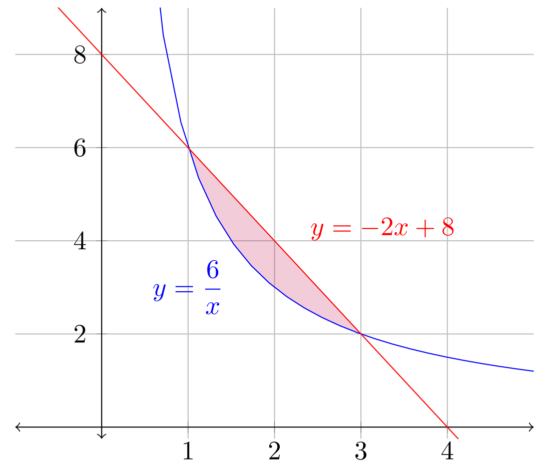
**DON'T COMPUTE IT!!!**




Area $=$ ____________________

26. Set up the integral that computes the **AREA** shown to the right with respect to $y$.

**DON'T COMPUTE IT!!!**




Area $=$ ____________________

27. Set up the integral that computes the **AREA** with respect to $x$ of the region bounded by

$$y = \frac{2}{x} \quad \text{and} \quad y = -x + 3$$

Area = ______________________

28. Find the area of the region bounded by $y = 6x - x^2$ and $y = 2x^2$.

Area = ______________________

29. Calculate the **AREA** of the region bounded by the following curves.

$$x = 100 - y^2 \quad \text{and} \quad x = 2y^2 - 8$$

Area = ______________________

30. Calculate the **AREA** of the region bounded by the following curves.

$$y = x^3 \quad \text{and} \quad y = x^2$$

Area = ______________________

31. After $t$ hours studying, one student is working $Q_1(t) = 25 + 9t - t^2$ problems per hour, and a second student is working on $Q_2(t) = 5 - t + t^2$ problems per hour. How many more problems will the first student have done than the second student after 10 hours?

Answer:______________________________

32. Set up the integral that computes the **VOLUME** of the region bounded by

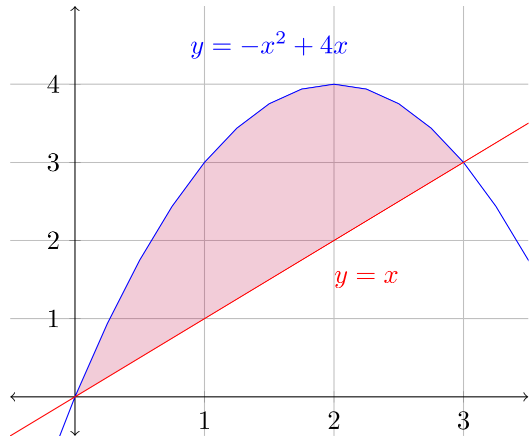
$$y = x + 8, \quad \text{and} \quad y = (x - 4)^2$$

about the x-axis

Volume $=$ ______________________________

33. Let $R$ be the region shown below. Set up the integral that computes the **VOLUME** as $R$ is rotated around the x-axis.

**DON'T COMPUTE IT!!!**




Volume $=$ ____________________

34. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{16 - x}, \quad y = 0 \quad \text{and} \quad x = 0$$

about the y-axis

Volume $=$ ____________________

35. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 0 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis

Volume $=$ ______________________

36. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y = \sqrt{x}$$

about the y-axis

Volume $=$ ______________________

37. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y^2 = X$$

about the x-axis

Volume $=$ ______________________________

38. Find the **VOLUME** of the region bounded by

$$y = x - x^2, \quad \text{and} \quad y = 0$$

around the x-axis

Volume $=$ ______________________________

39. Find the **VOLUME** of the region bounded by

$$y = 10x, \quad x = 0, \quad y = 10$$

around the y-axis

Volume $=$ ______________________

40. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 3, \quad x = 0, \quad y = 9$$

around the y-axis

Volume $=$ ______________________

41. Find the **VOLUME** of the region bounded by

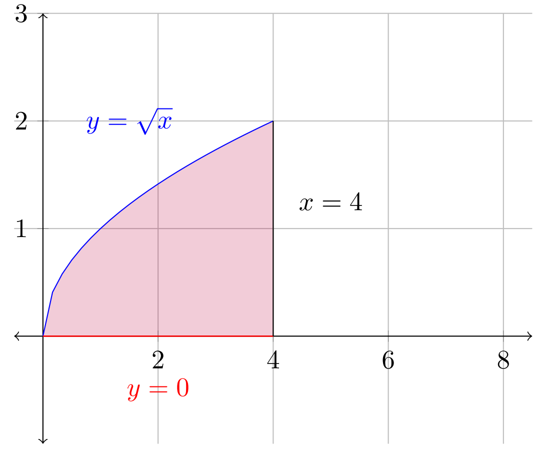
$$x + 3y = 9, \quad x = 0, \quad y = 0$$

around the y-axis

Volume $=$ ____________________

42. Let $R$ be the region shown to the right. Set up the integral that computes the **VOLUME** as $R$ is rotated around the line $x = 4$.

**DON'T COMPUTE IT!!!**




Volume $=$ ____________________

43. Set up the integral needed to find the volume of the solid obtained when the region bounded by

$$y = 2 - x^2 \quad \text{and} \quad y = x^2$$

is rotated about the line $y = 3$.

Volume $=$ ____________________________

44. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Volume $=$ ____________________________

45. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

$y \quad = \quad$ ______________________________

46. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

$y \quad = \quad$ ______________________________

47. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

$y \quad = \quad$ ______________________________

48. Let $y$ denote the mass of a radioactive substance at time $t$. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time $t$ in hours does half the original mass remain? Round your answer to 3 decimal places.

$t \quad = \quad$ ______________________________

49. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

$y =$ ______________________

50. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$y =$ ______________________

51. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$y =$ ____________________

52. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$y =$ ____________________

53. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \text{ and } y(0) = 4$$

$y =$ ________________________________

54. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \text{ and } y(0) = 1$$

$y =$ ________________________________

55. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \quad \text{where} \quad y = 10 \quad \text{when} \quad x = 2$$

Find the value of the integration constant, $C$.

$C =$ ______________________________

56. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right) y = 10\ln(x)$$

$u(x) =$ ______________________________

57. What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = (x+1)e^{x^2}$$

$u(x) =$ ____________________

58. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) =$ ____________________

59. Solve the initial value problem.

$$x^4y' + 4x^3 \cdot y = 10x^9 \ \text{ with } \ f(1) = 23$$

$y =$ ________________________________

60. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \dots$$

Answer:______________________________

(b) Use the sum from (a) and compute the sum.

Answer:______________________________

61. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \underline{\hspace{6cm}}$$

62. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^{n}$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^{n} =$ ______________________________

63. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ ______________________________

64. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \underline{\hspace{8cm}}$$

65. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \underline{\hspace{8cm}}$$

66. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

$R =$ ________________________

67. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

$R =$ ________________________

68. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$\dfrac{3}{1+2x} =$ ______________________________

$R =$ ______________________________

69. Express $f(x) = \dfrac{5x}{3+2x^2}$ as a power series and determine it's radius of converge.

$\dfrac{5x}{3+2x^2} =$ ______________________________

$R =$ ______________________________

70. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$\int \sin(x^{3/2})\,dx =$ ____________________

71. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx$$

$\int_0^{0.11} \frac{1}{1+x^4}\,dx \approx$ ____________________

72. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\,dx$$

$\int_0^{0.23} e^{-x^2}\,dx \approx$ ____________________

73. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ ____________________

74. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$\ln(1.56) \approx$ ______________________

75. Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

Domain = ______________________

76. Find the domain of

$$f(x,y) = \frac{\ln(x^2-y+3)}{\sqrt{x-6}}$$

Domain = ______________________

77. Describe the indicated level curves $f(x,y) = C$

$$f(x,y) = \ln(x^2 + y^2) \;\; C = \ln(36)$$

(a) Parabola with vertices at $(0,0)$

(b) Circle with center at $(0,\ln(36))$ and radius 6

(c) Parabola with vertices at $(0,\ln(36))$

(d) Circle with center at $(0,0)$ and radius 6

(e) Increasing Logarithm Function

78. What do the level curves for the following function look like?

$$f(x,y) = \ln(y - e^{5x})$$

(a) Increasing exponential functions

(b) Rational Functions with x-axis symmetry

(c) Natural logarithm functions

(d) Decreasing exponential functions

(e) Rational Functions with y-axis symmetry

79. What do the level curves for the following function look like?

$$f(x,y) = \sqrt{y + 4x^2}$$

(a) Lines

(b) Parabolas

(c) Circles

(d) Point at the origin

(e) Ellipses

(f) Hyperbolas

80. Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y^2-1}}$$

$f_x(6,5)$= ____________________

81. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$f_x(x,y) =$ ____________________

$f_y(x,y) =$ ____________________

82. Find the first order partial derivatives of $f(x, y) = (xy - 1)^2$

$f_x(x, y) =$ ______________________

$f_y(x, y) =$ ______________________

83. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$f_x(x, y) =$ ______________________

$f_y(x, y) =$ ______________________

84. Find the first order partial derivatives of $f(x, y) = y\cos(x^2 y)$

$f_x(x, y) =$ ______________________________

$f_y(x, y) =$ ______________________________

85. Given the function $f(x, y) = 4x^5 \tan(3y)$, compute $f_{xy}(2, \pi/3)$

$f_{xy}(2, \pi/3) =$ ______________________________

86. Find the second order partial derivatives of

$$f(x, y) = x^2 y \ln(7x)$$

$f_{xx}(x, y) =$ ____________________

$f_{xy}(x, y) =$ ____________________

$f_{yy}(x, y) =$ ____________________

87. Find the discriminant of

$$f(x, y) = e^x \sin(y)$$

Simplify your answer. Note: $\sin^2(y) + \cos^2(y) = 1$.

$D(x, y) =$ ______________________

88. Using the information in the table below, classify the critical points for the function $g(x, y)$.

| $(a, b)$ | $g_{xx}(a, b)$ | $g_{yy}(a, b)$ | $g_{xy}(a, b)$ |
|---|---|---|---|
| $(4, 5)$ | 0 | 4 | $-2$ |
| $(5, -10)$ | 5 | $-10$ | 6 |
| $(10, 10)$ | $-4$ | $-6$ | $-4$ |
| $(7, 9)$ | 5 | 7 | 4 |
| $(4, 8)$ | 2 | 2 | 2 |

(4,5) is ______________________

(5,-10) is ______________________

(10,10) is ______________________

(7,9) is ______________________

(4,8) is ______________________

89. Classify the critical points of the function $f(x, y)$ given the partial derivatives:

$$f_x(x, y) = x - y \qquad f_y(x, y) = y^3 - x$$

(a) Two saddle points and one local minimum
(b) Two saddle points and one local maximum
(c) One saddle point, one local maximum, and one local minimum
(d) Three saddle points
(e) Two local minimums and one saddle point

Answer:________________________________________

90. The critical points for a function $f(x, y)$ are (1,1) and (2,4). Given that the partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 7x - 3y \qquad f_y(x, y) = 4x^2 - 6y$$

Classify each critical point as a maximum, minimum, or saddle point.

(1,1) is ________________________________________

(2,4) is ________________________________________

91. Fleet feet stores two most sold running shoes brands are Aesics and Brookes. The total venue from selling $x$ pairs of Aesics and $y$ pairs of Brookes is given by

$$R(x, y) = -10x^2 - 16y^2 - 4xy + 84 + 204y$$

where $x$ and $y$ are in **thousands of units**. Determine the number of Brookes shoes to be sold to maximize the revenue.

The # of Brookes shoes sold is ________________

92. Find the point(s) $(x, y)$ where the function $f(x, y) = 3x^2 + 4xy + 6x - 15$ attains maximal value, subject to the constraint $x + y = 10$.

(x,y) = ________________________________

93. Find the minimum of the function using LaGrange Multipliers of the function $f(x, y) = 2x^2 + 4y^2$ subject to the constraint $x^2 + y^2 = 1$.

Minimum Value = ______________________

94. Find the minimum value of the function $f(x, y) = 2x^2y - 3y^2$ subject to the constraint $x^2 + 2y = 1$.

Minimum Value = ______________________

95. Locate and classify the points that maximize and minimize the function $f(x, y) = 5x^2 + 10y$ subject to the constraint $5x^2 + 5y^2 = 5$.

Minimum Value occurs at ____________________

Maximum Value occurs at ____________________

96. We are baking a tasty treat where customer satisfaction is given by $S(x, y) = 6x^{3/2}y$. Here, $x$ and $y$ are the amount of sugar and spice respectively. If the sugar and spice we use must satisfy $9x + y = 4$, what is the maximum customer satisfaction we can achieve? (Note: the function is defined only for $x \geq 0$ and $y \geq 0$.) Round your answer to 2 decimal places.

Maximum Value = ____________________

97. Determine the average value of $f(x, y) = xy$ over

(a) the rectangle formed by the vertices (-1,0), (1,0), (-1,3), and (1,3)

Answer:________________________

(b) the triangle formed by the vertices (0,0), (2,0), and (2,2).

Answer:________________________

98. Find the bounds for the integral $\iint\limits_R 5e^x \sin(y)\, dA$ where $R$ is a triangle with vertices (0,0), (1,2), and (0,2).

**DON"T COMPUTE!!!**

Answer:______________________________

99. Evaluate the double integral

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x)\, dy\, dx$$

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x)\, dy\, dx = \underline{\hspace{4cm}}$$

100. Evaluate the double integral

$$\int_0^{\pi/2} \int_0^1 12x^3 \sin(y)\, dx\, dy$$

$$\int_0^1 \int_0^{\pi/2} 12x^3 \sin(y)\, dx\, dy = \rule{5cm}{0.4pt}$$

101. Evaluate the double integral

$$\int_0^4 \int_2^y (y+x)\, dx\, dy$$

$$\int_0^4 \int_2^y (y+x)\, dx\, dy = \rule{5cm}{0.4pt}$$

102. Evaluate the double integral

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\, dy\, dx$$

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\, dy\, dx = \rule{5cm}{0.4pt}$$

103. Switch the order of integration on the follow integral

$$\int_0^6 \int_{x^2}^{36} f(x,y)\, dy\, dx$$

Answer:____________________________________

104. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx$$

(Hint: Change the order of integration)

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx = \underline{\hspace{5cm}}$$

105. Evaluate the double integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy$$

Round your answer to 2 decimal places.

(Hint: Change the order of integration)

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy = \underline{\hspace{5cm}}$$