Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: Solutions

1. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where $t$ is time in hours after 9:00 am and the rate $r(t)$ is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

10:00am ⇒ 1 hr
1:00pm ⇒ 4 hrs

$$\Rightarrow \int_1^4 6t^{1/2}\,dt = 6\cdot\frac{2}{3}t^{3/2}\Big]_1^4 = 4t^{3/2}\Big]_1^4 = 28$$

Answer: $\boxed{28}$

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Solve $\int_0^t 6t^{1/2}\,dt = 121$

$$4t^{3/2} = 121$$

$$t^{3/2} = \frac{121}{4}$$

$$t = \left(\frac{121}{4}\right)^{2/3}$$

Answer: $\boxed{\left(\frac{121}{4}\right)^{2/3}}$

2. Which derivative rule is undone by integration by substitution?

(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these

3. Which derivative rule is undone by integration by parts?

(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these

4. What would be the best substitution to make the solve the given integral?

$$\int e^{2x} \cos(e^{2x}) \sin^3(e^{2x})\, dx$$

Check du is in the integral.

$u =$ $\boxed{\sin(e^{2x})}$

5. What would be the best substitution to make the solve the given integral?

$$\int \sec^2(5x) e^{\tan(5x)}\, dx$$

Check du is in the integral.

$u =$ $\boxed{\tan(5x)}$

6. Find the area under the curve $y = 14e^{7x}$ for $0 \leq x \leq 4$.

$$A = \int_0^4 14e^{7x}\,dx \quad \frac{u = 7x}{du = 7dx} \quad \int 2e^u\,du$$

$$= 2e^u = 2e^{7x}\Big]_0^4 = 2e^{28} - 2$$

Area = $\boxed{2e^{28} - 2}$

7. Evaluate the definite integral.

$$\int_0^2 (5e^{2x} + 8)\,dx$$

$$\underbrace{\int_0^2 5e^{2x}\,dx}_{u\text{-sub}} + \int_0^2 8\,dx = \frac{5}{2}e^{2x}\Big]_0^2 + 8x\Big]_0^2$$

$$= \frac{5}{2}(e^4 - e^0) + 8(2-0)$$

$$= \frac{5}{2}e^4 - \frac{5}{2} + 16$$

$$= \frac{5}{2}e^4 - \frac{27}{2}$$

$\int_0^2 (5e^{2x} + 8)\,dx =$ $\boxed{\frac{5}{2}e^4 + \frac{27}{2}}$

8. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx$$

$u = x-1$ $\quad dv = \sin(x)\,dx$

$du = dx$ $\quad v = -\cos(x)$

$$uv - \int v\,du = -(x-1)\cos x \Big]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)\,dx$$

$$= -(x-1)\cos x \Big]_0^{\pi/2} + \sin(x)\Big]_0^{\pi/2}$$

$$= \cancel{-\left(\tfrac{\pi}{2}-1\right)\cos\left(\tfrac{\pi}{2}\right)} \to 0 - \left[-(0-1)\cos(0)\right]$$

$$+ \sin\left(\tfrac{\pi}{2}\right) - \cancel{\sin(0)} \to 0$$

$$= -1 + 1 = 0$$

$\int_0^{\pi/2} (x-1)\sin(x)\,dx =$ $\boxed{0}$

9. Evaluate the indefinite integral.

$$\int 9x^3 e^{-x^4}\,dx$$

$u = -x^4$

$du = -4x^3\,dx$

$\frac{du}{-4x^3} = dx$

$$\int 9x^3 e^u \frac{du}{-4x^3} = -\frac{9}{4}\int e^u\,du$$

$$= -\frac{9}{4}e^u = -\frac{9}{4}e^{-x^4} + C$$

$\int 9x^3 e^{-x^4}\,dx =$ $\boxed{-\frac{9}{4}e^{-x^4} + C}$

10. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that $t$ hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$L'(t) = \sqrt{3t+2} \quad \text{gallows per hour.}$$

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.

i.e. $\int_0^4 (3t+2)^{1/2}\,dt$ $\quad \dfrac{u=3t+2}{du=3\,dt}$ $\quad \int u^{1/2}\dfrac{du}{3}$

$\dfrac{du}{3}=dt$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} = \frac{2}{9}(3t+2)^{3/2}\Big]_0^4$$

$$\approx 11.0122$$

Answer: $\boxed{11.0122}$

11. Evaluate

$$\int 3x\ln(x^7)\,dx$$

Rewrite $\int 3x(7\ln(x))\,dx = \int 21x\ln x\,dx$

$\dfrac{u=21\ln(x)}{du=\frac{21}{x}\,dx}$ $\quad \dfrac{dv=x\,dx}{v=\frac{x^2}{2}}$ $\quad uv-\int v\,du$

$$= \frac{21x^2\ln x}{2} - \int \frac{x^2}{2}\cdot\frac{21}{x}\,dx$$

$$= \frac{21x^2\ln x}{2} - \int \frac{21}{2}x\,dx$$

$$= \frac{21x^2\ln x}{2} - \frac{21}{2}\cdot\frac{x^2}{2} + C$$

$$= \frac{21x^2\ln x}{2} - \frac{21x^2}{4} + C$$

$\int 3x\ln(x^7)\,dx = \boxed{\dfrac{21x^2\ln x}{2} - \dfrac{21x^2}{4} + C}$

12. Evaluate the indefinite integral

$$\int \frac{\ln(5x)}{x}\,dx$$

$u = \ln(5x)$

$du = \frac{1}{5x} \cdot 5\,dx$

$du = \frac{1}{x}\,dx$

$$\int u\,du = \frac{u^2}{2} = \frac{(\ln(5x))^2}{2} + C$$

$$\int \frac{\ln(5x)}{x}\,dx = \boxed{\frac{(\ln(5x))^2}{2} + C}$$

13. Evaluate

$$\int_1^e \frac{\ln(x^4)}{x}\,dx$$

Rewrite $\int_1^e \frac{4\ln x}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$ $\quad \int 4u\,du = \frac{4u^2}{2} = 2u^2 = 2(\ln x)^2\Big]_1^e$

$$= \underbrace{2(\ln e)^2}_{2} - \underbrace{2(\ln 1)^2}_{0}$$

$$= 2$$

$$\int_1^e \frac{\ln(x^4)}{x}\,dx = \boxed{2}$$

14. The population of pink elephants in Dumbo's dreams, in hundreds, $t$ years after the year 1980 is given by

$$P(t) = \frac{e^{5t}}{1+e^{5t}}$$

What is the average population during the decade between 1980 and 2000?

i.e. $\frac{1}{2000-1980}\int_0^{20} \frac{e^{5t}}{1+e^{5t}}\,dt$ $\quad \frac{u = 1+e^{5t}}{du = 5e^{5t}dt}$ $\quad \frac{1}{20}\int \frac{e^{5t}}{u}\cdot\frac{du}{5e^{5t}}$

$\frac{du}{5e^{5t}} = dt$

$= \frac{1}{100}\int \frac{du}{u}$

$= \frac{1}{100}\ln|u|$

$= \frac{1}{100}\ln|1+e^{5t}|\Big]_0^{20}$

$\approx 0.9931$

Answer: $\boxed{0.9931 \text{ hundreds or } 993}$

15. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{3x+1}{x^2(x+1)^2(x^2+1)}$$

(A)
$$\frac{A}{x^2} + \frac{B}{(x+1)^2} + \frac{C}{x^2+1}$$

(B)
$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{E}{x^2+1}$$

(C)
$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2} + \frac{Ex+F}{x^2+1}$$

(D)
$$\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{D}{x+1} + \frac{Ex+F}{(x+1)^2} + \frac{Gx+H}{x^2+1}$$

(E)
$$\frac{A}{x} + \frac{B}{(x+1)^2} + \frac{C}{x^2+1}$$

16. Determine the partial fraction decomposition of

$$\frac{7x^2+9}{x(x^2+3)}$$

$$\frac{A}{x}+\frac{Bx+C}{x^2+3}=\frac{A(x^2+3)+x(Bx+C)}{x(x^2+3)}$$
$$=\frac{Ax^2+3A+Bx^2+Cx}{x(x^2+3)}$$
$$=\frac{(A+B)x^2+Cx+3A}{x(x^2+3)}$$

$$(A+B)x^2+Cx+3A=7x^2+0x+9$$
$$\begin{cases} A+B=7 \\ C=0 \\ 3A=9 \to A=3 \end{cases}$$
So $B=4$

Answer: $\boxed{\frac{3}{x}+\frac{4x}{x^2+3}}$

17. Evaluate $\displaystyle\int \frac{5x^2+9}{x^2(x+3)}\,dx$

$$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}=\frac{Ax(x+3)+B(x+3)+Cx^2}{x^2(x+3)}$$
$$=\frac{Ax^2+3Ax+Bx+3B+Cx^2}{x^2(x+3)}=\frac{(A+C)x^2+(3A+B)x+3B}{x^2(x+3)}$$

$$(A+C)x^2+(3A+B)x+3B=5x^2+0x+9$$
$$\begin{cases} A+C=5 \\ 3A+B=0 \\ 3B=9 \to B=3 \end{cases}$$

$3A+B=0$
$3A+3=0$
$3A=-3$
$A=-1$

$A+C=5$
$-1+C=5$
$C=6$

$$\int -\frac{1}{x}\,dx+\int \frac{3}{x^2}\,dx+\int \frac{6}{x+3}\,dx=$$

$$\int \frac{5x^2+9}{x^2(x+3)}\,dx= \boxed{-\ln|x|-\frac{3}{x}+6\ln|x+3|+C}$$

18. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1-\cos x}\, dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$1 - \cos x = 0$

$1 = \cos x$

$x = 0, \pi, 2\pi$

19. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \tan(x)\, dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$\tan x = \dfrac{\sin x}{\cos x}$

$\cos x = 0$

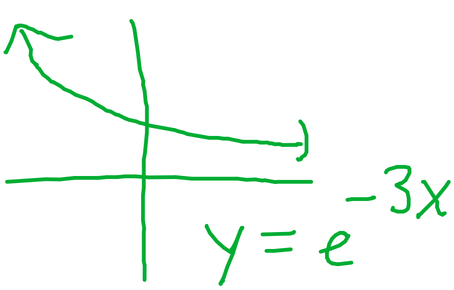
$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \ldots$

20. Evaluate the following integral;

$$\int_0^{\infty} e^{-3x} dx$$

$$\int_0^\infty e^{-3x} dx = \lim_{N\to\infty} \int_0^N e^{-3x} dx = \lim_{N\to\infty} \left(\frac{e^{-3x}}{-3}\right)\Bigg]_0^N$$

$$= \lim_{N\to\infty} \left(\frac{e^{-3N}}{-3} + \frac{1}{3}\right) = 0 + 1/3$$




$$\int_0^{\infty} e^{-3x} dx = \boxed{1/3}$$

21. Evaluate the following integral;

$$\int_1^\infty \frac{5}{\sqrt{x}}dx$$

$$\int_1^\infty \frac{5}{\sqrt{x}}dx = \lim_{N\to\infty}\int_1^N 5x^{-1/2}dx = \lim_{N\to\infty}\left(5\cdot 2x^{1/2}\right)\Big]_1^N$$

$$= \lim_{N\to\infty}\left(10(N)^{1/2} - 10\right) = \infty$$

$$\int_1^\infty \frac{5}{\sqrt{x}}dx = \boxed{\infty}$$

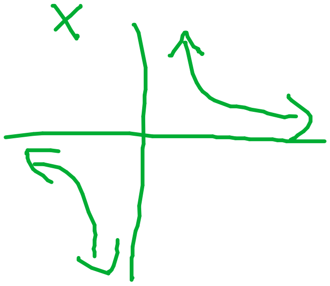
22. Evaluate the following integral;

$$\int_1^\infty \frac{3}{x^2}dx$$

$$\int_1^\infty \frac{3}{x^2}dx = \lim_{N\to\infty}\int_1^N 3x^{-2}dx = \lim_{N\to\infty}\left(\frac{3x^{-1}}{-1}\right]_1^N$$

$$= \lim_{N\to\infty}\left(-\frac{3}{x}\right)\Big]_1^N = \lim_{N\to\infty}\left(-\frac{3}{x} + \frac{3}{1}\right)$$

$y = -\frac{1}{x}$



$$\int_1^\infty \frac{3}{x^2}dx = \boxed{3}$$

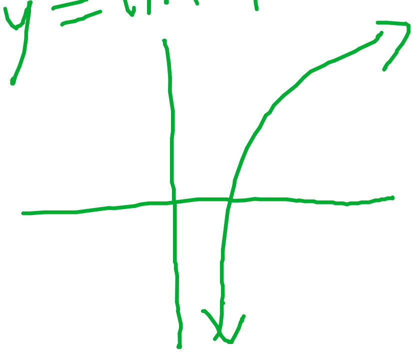
23. Evaluate the following integral;

$$\int_1^\infty \frac{10}{x}dx$$

$$\int_1^\infty \frac{10}{x}dx = \lim_{N\to\infty}\int_1^N \frac{10}{x}dx = \lim_{N\to\infty}\left(10\ln|x|\right)\Big]_1^N$$

$$= \lim_{N\to\infty}\left(10\ln|N| - 0\right)$$

$y = \ln|x|$



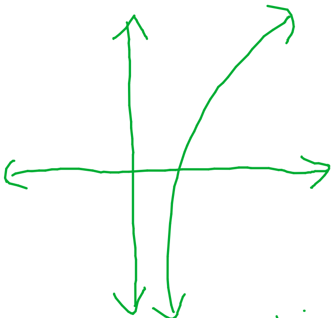
$$\int_1^\infty \frac{10}{x}dx = \boxed{\infty}$$

24. Evaluate the definite integral

$$\int_2^\infty \frac{dx}{5x+2}$$

$\lim_{N\to\infty}\int_2^N \frac{dx}{5x+2}$ $\quad u=5x+2,\ du=5\,dx,\ \frac{du}{5}=dx$ $\quad \lim_{N\to\infty}\int \frac{1}{5}\frac{1}{u}\,du = \lim_{N\to\infty}\frac{1}{5}\ln|u| = \lim_{N\to\infty}\frac{1}{5}\ln|5x+2|\Big]_2^N$

$= \lim_{N\to\infty}\left(\frac{1}{5}\ln|5N+2| - \frac{1}{5}\ln|12|\right) = \infty$

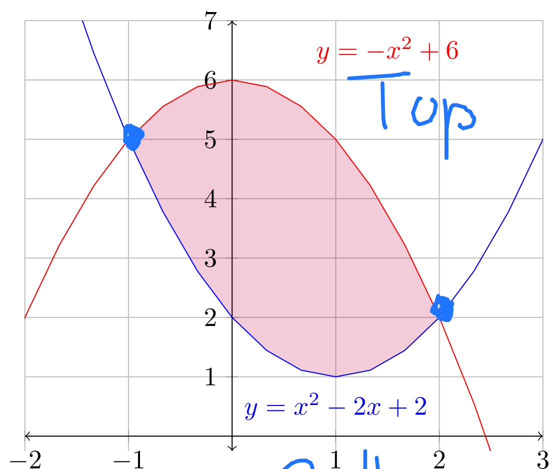


ln-function

$$\int_2^\infty \frac{dx}{5x+2} = \boxed{\infty}$$

25. Set up the integral that computes the **AREA** shown to the right with respect to $x$.

**DON'T COMPUTE IT!!!**




Top

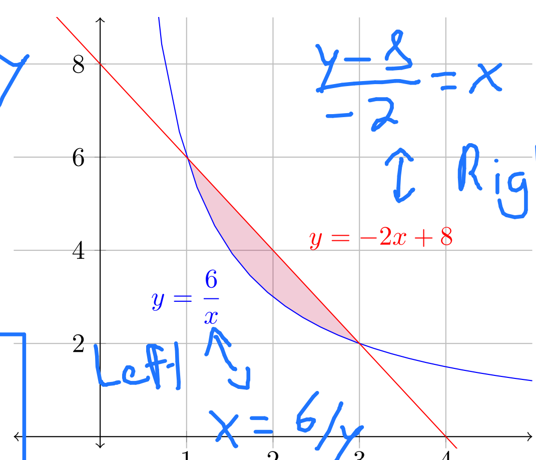
Bottom

$$\boxed{\int_{-1}^{2}(-x^2+6)-(x^2-2x+2)\,dx}$$

Area = ____________________

26. Set up the integral that computes the **AREA** shown to the right with respect to $y$. $\to dy$

**DON'T COMPUTE IT!!!**




$\frac{y-8}{-2}=x$

Right

Left

$x = 6/y$

Area = $\boxed{\int_2^6 \left(\frac{y-8}{-2}\right) - \frac{6}{y}\,dy}$

27. Set up the integral that computes the **AREA** with respect to $x$ of the region bounded by

$$y = \frac{2}{x} \quad \text{and} \quad y = -x + 3$$

$\to dx$ problem

Bounds:

$\frac{2}{x} = -x+3$

$2 = -x^2+3x$

$x^2-3x+2=0$

$(x-1)(x-2)=0$

$x=1,2$

Test Pt: $x=1.5$

$y=\frac{2}{x} \Rightarrow y=\frac{2}{1.5}=\frac{4}{3}\approx 1.33 \to$ Bottom

$y=-x+3 \Rightarrow y=-1.5+3=1.5 \to$ Top

Area = $\boxed{\int_1^2 \left(-x+3-\frac{2}{x}\right)dx}$

28. Find the area of the region bounded by $y = 6x - x^2$ and $y = 2x^2$.

Bounds:

$6x-x^2=2x^2$

$6x-3x^2=0$

$3x(2-x)=0$

$x=0,2$

Test Pt: $x=1$

$y=6x-x^2 \Rightarrow y=5 \to$ Top

$y=2x^2 \Rightarrow y=2 \to$ Bottom

$A=\int_0^2 [(6x-x^2)-2x^2]dx$

$=\int_0^2 (6x-3x^2)dx$

$=(3x^2-x^3)\Big]_0^2 = 4$

Area = $\boxed{4}$

29. Calculate the **AREA** of the region bounded by the following curves.

$$x = 100 - y^2 \quad \text{and} \quad x = 2y^2 - 8$$

Bounds:

$100 - y^2 = 2y^2 - 8$

$108 = 3y^2$

$36 = y^2$

$y = \pm 6$

Test Pt: $y = 0$

$x = 100 - y^2 \rightarrow x = 0 \rightarrow$ Right

$x = 2y^2 - 8 \rightarrow x = -8 \rightarrow$ Left

$A = \int_{-6}^{6} (100 - y^2) - (2y^2 - 8)\, dy$

$= \int_{-6}^{6} (108 - 3y^2)\, dy$

$= (108y - y^3)\Big]_{-6}^{6}$

$= 864$

Area = $\boxed{864}$

30. Calculate the **AREA** of the region bounded by the following curves.

$$y = x^3 \quad \text{and} \quad y = x^2$$

Bounds:

$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0, 1$

Test Pt: $x = \frac{1}{2}$

$y = x^3 \rightarrow y = \frac{1}{8} \rightarrow$ Bottom

$y = x^2 \rightarrow y = \frac{1}{4} \rightarrow$ Top

$A = \int_0^1 (x^2 - x^3)\, dx$

$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big]_0^1$

$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

Area = $\boxed{1/12}$

31. After $t$ hours studying, one student is working $Q_1(t) = 25 + 9t - t^2$ problems per hour, and a second student is working on $Q_2(t) = 5 - t + t^2$ problems per hour. How many more problems will the first student have done than the second student after 10 hours?

$$\int_0^{10} Q_1(t) - Q_2(t)\, dt$$

$$= \int_0^{10} (25 + 9t - t^2) - (5 - t + t^2)\, dt$$

$$= \int_0^{10} (20 + 10t - 2t^2)\, dt$$

$$= \left(20t + 5t^2 - \frac{2}{3}t^3\right)\Big]_0^{10}$$

$$= \frac{100}{3}$$

Answer: $\boxed{100/3}$

32. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x + 8, \quad \text{and} \quad y = (x-4)^2$$

about the x-axis

Bounds:

$x + 8 = (x-4)^2$

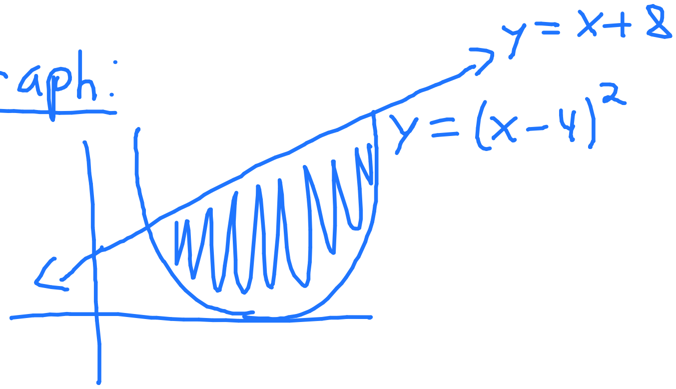
$x + 8 = x^2 - 8x + 16$

$0 = x^2 - 9x + 8$

$0 = (x-8)(x-1)$

$x = 1, 8$

Graph:




Volume $= \boxed{\pi \int_1^8 \left[(x+8)^2 - (x-4)^4\right] dx}$

33. Let $R$ be the region shown below. Set up the integral that computes the **VOLUME** as $R$ is rotated around the x-axis.

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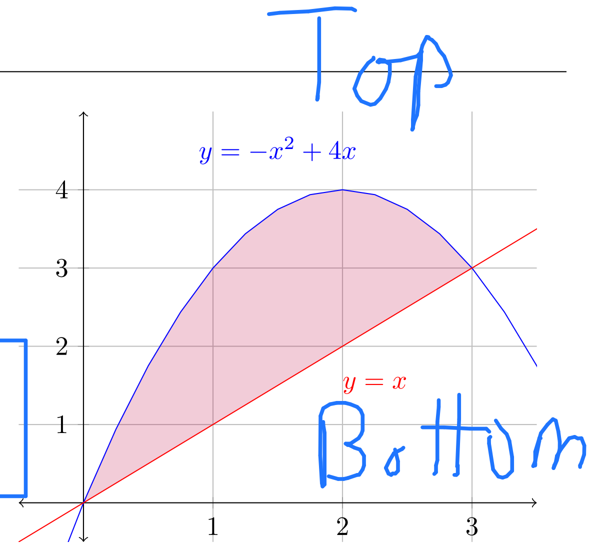



$$\boxed{\pi \int_0^3 \left[(-x^2+4x)^2 - (x)^2\right] dx}$$

Volume $=$ ____________________

34. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{16 - x}, \quad y = 0 \quad \text{and} \quad x = 0$$

about the y-axis $\Rightarrow$ dy problem

Disk

$y = \sqrt{16-x}$

$y^2 = 16 - x$

$x = 16 - y^2$

Bounds: Given $y=0$

Plug $x=0$ into $y = \sqrt{16-x}$

$y = \sqrt{16-x}$

$y = \sqrt{16}$

$y = 4$

Volume $=$ $\boxed{\pi \int_0^4 (16-y^2)^2 \, dy}$

35. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 0 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis


Disk

$$V = \pi \int_1^3 (7x)^2\,dx$$

$$= \pi \int_1^3 49x^2\,dx$$

$$= \pi \left(\frac{49x^3}{3}\right)\Big]_1^3$$

$$= \frac{49\pi}{3}(3^3 - 1)$$

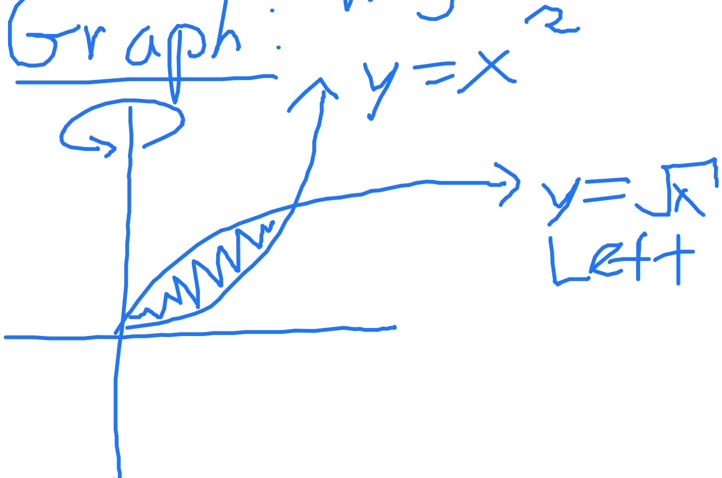
Volume $= \boxed{\frac{1274}{3}\pi}$

36. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y = \sqrt{x}$$

about the y-axis

Graph:




Bounds:

$$\sqrt{y} = y^2$$

$$y = y^4$$

$$0 = y^4 - y$$

$$0 = y(y^3 - 1)$$

$$y = 0, 1$$

But y-axis ⇒ dy

Right → $y = x^2 \to x = \sqrt{y}$

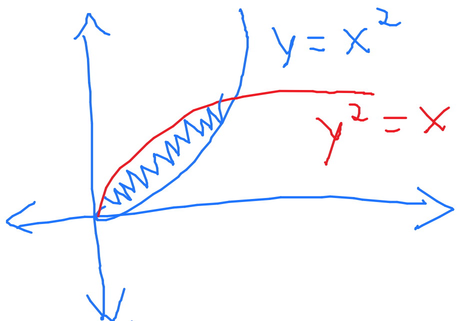
Left → $y = \sqrt{x} \to x = y^2$

Volume $= \boxed{\pi \int_0^1 \left[(\sqrt{y})^2 - (y^2)^2\right] dy}$

37. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y^2 = x$$

about the x-axis $\rightarrow$ dx problem




$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$$

**Bounds**

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

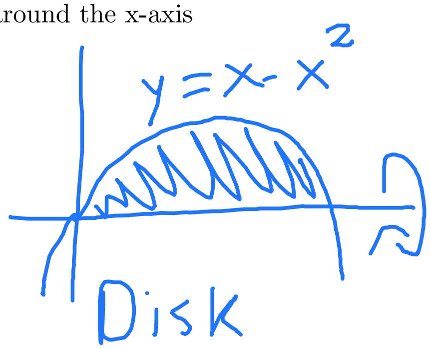
$x = 0, 1$

Volume = $\boxed{\pi \int_0^1 (x - x^4) \, dx}$

38. Find the **VOLUME** of the region bounded by

$$y = x - x^2, \quad \text{and} \quad y = 0$$

around the x-axis




$$V = \pi \int_0^1 (x - x^2)^2 \, dx$$

$$= \pi \int_0^1 (x^2 - 2x^3 + x^4) \, dx$$

$$= \pi \left( \frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \Big]_0^1$$

$$= \frac{\pi}{30}$$

**Bounds:**

$x - x^2 = 0$

$x(1 - x) = 0$

$x = 0, 1$

Volume = $\boxed{\pi/30}$

39. Find the **VOLUME** of the region bounded by

$$y = 10x, \quad x = 0, \quad y = 10$$

around the y-axis

Graph:


Disk

But y-axis ⇒ dy problem

$y = 10x$

$\frac{y}{10} = x$

$V = \pi \int_0^{10} \left(\frac{y}{10}\right)^2 dy$

$= \pi \int_0^{10} \frac{y^2}{100}\, dy$

$= \frac{\pi}{100}\left(\frac{y^3}{3}\right)\Big]_0^{10}$

$= \frac{10\pi}{3}$

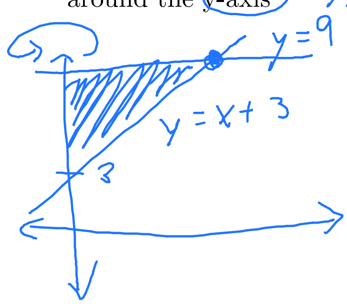
Volume = $\boxed{10\pi/3}$

40. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 3, \quad x = 0, \quad y = 9$$

$x = y - 3$

around the y-axis → dy problem.




$V = \pi \int_3^9 (y-3)^2 dy$

$= \pi \int_3^9 (y^2 - 6y + 9)\, dy$

$= \pi\left(\frac{y^3}{3} - 3y^2 + 9y\right)\Big]_3^9$

Volume = $\boxed{72\pi}$

41. Find the **VOLUME** of the region bounded by

$$x + 3y = 9, \quad x = 0, \quad y = 0$$

around the y-axis

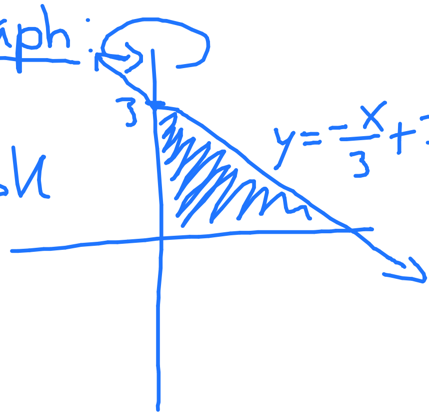
$x + 3y = 9$

$3y = -x + 9$

$y = \frac{-x}{3} + 3$

Graph:

Disk




But y-axis $\Rightarrow dy$

So $x + 3y = 9$

$x = 9 - 3y$

$$V = \pi \int_0^3 (9-3y)^2 \, dy$$

$$= \pi \int_0^3 (81 - 54y + 9y^2) \, dy$$

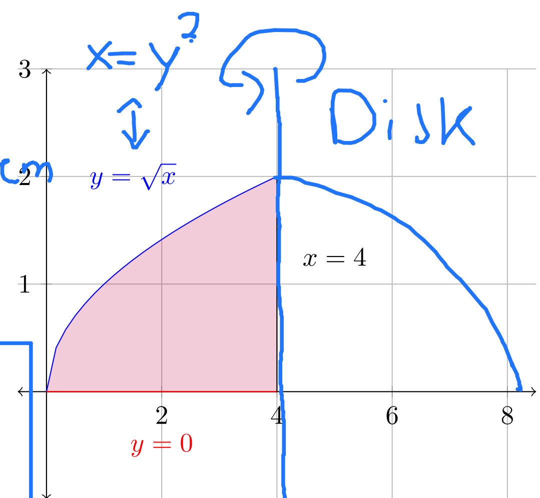
$$= \pi \left(81y - 27y^2 + 3y^3\right)\Big]_0^3$$

$$= 81\pi$$

Volume = $\boxed{81\pi}$

42. Let $R$ be the region shown to the right. Set up the integral that computes the **VOLUME** as $R$ is rotated around the line $x = 4$. $\Rightarrow$ dy problem

**DON'T COMPUTE IT!!!**




Volume = $\boxed{\pi \int_0^2 (y^2 - 4)^2 \, dy}$

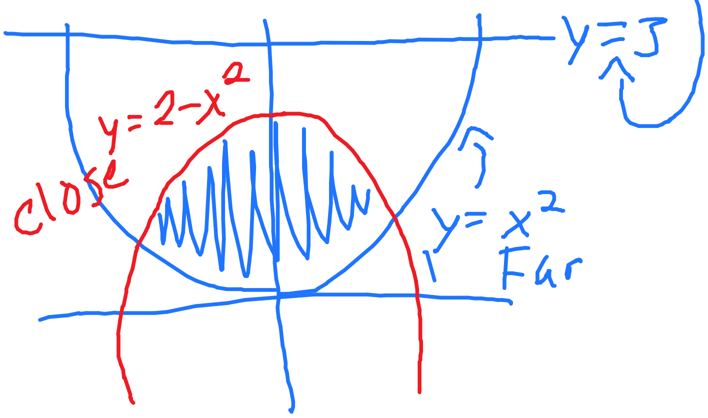
43. Set up the integral needed to find the volume of the solid obtained when the region bounded by

$$y = 2 - x^2 \quad \text{and} \quad y = x^2$$

is rotated about the line $y = 3$.

$y = 3 \Rightarrow dx$ problem

Graph      Washer




Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

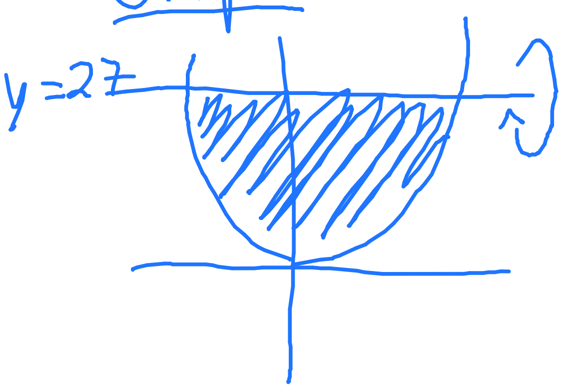
Volume $=$ $\boxed{\pi \int_{-1}^{1} (2-x^2-3)^2 - (x^2-3)^2 \, dx}$

44. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Graph      Disk




$y = 27 \Rightarrow dx$ problem

Bounds: Given $x = 0$

$27 = 3x^2$

$9 = x^2 \rightarrow x = 3$

$V = \pi \int_0^3 (3x^2 - 27)^2 \, dx$

$= \pi \int_0^3 (9x^4 - 162x^2 + 729) \, dx$

$= \pi \left( \frac{9x^5}{5} - 54x^3 + 729x \right) \Big]_0^3$

$= 11664.4\pi$

Volume $=$ $\boxed{\frac{8322}{5}\pi}$

45. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

Rewrite: $y\,dy = 3x^2\,dx$

$$\int y\,dy = \int 3x^2\,dx$$

$$\frac{y^2}{2} = x^3 + c$$

$$y^2 = 2x^3 + c$$

$$y = \pm\sqrt{2x^3+c}$$

$y = \boxed{\pm\sqrt{2x^3+c}}$

46. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

Rewrite $dy = 5y\,dx$

$$\frac{dy}{y} = 5dx$$

$$\int \frac{dy}{y} = \int 5dx$$

$$\ln|y| = 5x + c$$

$$|y| = e^{5x+c}$$

$$\pm y = e^c e^{5x}$$

$$y = \underbrace{\pm e^c}_{c} e^{5x}$$

$$y = ce^{5x}$$

Or memorize

$$\frac{dy}{dx} = ky \Rightarrow y = Ce^{kx}$$

$y = \boxed{Ce^{5x}}$

47. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

Rewrite: $y\,dy = -x\,dx$

$\int y\,dy = \int -x\,dx$

$\frac{y^2}{2} = \frac{-x^2}{2} + C$

$y^2 = -x^2 + C$

$y = \pm\sqrt{C - x^2}$

$y$ = $\boxed{\pm\sqrt{C-x^2}}$

48. Let $y$ denote the mass of a radioactive substance at time $t$. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time $t$ in hours does half the original mass remain? Round your answer to 3 decimal places.

$y' = -18y \Rightarrow y = Ce^{-18t}$

$y(0) = 20 \Rightarrow 20 = Ce^{-18(0)}$

$20 = C \qquad \Rightarrow y = 20e^{-18t}$

We want solve $\frac{1}{2}(20) = y(t)$ for $t$.

$10 = 20e^{-18t}$

$1/2 = e^{-18t}$

$\ln(1/2) = -18t$

$\frac{\ln(1/2)}{-18} = t$

$t$ = $\boxed{0.039}$

49. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

Note there are 2 ways to do this problem.

① Separation of Variables

② First-Order Linear Eqn

By method 1,

$$\frac{dy}{dt} = 15y$$

$$\frac{dy}{y} = 15\,dt$$

$$\int \frac{dy}{y} = \int 15\,dt$$

$$\ln|y| = 15t + C$$

$$y = e^{15t+C}$$

$$y = e^{C}e^{15t}$$

$$y = Ce^{15t}$$

$y =$ $\boxed{Ce^{15t}}$

50. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$$y\,dy = 3\,dx$$

$$\int y\,dy = \int 3\,dx$$

$$\frac{y^2}{2} = 3x + C$$

$$y^2 = 6x + 2C$$

$$y^2 = 6x + C$$

$$y = \pm\sqrt{6x+C}$$

$y =$ $\boxed{\pm\sqrt{6x+C}}$

51. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$$\frac{dy}{y} = 3x^2\,dx$$

$$\int \frac{dy}{y} = \int 3x^2\,dx$$

$$\ln|y| = x^3 + C$$

$$y = e^{x^3 + C}$$

$$y = e^C e^{x^3}$$

$$y = Ce^{x^3}$$

$y =$ $\boxed{Ce^{x^3}}$

52. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$$dy = 8e^{-4t}e^{-y}\,dt$$

$$e^y\,dy = 8e^{-4t}\,dt$$

$$\int e^y\,dy = \int 8e^{-4t}\,dt$$

$$e^y = \frac{8}{-4}e^{-4t} + C$$

$$e^y = -2e^{-4t} + C$$

$$y = \ln(-2e^{-4t} + C)$$

$y =$ $\boxed{\ln(-2e^{-4t} + C)}$

53. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$2y\,dy = (3x+2)\,dx$

$\int 2y\,dy = \int (3x+2)\,dx$

$y^2 = \frac{3x^2}{2} + 2x + C$

So $y^2 = \frac{3x^2}{2} + 2x + 16$

$y = \pm\sqrt{\frac{3x^2}{2} + 2x + 16}$

when $y(0) = 4$

$4^2 = 0 + 0 + C$

$16 = C$

$y =$ $\boxed{\pm\sqrt{\frac{3x^2}{2} + 2x + 16}}$

54. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$\frac{dy}{y} = \frac{5}{6x+3}\,dx$

$\int \frac{dy}{y} = \int \frac{5}{6x+3}\,dx$

$\ln|y| = \frac{5}{6}\ln|6x+3| + C$

$y = \exp\left[\frac{5}{6}\ln|6x+3| + C\right]$

$y = e^C \exp\left[\ln|6x+3|^{5/6}\right]$

$y = C \cdot |6x+3|^{5/6}$

When $y(0) = 1$

$1 = C \cdot |6(0)+3|^{5/6}$

$1 = C \cdot 3^{5/6}$

$C = 3^{-5/6}$

$y =$ $\boxed{3^{-5/6} \cdot |6x+3|^{5/6}}$

55. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2 e^{-x^3} \quad \text{where } \ y = 10 \ \text{ when } \ x = 2$$

Find the value of the integration constant, $C$.

$dy = 11x^2 e^{-x^3} dx$

$\int dy = \underbrace{\int 11x^2 e^{-x^3} dx}$

$u = -x^3$

$du = -3x^2 dx$

$y = \int -\frac{11}{3} e^u du$

$y = -\frac{11}{3} e^{-x^3} + C$

When $y = 10$ and $x = 2$

$10 = -\frac{11}{3} e^{-2^3} + C$

$10 = -\frac{11}{3} e^{-8} + C$

$C = 10 + \frac{11}{3} e^{-8}$

$C =$ $\boxed{10 + \frac{11}{3} e^{-8}}$

56. What is the **integrating factor** of the following differential equation?

$$\frac{2y' + \left(\frac{6}{x}\right) y}{2} = \frac{10\ln(x)}{2}$$

$y' + \frac{3}{x} y = 5\ln x$

$P(x) = \frac{3}{x} \quad Q(x) = 5\ln x$

$u(x) = \exp\left[\int \frac{3}{x} dx\right]$

$= \exp[3\ln x]$

$= \exp[\ln x^3]$

$= x^3$

$u(x) =$ $\boxed{x^3}$

57. What is the **integrating factor** of the following differential equation?

$$\underbrace{(x+1)\frac{dy}{dx} - 2(x^2+x)y}_{(x+1)} = \underbrace{(x+1)e^{x^2}}_{(x+1)}$$

$$\frac{dy}{dx} - \frac{2x(x+1)}{(x+1)}y = e^{x^2}$$

$$\frac{dy}{dx} + (-2x)\cdot y = e^{x^2}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$

$$= \exp\left[\int -2x\,dx\right]$$

$$= \exp\left[-x^2\right]$$

$u(x) =$ $\boxed{e^{-x^2}}$

58. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x)\cdot y = \sin^2(x)$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$

$$= \exp\left[\int \cot x\,dx\right]$$

$$= \exp\left[\int \frac{\cos x}{\sin x}\,dx\right]$$

$u = \sin x$

$du = \cos x\,dx$

$$= \exp\left[\int \frac{du}{u}\right]$$

$$= \exp\left[\ln u\right]$$

$$u(x) = \exp\left[\ln \sin x\right]$$

$$= \sin x$$

$u(x) =$ $\boxed{\sin x}$

59. Solve the initial value problem.

$$x^4 y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$$\frac{x^4 y' + 4x^3 y}{x^4} = \frac{10x^9}{x^4}$$

$$y' + \frac{4}{x} \cdot y = 10x^5$$

$$P(x) = \frac{4}{x} \quad Q(x) = 10x^5$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int \frac{4}{x}\,dx\right]$$
$$= \exp[4\ln x]$$
$$= \exp[\ln x^4]$$
$$= x^4$$

$$y \cdot u(x) = \int Q(x)u(x)\,dx + C$$
$$y \cdot x^4 = \int 10x^5 x^4\,dx + C$$
$$y \cdot x^4 = \int 10x^9\,dx + C$$
$$y \cdot x^4 = x^{10} + C$$
$$y = \frac{x^{10}}{x^4} + \frac{C}{x^4}$$
$$y = x^6 + \frac{C}{x^4}$$

$$23 = 1 + \frac{C}{1}$$
$$22 = C$$
$$y = x^6 + \frac{22}{x^4}$$

$y =$ $\boxed{x^6 + \frac{22}{x^4}}$

60. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \ldots$$

$$= 1 - \frac{6}{10} + \frac{36}{100} - \frac{216}{1000} + \ldots$$

$$= 1 - \frac{6}{10} + \left(\frac{6}{10}\right)^2 - \left(\frac{6}{10}\right)^3 + \ldots$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n$$

Answer: $\boxed{\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n}$

(b) Use the sum from (a) and compute the sum.

$$\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n = \frac{1}{1-(-6/10)} = \frac{1}{1+6/10} = \frac{1}{16/10} = \frac{10}{16} = \frac{5}{8}$$

Answer: $\boxed{5/8}$

61. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Note $r = 3/2$ and

$\left|\frac{3}{2}\right| < 1$ is false

So the sum diverges

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \boxed{\text{diverges}}$$

62. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$$= \frac{6}{1-(-1/9)}$$

$$= \frac{6}{1+1/9}$$

$$= \frac{6}{10/9}$$

$$= 6 \cdot \frac{9}{10}$$

$$= 3 \cdot \frac{9}{5} = \frac{27}{5}$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ $\boxed{27/5}$

63. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$$= \sum_{n=0}^{\infty} 7\left(\frac{1}{4}\right)^n$$

$$= \frac{7}{1-1/4}$$

$$= \frac{7}{3/4}$$

$$= 7 \cdot \frac{4}{3} = \frac{28}{3}$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ $\boxed{28/3}$

64. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$= \frac{5^3}{6} + \frac{5^4}{6^2} + \frac{5^5}{6^3} + \cdots$$

$$= \frac{5^3}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \cdots\right)$$

$$= \frac{125}{6}\sum_{n=0}^{\infty}\left(\frac{5}{6}\right)^n = \frac{125}{6} \cdot \frac{1}{1-5/6}$$

$$= \frac{125}{6} \cdot \frac{1}{1/6} = \frac{125}{6} \cdot \frac{6}{1} = 125$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \underline{\boxed{125}}$$

65. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n}{3 \cdot 3^{2n}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3}\left(-\frac{2}{9}\right)^n$$

$$= \frac{1/3}{1-(-2/9)}$$

$$= \frac{1/3}{1+2/9}$$

$$= \frac{1/3}{11/9}$$

$$= \frac{1}{3} \cdot \frac{9}{11}$$

$$= 3/11$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \underline{\boxed{3/11}}$$

66. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

Remember

$\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square}$ where $|\square| < 1$

$|-2x| < 1$

$|2x| < 1$

$2|x| < 1$

$|x| < 1/2 = R$

$R =$ $\boxed{1/2}$

67. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

Remember

$\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square}$ where $|\square| < 1$

$|7x^2| < 1$

$7|x^2| < 1$

$|x^2| < 1/7$

$-1/7 < x^2 < 1/7$

By algebra

$x^2 < 1/7$

$x < \pm\sqrt{1/7}$

$|x| < \sqrt{1/7}$

$R =$ $\boxed{\sqrt{1/7}}$

68. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$$\frac{3}{1+2x} = \frac{3}{1}\cdot\frac{1}{1+2x} = \frac{3}{1}\cdot\frac{1}{1-(-2x)}$$

$$\frac{1}{1-(-2x)} = \sum_{n=0}^{\infty}(-2x)^n \text{ where } |-2x|<1$$

$$f(x) = \frac{3}{1-(-2x)} = 3\sum_{n=0}^{\infty}(-2x)^n \text{ where } 2|x|<1$$

$$= \sum_{n=0}^{\infty} 3(-1)^n 2^n x^n \text{ where } |x|<1/2$$

$\dfrac{3}{1+2x} =$ $\boxed{\sum_{n=0}^{\infty} 3(-1)^n 2^n x^n}$

$R =$ $\boxed{1/2}$

69. Express $f(x) = \dfrac{5x}{3+2x^2}$ as a power series and determine it's radius of converge.

$$\frac{5x}{3(1+2x^2/3)} = \frac{5x}{3}\cdot\frac{1}{1-(-(2x^2/3))}$$

$$\frac{1}{1-(-2x^2/3)} = \sum_{n=0}^{\infty}\left(-\frac{2x^2}{3}\right)^n \text{ where } \left|-\frac{2x^2}{3}\right|<1$$

$$\frac{2}{3}|x^2|<1$$

$$|x^2|<\frac{3}{2}$$

$$-\frac{3}{2}<x^2<\frac{3}{2}$$

By algebra

$$x^2<\frac{3}{2}$$

$$-\sqrt{\frac{3}{2}}<x<\sqrt{\frac{3}{2}}$$

$$|x|<\sqrt{\frac{3}{2}}$$

$$f(x) = \frac{5x}{3}\cdot\frac{1}{1-(-2x^2/3)} = \frac{5x}{3}\sum_{n=0}^{\infty}\left(\frac{-2x^2}{3}\right)^n$$

$$f(x) = \frac{5x}{3}\sum_{n=0}^{\infty}\frac{(-1)^n 2^n x^{2n}}{3^n}$$

$$f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n\cdot 5\cdot x^{2n+1}}{3^{n+1}}$$

$\dfrac{5x}{3+2x^2} =$ $\boxed{\sum_{n=0}^{\infty}\frac{(-1)^n 2^n\cdot 5\cdot x^{2n+1}}{3^{n+1}}}$

$R =$ $\boxed{\sqrt{3/2}}$

70. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\sin(x^{3/2}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x^{3/2})^{2n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}$$

$$\int \sin(x^{3/2})\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{3n+3/2}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{3n+5/2}}{3n+5/2}$$

$$= \frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6\cdot(3+5/2)} + \frac{x^{17/5}}{5!\,(6+5/2)}$$

$\int \sin(x^{3/2})\,dx = \boxed{\frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6\cdot(3+5/2)} + \frac{x^{17/5}}{5!\,(6+5/2)}}$

71. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx$$

$$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = \sum_{n=0}^{\infty} (-x^4)^n = \sum_{n=0}^{\infty} (-1)^n x^{4n}$$

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx = \int_0^{0.11} \sum_{n=0}^{\infty} (-1)^n x^{4n}\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \int_0^{0.11} x^{4n}\,dx$$

$$= \sum_{n=0}^{\infty} \left.\frac{(-1)^n x^{4n+1}}{4n+1}\right]_0^{0.11}$$

$$= \left.\left(x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13}\right)\right]_0^{0.11}$$

$\int_0^{0.11} \frac{1}{1+x^4}\,dx \approx \boxed{0.11000}$

72. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$$

$$\int_0^{0.23} e^{-x^2}dx = \int_0^{0.23} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^{0.23} x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{2n+1}}{2n+1}\Bigg]_0^{0.23}$$

$$= \left(\frac{x}{0!} - \frac{x^3}{1!(3)} + \frac{x^5}{2!(5)}\right)\Bigg]_0^{0.23}$$

$$= \left(x - \frac{x^3}{3} + \frac{x^5}{10}\right)\Bigg]_0^{0.23}$$

$\int_0^{0.23} e^{-x^2}\,dx \approx$ $\boxed{0.226}$

73. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x^{1/2})^{2n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$f(x) = 4x\cos(\sqrt{x}) = 4x \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1}$$

$$\int_0^{0.45} 4x\cos(\sqrt{x})dx = \int_0^{0.45} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\int_0^{0.45} x^{n+1}\,dx$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\frac{x^{n+2}}{n+2}\Bigg]_0^{0.45}$$

$$= \left(\frac{4x^2}{0!(2)} - \frac{4x^3}{2!(3)} + \frac{4x^4}{4!(4)} - \frac{4x^5}{6!(5)}\right)\Bigg]_0^{0.45}$$

$$= \left(2x^2 - \frac{2x^3}{3} + \frac{x^4}{6} - \frac{x^5}{900}\right)\Bigg]_0^{0.45}$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ $\boxed{0.35106}$

74. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$

Note $1.56 = 1 + 0.56$

$\ln(1+0.56) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (0.56)^n$

$= 0.56 - \frac{(0.56)^2}{2} + \frac{(0.56)^3}{3}$

$\ln(1.56) \approx$ $\boxed{0.46174}$

75. Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

$\sqrt{?} \to ? \geq 0$

$\sqrt{x+y-1} \to x+y-1 \geq 0$

$x+y \geq 1$

$\ln(?) \to ? > 0$

$\ln(y-11) \to y-11 > 0$

$y > 11$

$\frac{1}{?} \to ? \neq 0$

$\ln(y-11) - 9 \neq 0$

$\ln(y-11) \neq 9$

$y - 11 \neq e^9$

$y \neq e^9 + 11$

Domain = $\boxed{\{(x,y) \mid x+y \geq 1,\ y > 11,\ y \neq 11+e^9\}}$

76. Find the domain of

$$f(x,y) = \frac{\ln(x^2-y+3)}{\sqrt{x-6}}$$

$\ln(?) \to ? > 0$

$\ln(x^2-y+3) \to x^2-y+3 > 0$

$x^2+3 > y$

$\frac{1}{\sqrt{?}} \to ? > 0$

$\frac{1}{\sqrt{x-6}} \to x-6 > 0$

$x > 6$

Domain = $\boxed{\{(x,y) \mid x > 6,\ x^2+3 > y\}}$

77. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \quad C = \ln(36)$$

$\ln(x^2+y^2)=\ln(36)$
$x^2+y^2=36$
$x^2+y^2=6^2$

(a) Parabola with vertices at $(0, 0)$
(b) Circle with center at $(0, \ln(36))$ and radius 6
(c) Parabola with vertices at $(0, \ln(36))$
(d) Circle with center at $(0, 0)$ and radius 6
(e) Increasing Logarithm Function

78. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

$\ln(y-e^{5x})=C$
$y-e^{5x}=e^C$
$y-e^{5x}=C$
$y=e^{5x}+C$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

79. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

$\sqrt{y+4x^2}=C$
$y+4x^2=C^2$
$y+4x^2=C$
$y=-4x^2+C$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

80. Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y^2-1}}$$

$$f_x(x,y) = \frac{d}{dx}\left(\frac{(6x-6y)^2}{\sqrt{y^2-1}}\right)$$

$$= \frac{1}{\sqrt{y^2-1}} \frac{d}{dx}\left((6x-6y)^2\right)$$

$$= \frac{1}{\sqrt{y^2-1}} \cdot 2(6x-6y)\frac{d}{dx}(6x+6y)$$

$$= \frac{1}{\sqrt{y^2-1}} \cdot 2(6x-6y)\cdot 6$$

$$= \frac{72x-72y}{\sqrt{y^2-1}}$$

$f_x(6,5)=$ $\boxed{72/\sqrt{24}}$

81. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$$f(x,y) = 3x^2 \cdot \frac{y^3}{(y-1)^2}$$

$$f_x(x,y) = \frac{d}{dx}\left(3x^2 \cdot \frac{y^3}{(y-1)^2}\right) = \frac{y^3}{(y-1)^2}\cdot\frac{d}{dx}\left(3x^2\right) = \frac{y^3}{(y-1)^2}\cdot 6x$$

$$f_y(x,y) = \frac{d}{dy}\left(3x^2 \cdot \frac{y^3}{(y-1)^2}\right) = 3x^2\frac{d}{dy}\left(\frac{y^3}{(y-1)^2}\right) = 3x^2\left(\frac{3y^2(y-1)^2 - y^3\cdot 2(y-1)}{(y-1)^4}\right)$$

$$= 3x^2\left(\frac{\cancel{(y-1)}\left[3y^2(y-1) - 2y^3\right]}{(y-1)^{\cancel{4}\,3}}\right) = \frac{3x^2(3y^3-3y^2-2y^3)}{(y-1)^3}$$

$$= \frac{3x^2(y^3-3y^2)}{(y-1)^3}$$

$f_x(x,y) =$ $6xy^3/(y-1)^2$

$f_y(x,y) =$ $\dfrac{3x^2(y^3-3y^2)}{(y-1)^3}$

82. Find the first order partial derivatives of $f(x, y) = (xy - 1)^2$

$$f_x(x,y) = \frac{d}{dx}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dx}(xy-1)$$
$$= 2(xy-1)\,y$$
$$= 2xy^2 - 2y$$

$$f_y(x,y) = \frac{d}{dy}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dy}(xy-1)$$
$$= 2(xy-1)\,x$$
$$= 2x^2y - 2x$$

$f_x(x, y) =$ $\boxed{2xy^2 - 2y}$

$f_y(x, y) =$ $\boxed{2x^2y - 2x}$

83. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$$f_x(x,y) = \frac{d}{dx}(x)\,e^{x^2+xy+y^2} + x\frac{d}{dx}\left(e^{x^2+xy+y^2}\right)$$
$$= e^{x^2+xy+y^2} + x\left(e^{x^2+xy+y^2}\right)(2x+y)$$
$$= (1+2x^2+xy)e^{x^2+xy+y^2}$$

$$f_y(x,y) = x\frac{d}{dy}\left(e^{x^2+xy+y^2}\right) = x\left(e^{x^2+xy+y^2}\right)(x+2y)$$
$$= (x^2+2xy)e^{x^2+xy+y^2}$$

$f_x(x, y) =$ $\boxed{(1+2x^2+xy)e^{x^2+xy+y^2}}$

$f_y(x, y) =$ $\boxed{(x^2+2xy)e^{x^2+xy+y^2}}$

84. Find the first order partial derivatives of $f(x, y) = y\cos(x^2y)$

$$f_x(x,y) = y\frac{d}{dx}\left(\cos(x^2y)\right) = y\left(-\sin(x^2y)\right)\frac{d}{dx}(x^2y) = -y\sin(x^2y)\left[2xy\right]$$
$$= -2xy^2\sin(x^2y)$$

$$f_y(x,y) = \frac{d}{dy}(y)\cos(x^2y) + y\frac{d}{dy}\left(\cos(x^2y)\right)$$
$$= \cos(x^2y) + y\left(-\sin(x^2y)\right)\frac{d}{dy}(x^2y)$$
$$= \cos(x^2y) - y\sin(x^2y)\left[x^2\right]$$
$$= \cos(x^2y) - x^2y\sin(x^2y)$$

$f_x(x, y) =$ $-2xy^2\sin(x^2y)$

$f_y(x, y) =$ $\cos(x^2y) - x^2y\sin(x^2y)$

85. Given the function $f(x, y) = 4x^5\tan(3y)$, compute $f_{xy}(2, \pi/3)$

$$f_x(x,y) = \frac{d}{dx}\left(4x^5\tan(3y)\right) = \tan(3y)\cdot\frac{d}{dx}(4x^5)$$
$$= \tan(3y)\cdot(20x^4)$$

$$f_{xy}(x,y) = \frac{d}{dy}\left(f_x(x,y)\right) = \frac{d}{dy}\left(\tan(3y)\cdot(20x^4)\right) = 20x^4\frac{d}{dy}\left(\tan(3y)\right)$$
$$= 20x^4\cdot\sec^2(3y)\cdot 3$$
$$= 60x^4\sec^2(3y)$$

$$f_{xy}(2,\pi/3) = 60(2)^4\sec^2(3\pi/3)$$
$$= 60(16)\sec^2(\pi)$$
$$= 960$$

$f_{xy}(2, \pi/3) =$ $\boxed{960}$

86. Find the second order partial derivatives of

$$f(x,y) = x^2 y \ln(7x)$$

$$f(x,y) = (x^2 \ln(7x))\, y$$

$$f_x(x,y) = \frac{d}{dx}\left((x^2 \ln(7x)) \cdot y\right) = y \frac{d}{dx}\left(x^2 \ln(7x)\right)$$

$$= y\left(2x\ln(7x) + x^2 \frac{1}{7x} \cdot 7\right) = y(2x\ln(7x) + x)$$

$$f_{xx}(x,y) = \frac{d}{dx}\left(y(2x\ln(7x)+x)\right) = y\frac{d}{dx}(2x\ln(7x)+x)$$

$$= y\left(2\ln(7x) + 2x \cdot \frac{1}{7x} \cdot 7 + 1\right) = y(2\ln(7x) + 2 + 1)$$

$$= y(2\ln(7x)+3)$$

$$f_{xy}(x,y) = \frac{d}{dy}\left(y(2x\ln(7x)+x)\right) = (2x\ln(7x)+x)\frac{d}{dy}(y)$$

$$= 2x\ln(7x) + x$$

$$f_y(x,y) = \frac{d}{dy}\left((x^2\ln(7x)) \cdot y\right) = (x^2\ln(7x))\frac{d}{dy}(y) = x^2\ln(7x)$$

$$f_{yy}(x,y) = \frac{d}{dy}(x^2\ln(7x)) = 0$$

$f_{xx}(x,y) =$ $(2\ln(7x)+3)\,y$

$f_{xy}(x,y) =$ $2x\ln(7x)+x$

$f_{yy}(x,y) =$ $0$

87. Find the discriminant of

$$f(x,y) = e^x \sin(y)$$

Simplify your answer. Note: $\sin^2(y) + \cos^2(y) = 1$.

$f_x(x,y) = e^x \sin(y)$

$f_{xx}(x,y) = e^x \sin(y)$

$f_{xy}(x,y) = e^x \cos(y)$

$f_y(x,y) = e^x \cos(y)$

$f_{yy}(x,y) = -e^x \sin(y)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= (e^x \sin(y))(-e^x \sin(y)) - (e^x \cos(y))^2$

$= -e^{2x} \sin^2(y) - e^{2x} \cos^2(y)$

$= -e^{2x}(\sin^2(y) + \cos^2(y))$

$= -e^{2x}(1)$

$D(x,y) =$ $\boxed{-e^{2x}}$

88. Using the information in the table below, classify the critical points for the function $g(x,y)$.

| $(a,b)$ | $g_{xx}(a,b)$ | $g_{yy}(a,b)$ | $g_{xy}(a,b)$ |
|---|---|---|---|
| $(4,5)$ | 0 | 4 | $-2$ |
| $(5,-10)$ | 5 | $-10$ | 6 |
| $(10,10)$ | $-4$ | $-6$ | $-4$ |
| $(7,9)$ | 5 | 7 | 4 |
| $(4,8)$ | 2 | 2 | 2 |

$D(4,5) = (0)(4) - (-2)^2 = -4 < 0 \to$ saddle pt

$D(5,-10) = (5)(-10) - 6^2 = -86 < 0 \to$ saddle pt

$D(10,10) = (-4)(-6) - (-4)^2 = 8 > 0 \to$ relative

$g_{xx} = -4 < 0 \to$ max

$D(7,9) = (5)(7) - (4)^2 = 19 > 0 \to$ relative

$g_{xx} = 5 > 0 \to$ min

$D(4,8) = (2)(2) - 2^2 = 0$

$\downarrow$

Inconclusive

(4,5) is saddle pt

(5,-10) is saddle pt

(10,10) is relative max

(7,9) is relative min

(4,8) is inconclusive

89. Classify the critical points of the function $f(x,y)$ given the partial derivatives:

$$f_x(x,y) = x - y \qquad f_y(x,y) = y^3 - x$$

(a) Two saddle points and one local minimum

(b) Two saddle points and one local maximum

(c) One saddle point, one local maximum, and one local minimum

(d) Three saddle points

(e) Two local minimums and one saddle point

$f_x = 0$
$x - y = 0$
$x = y$

$f_y = 0$
$y^3 - x = 0$
$y^3 = x$

$\begin{cases} x = y \\ y^3 = x \end{cases} \Rightarrow y = y^3$
$y - y^3 = 0$
$y(1-y^2) = 0$
$y = 0, \pm 1$

$f_x = x - y$ $\quad f_y = y^3 - x$
$f_{xx} = 1$ $\quad f_{yy} = 3y^2$
$f_{xy} = -1$

$D = f_{xx}f_{yy} - (f_{xy})^2$
$= (1)(3y^2) - (-1)^2$
$= 3y^2 - 1$

Note we don't need to find the x-values b/c D which we found on the left only has y's.

When $y = 0$, $D = -1 < 0 \rightarrow$ saddle

When $y = -1$, $D = 2 > 0 \rightarrow$ rel extrema
When $y = +1$, $D = 2 > 0 \rightarrow$ rel extrema

Check $f_{xx} = 1 > 0 \rightarrow$ rel mins @ $y = \pm 1$

Answer: __________

90. The critical points for a function $f(x,y)$ are (1,1) and (2,4). Given that the partial derivatives of $f(x,y)$ are

$$f_x(x,y) = 7x - 3y \qquad f_y(x,y) = 4x^2 - 6y$$

Classify each critical point as a maximum, minimum, or saddle point.

$f_x = 7x - 3y$ $\quad f_y = 4x^2 - 6y$
$f_{xx} = 7$ $\quad f_{yy} = -6$
$f_{xy} = -3$

$D = f_{xx}f_{yy} - (f_{xy})^2$
$= (7)(-6) - (-3)^2$
$= -42 - 9 = -51$

Since $D < 0$ always, both pts are saddle pts.

(1,1) is saddle pt

(2,4) is saddle pt

91. Fleet feet stores two most sold running shoes brands are Aesics and Brookes. The total venue from selling $x$ pairs of Aesics and $y$ pairs of Brookes is given by

$$R(x, y) = -10x^2 - 16y^2 - 4xy + 84 + 204y$$

where $x$ and $y$ are in **thousands of units**. Determine the number of Brookes shoes to be sold to maximize the revenue.

First find the critical pts.

$$\begin{cases} R_x = -20x - 4y = 0 & (1) \\ R_y = -32y - 4x + 204 = 0 & (2) \end{cases}$$

Divide (1) and (2) by $-4$.

$$\begin{cases} 5x + y = 0 & (1) \\ x + 8y - 51 = 0 & (2) \end{cases}$$

$$\Rightarrow \begin{cases} 5x + y = 0 & (1) \\ x + 8y = 51 & (2) \end{cases}$$

Multiply (2) by 5.

$$\Rightarrow \begin{cases} 5x + y = 0 & (1) \\ 5x + 40y = 255 & (2) \end{cases}$$

Subtract (1) and (2)

$-39y = -255$

$y \approx 6.5$

$\Rightarrow y = 7$

The # of Brookes shoes sold is $\boxed{7000}$

92. Find the point(s) $(x, y)$ where the function $f(x, y) = 3x^2 + 4xy + 6x - 15$ attains maximal value, subject to the constraint $x + y = 10$.

$f = 3x^2 + 4xy + 6x - 15$ $\qquad g = x + y = 10$

$f_x = 6x + 4y + 6$ $\qquad g_x = 1$

$f_y = 4x$ $\qquad g_y = 1$

System
$$\begin{cases} 6x + 4y + 6 = \lambda & (1) \\ 4x = \lambda & (2) \\ x + y = 10 & (3) \end{cases}$$

Set (1) = (2)

$6x + 4y + 6 = 4x$

$2x + 4y + 6 = 0$

$2x = -4y - 6$

$x = -2y - 3$

Plug $x = -2y - 3$ into (3)

$x + y = 10$

$-2y - 3 + y = 10$

$-y - 3 = 10$

$-y = 13$

$y = -13$

Plug $y = -13$ into $x = -2y - 3$.

$x = -2(-13) - 3$

$= 26 - 3$

$= 23$

(x,y) = $\boxed{(23, -13)}$

93. Find the minimum of the function using LaGrange Multipliers of the function $f(x,y) = 2x^2 + 4y^2$ subject to the constraint $x^2 + y^2 = 1$.

$f = 2x^2 + 4y^2$
$f_x = 4x$
$f_y = 8y$

$g = x^2 + y^2 = 1$
$g_x = 2x$
$g_y = 2y$

System:
$$\begin{cases} 4x = 2x\lambda & (1) \\ 8y = 2y\lambda & (2) \\ x^2 + y^2 = 1 & (3) \end{cases}$$

Solve (1).
$4x = 2x\lambda$
$4x - 2x\lambda = 0$
$2x(1-\lambda) = 0$
$x = 0,\ \lambda = 1$

Plug $x=0$ into (3)
$0^2 + y^2 = 1$
$y = \pm 1$
Pts: $(0,1), (0,-1)$

Plug $\lambda = 1$ into (2)
$8y = 2y$
only true when $y=0$

Plug $y=0$ into (3)
$x^2 + 0^2 = 1$
$x = \pm 1$
Pts: $(1,0), (-1,0)$

Now plug the pts into $f(x,y) = 2x^2 + 4y^2$
$f(0,1) = 4$
$f(0,-1) = 4$
$\left.\begin{array}{l} f(1,0) = 2 \\ f(-1,0) = 2 \end{array}\right\} \to$ Min

Minimum Value = $\boxed{2}$

94. Find the minimum value of the function $f(x,y) = 2x^2y - 3y^2$ subject to the constraint $x^2 + 2y = 1$.

$f = 2x^2y - 3y^2$
$f_x = 4xy$
$f_y = 2x^2 - 6y$

$g = x^2 + 2y = 1$
$g_x = 2x$
$g_y = 2$

System
$$\begin{cases} 4xy = 2x\lambda & (1) \\ 2x^2 - 6y = 2\lambda & (2) \\ x^2 + 2y = 1 & (3) \end{cases}$$

Solve (1)
$4xy - 2x\lambda = 0$
$2x(2y - \lambda) = 0$
$x = 0,\ \lambda = 2y$

Plug $x=0$ into (3)
$0^2 + 2y = 1$
$y = 1/2$
Pts: $(0, 1/2)$

Plug $\lambda = 2y$ into (2)
$2x^2 - 6y = 2(2y)$
$2x^2 - 6y = 4y$
$2x^2 = 10y$
$x^2 = 5y$

Plug $x^2 = 5y$ into (3)
$5y + 2y = 1$
$7y = 1$
$y = 1/7$

Plug $y = 1/7$ into $x^2 = 5y$
$x^2 = \frac{5}{7}$
$x = \pm\sqrt{\frac{5}{7}}$
Pts: $\left(\sqrt{\frac{5}{7}}, \frac{1}{7}\right), \left(-\sqrt{\frac{5}{7}}, \frac{1}{7}\right)$

Test for Min
$f(0, 1/2) = -3/4$
$f\left(\pm\sqrt{\frac{5}{7}}, \frac{1}{7}\right) = \frac{1}{7}$

Minimum Value = $\boxed{-3/4}$

95. Locate and classify the points that maximize and minimize the function $f(x, y) = 5x^2 + 10y$ subject to the constraint $5x^2 + 5y^2 = 5$.

$f = 5x^2 + 10y$ $\quad g = 5x^2 + 5y^2 = 5$

$f_x = 10x$ $\quad g_x = 10x$

$f_y = 10$ $\quad g_y = 10y$

System: $\begin{cases} 10x = 10x\lambda & ① \\ 10 = 10y\lambda & ② \\ 5x^2 + 5y^2 = 5 & ③ \end{cases}$

Solve ①

$10x - 10x\lambda = 0$

$10x(1-\lambda) = 0$

$x = 0, \ \lambda = 1$

Plug $x = 0$ into ③

$5y^2 = 5$

$y^2 = 1$

$y = \pm 1$

Pts: $(0,1)(0,-1)$

Plug $\lambda = 1$ into ②

$10 = 10y$

$y = 1$

plug $y = 1$ into ③

$5x^2 + 5 = 5$

$5x^2 = 0$

$x = 0$

Pt: $(0,1)$ again

Test w/ $f(x,y)$

$f(0,-1) = -10$

$f(0,1) = 10$

Minimum Value occurs at $\boxed{-10}$

Maximum Value occurs at $\boxed{10}$

96. We are baking a tasty treat where customer satisfaction is given by $S(x, y) = 6x^{3/2}y$ . Here, $x$ and $y$ are the amount of sugar and spice respectively. If the sugar and spice we use must satisfy $9x + y = 4$, what is the maximum customer satisfaction we can achieve? (Note: the function is defined only for $x \geq 0$ and $y \geq 0$.) Round your answer to 2 decimal places.

$S = 6x^{3/2}y$ $\quad g = 9x + y = 4$

$S_x = 9x^{1/2}y$ $\quad g_x = 9$

$S_y = 6x^{3/2}$ $\quad g_y = 1$

System: $\begin{cases} 9x^{1/2}y = 9\lambda & ① \\ 6x^{3/2} = \lambda & ② \\ 9x^2 + y^2 = 4 & ③ \end{cases}$

Plug ② in ①

$9x^{1/2}y = 9(6x^{3/2})$

$x^{1/2}y = 6x^{3/2}$

$x^{1/2}y - 6x^{3/2} = 0$

$x^{1/2}(y - 6x) = 0$

$x = 0, \ y = 6x$

Plug $x = 0$ into ③

$0 + y = 4$

Pt: $(0,4)$

Plug $y = 6x$ into

$9x + 6x = 4$

$15x = 4$

$x = \frac{4}{15}$

Plug $x = \frac{4}{15}$ into $y = 6x$

$y = \frac{8}{5}$

Pt: $\left(\frac{4}{15}, \frac{8}{5}\right)$

Test for max

$S(0,4) = 0$

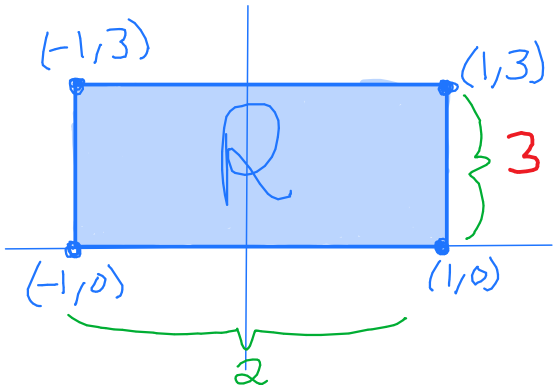
$S\left(\frac{4}{15}, \frac{8}{5}\right) \approx 1.32$

max

Maximum Value = $\boxed{1.32}$

97. Determine the average value of $f(x, y) = xy$ over

(a) the rectangle formed by the vertices (-1,0), (1,0), (-1,3), and (1,3)




Area of $R = 3$

$$f_{AVE} = \frac{1}{3}\int_{x=-1}^{x=1}\int_{y=0}^{y=3} xy\,dy\,dx$$

$$= \frac{1}{3}\int_{x=-1}^{x=1} x\left(\int_{y=0}^{y=3} y\,dy\right)dx$$

$$= \frac{1}{3}\int_{x=-1}^{x=1} x\left(\frac{y^2}{2}\Big]_{y=0}^{y=3}\right)dx$$

$$= \frac{1}{3}\cdot\frac{9}{2}\int_{x=-1}^{x=1} x\,dx$$

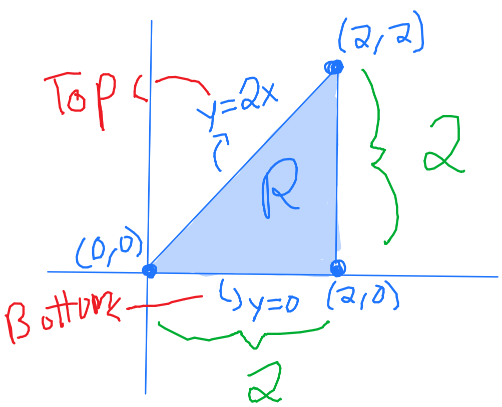
$$= \frac{3}{2}\,\frac{x^2}{2}\Big]_{x=-1}^{x=1}$$

$$= \frac{3}{2}\left(\frac{1}{2}-\frac{1}{2}\right)$$

$$= 0$$

Answer: $\boxed{0}$

(b) the triangle formed by the vertices (0,0), (2,0), and (2,2).




Area of $R = \frac{1}{2}(2)(2) = 2$

Bounds $\quad 0 \le y \le 2x$

$\quad 0 \le x \le 2$

$$f_{AVE}(x,y) = \frac{1}{2}\int_{x=0}^{x=2}\int_{y=0}^{y=2x} xy\,dy\,dx$$

$$= \frac{1}{2}\int_{x=0}^{x=2} x\left(\int_{y=0}^{y=2x} y\,dy\right)dx$$

$$= \frac{1}{2}\int_{x=0}^{x=2} x\left(\frac{y^2}{2}\Big]_{y=0}^{y=2x}\right)dx$$

$$= \frac{1}{2}\int_{x=0}^{x=2} x(2x^2)\,dx$$

$$= \int_{x=0}^{x=2} x^3\,dx$$

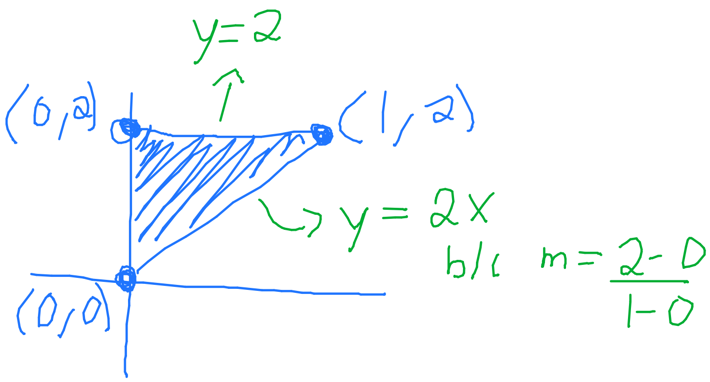
$$= \frac{x^4}{4}\Big]_{x=0}^{x=2}$$

$$= 2$$

Answer: $\boxed{2}$

98. Find the bounds for the integral $\iint_R 5e^x \sin(y)\, dA$ where $R$ is a triangle with vertices (0,0), (1,2), and (0, 2).

**DON"T COMPUTE!!!**




Hence

$$\int_0^1 \int_{2x}^2 5e^x \sin(y)\, dy\, dx$$

Answer: $\boxed{\int_0^1 \int_{2x}^2 5e^x \sin(y)\, dy\, dx}$

99. Evaluate the double integral

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x)\, dy\, dx$$

$$\int_0^{\pi/3} \sec^2(x) \left( \int_0^2 25y^4\, dy \right) dx$$

$$= \int_0^{\pi/3} \sec^2(x) \left( 5y^5 \Big]_0^2 \right) dx$$

$$= \int_0^{\pi/3} \sec^2(x)\, (20)\, dx$$

$$= 20 \int_0^{\pi/3} \sec^2(x)\, dx$$

$$= 20 \tan x \Big]_0^{\pi/3}$$

$$= 20\sqrt{3}$$

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x)\, dy\, dx = \boxed{20\sqrt{3}}$$

100. Evaluate the double integral

$$\int_0^{\frac{\pi}{2}} \int_0^{1} 12x^3 \sin(y)\, dx\, dy$$

$$= \int_0^{\pi/2} \sin(y) \left( \int_0^1 12x^3\, dx \right) dy$$

$$= \int_0^{\pi/2} \sin(y) \left( 3x^4 \Big]_0^1 \right) dy$$

$$= \int_0^{\pi/2} \sin(y)\ (3)\, dy$$

$$= 3 \int_0^{\pi/2} \sin(y)\, dy$$

$$= -3\cos(y) \Big]_0^{\pi/2}$$

$$= -3\cos\left(\frac{\pi}{2}\right) - \left(-3\cos(0)\right)$$

$$= 0 - (-3)$$

$$= 3$$

$$\int_0^1 \int_0^{\pi/2} 12x^3 \sin(y)\, dx\, dy = \boxed{3}$$

101. Evaluate the double integral

$$\int_0^4 \int_2^y (y + x)\, dx\, dy$$

$$\int_{y=0}^{y=4} \int_{x=2}^{x=y} (y+x)\, dx\, dy$$

$$= \int_{y=0}^{y=4} \left( \left( xy + \frac{x^2}{2} \right) \Big]_{x=2}^{x=y} \right) dy$$

$$= \int_{y=0}^{y=4} \left( y^2 + \frac{y^2}{2} - (2y+2) \right) dy$$

$$= \int_{y=0}^{y=4} \left( \frac{3}{2} y^2 - 2y - 2 \right) dy$$

$$= \left( \frac{3}{2} \cdot \frac{y^3}{3} - \frac{2y^2}{2} - 2y \right) \Big]_{y=0}^{y=4}$$

$$= \left( \frac{y^3}{2} - y^2 - 2y \right) \Big]_{y=0}^{y=4}$$

$$= 8$$

$$\int_0^4 \int_2^y (y + x)\, dx\, dy = \boxed{8}$$

102. Evaluate the double integral

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\, dy\, dx$$

$$= \int_{x=1}^{x=2} \int_{y=1}^{y=x^2} x y^{-2}\, dy\, dx$$

$$= \int_{x=1}^{x=2} x \left( \int_{y=1}^{y=x^2} y^{-2}\, dy \right) dx$$

$$= \int_{x=1}^{x=2} x \left( -y^{-1} \Big]_{y=1}^{y=x^2} \right) dx$$

$$= \int_{x=1}^{x=2} x \left( -\frac{1}{y} \Big]_{y=1}^{y=x^2} \right) dx$$

$$= \int_{x=1}^{x=2} x \left( -\frac{1}{x^2} + 1 \right) dx$$

$$= \int_1^2 \left( x - \frac{1}{x} \right) dx$$

$$= \left( \frac{x^2}{2} - \ln(x) \right] _1^2$$

$$= \left(2 - \ln(2)\right) - \left(\frac{1}{2} - 0\right)$$

$$= \frac{3}{2} - \ln(2)$$

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\, dy\, dx = \boxed{\frac{3}{2} - \ln(2)}$$

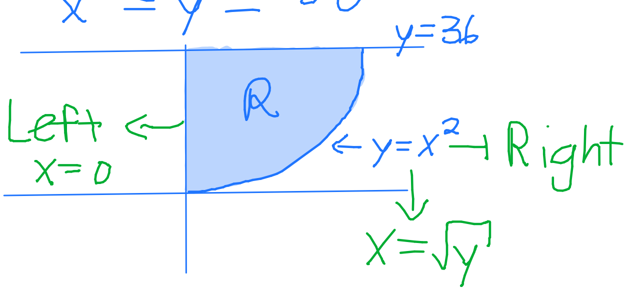
103. Switch the order of integration on the follow integral

$$\int_0^6 \int_{x^2}^{36} f(x, y)\, dy\, dx$$

The bounds tell me

$0 \leq x \leq 6$

$x^2 \leq y \leq 36$




So $0 \leq x \leq \sqrt{y}$

what does y range from?

$0 \leq y \leq 36$

Answer: $\boxed{\int_0^{36} \int_0^{\sqrt{y}} f(x,y)\, dx\, dy}$

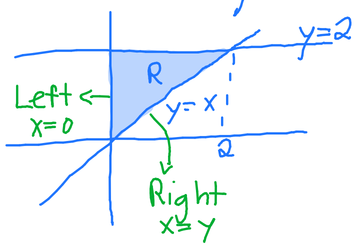
104. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx$$

(Hint: Change the order of integration)

Bounds: $0 \le x \le 2$

$x \le y \le 2$




So $0 \le y \le 2$

$0 \le x \le y$

$$\int_0^2 \int_x^2 4e^{y^2} dy\, dx$$

$$= \int_{y=0}^{y=2} \int_{x=0}^{x=y} 4e^{y^2} dx\, dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2} \left( \int_{x=0}^{x=y} dx \right) dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2} \left( x \Big]_{x=0}^{x=y} \right) dy$$

$$= \int_{y=0}^{y=2} 4ye^{y^2} dy$$

$$\frac{u = y^2}{du = 2y\,dy} \int 2e^u du$$

$$= 2e^u.$$

$$= 2e^{y^2} \Big]_{y=0}^{y=2}$$

$$= 2e^4 - 2$$

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx = \boxed{2e^4 - 2}$$

105. Evaluate the double integral

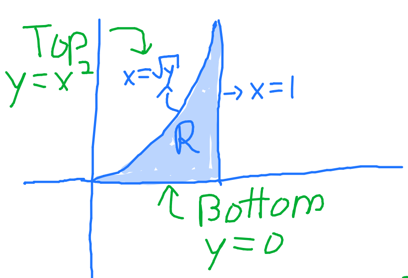
$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy$$

Round your answer to 2 decimal places.

(Hint: Change the order of integration)

Bounds: $0 \le y \le 1$

$\sqrt{y} \le x \le 1$




New Bounds: $0 \le y \le x^2$

$0 \le x \le 1$

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx\, dy$$

$$= \int_{x=0}^{x=1} \int_{y=0}^{y=x^2} \sin(x^3) dy\, dx$$

$$= \int_{x=0}^{x=1} \sin(x^3) \left( \int_{y=0}^{y=x^2} dy \right) dx$$

$$= \int_{x=0}^{x=1} \sin(x^3) \left( y \Big]_{y=0}^{y=x^2} \right) dx$$

$$= \int_{x=0}^{x=1} \sin(x^3) \cdot x^2 dx$$

$$\frac{u = x^3}{du = 3x^2 dx} \int \frac{1}{3} \sin(u) du$$

$$= -\frac{1}{3} \cos(u)$$

$$= -\frac{1}{3} \cos(x^3) \Big]_{x=0}^{x=1}$$

$$\approx 0.15$$

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy = \boxed{0.15}$$