Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:________________________________

1. [**10 pts**] Given the information in the table below, find and classify any critical points for the function $g(x, y)$.

(x_0, y_0)	$g(x_0, y_0)$	$g_x(x_0, y_0)$	$g_y(x_0, y_0)$	$g_{xx}(x_0, y_0)$	$g_{yy}(x_0, y_0)$	$g_{xy}(x_0, y_0)$
$(4, 5)$	1	-4	0	5	8	-3
$(5, -10)$	-10	0	0	5	-10	6
$(10, 10)$	0	0	0	-4	-6	-4
$(7, 9)$	4	0	0	5	7	4
$(4, 8)$	2	0	0	2	2	2

Solution: First check for each point that both g_x and g_y are 0.

- Hence $(4, 5)$ is not a critical point. [**2 pts**]

Next, let's compute the discriminant of each point.

- $\underline{(5, -10)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 5 \cdot (-10) - (6)^2 = -86$
- $\underline{(10, 10)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = (-4) \cdot (-6) - (-4)^2 = 8$
- $\underline{(7, 9)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 5 \cdot 7 - (4)^2 = 19$
- $\underline{(4, 8)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 2 \cdot 2 - (2)^2 = 0$

When $D > 0$, we have a relative extrema. Hence $(10, 10)$ and $(7, 9)$ are relative extrema. To determine whether they are maxs or mins, we need to check the sign of g_{xx}.

- $\underline{(10, 10)}$: $g_{xx} = -4 < 0$. Hence $(10, 10)$ is a relative max. [**2 pts**]
- $\underline{(7, 9)}$: $g_{xx} = 5 > 0$. Hence $(7, 9)$ is a relative min. [**1 pt**]

When $D < 0$, we have a saddle point. Hence $(5, -10)$ is a saddle point. [**2 pts**]

When $D = 0$, the test is inconclusive. Hence at $(4, 8)$ the test is inconclusive. [**2 pts**]