MA 16020 Exam 2 Study Guide: Cal 2

An improper integral is when
(1) we have $\pm\infty$ in the bounds, or
(2) we have a discontinuity within the bounds
↓
Check if the integrand is undefined and check if that value is in the interval.

When computing them, rewrite with a limit

ex. $\int_0^{\infty} e^{-x}\,dx = \lim_{N\to\infty} \int_0^{N} e^{-x}\,dx$

To review limit check MA 16020 Exam 2 Study Guide: Cal 1.

Area Between Two Curves

The area between two curves can be described two ways:

$A = \int_a^b (\text{Top} - \text{Bottom})\,dx$ → You want $y = \text{something}_x$ for Top and Bottom

or $A = \int_c^d (\text{Right} - \text{Left})\,dy$ → You want $x = \text{something}_y$ for Right and Left

Volume of Solids of Revolution

Read the problem to see if a particular method is asked for. Plus try to draw the regions.

When the region "hugs" the line of rotation ⇒ Disk

- x-axis ⇒ dx problem ⇒ $V = \int_a^b \pi (f(x))^2 dx$
- y-axis ⇒ dy problem ⇒ $V = \int_c^d \pi (g(y))^2 dy$
- the line $y = \#$ ⇒ dx problem ⇒ $V = \int_a^b \pi (f(x) - \#)^2 dx$
- the line $x = \#$ ⇒ dy problem ⇒ $V = \int_c^d \pi (g(y) - \#)^2 dy$

When there is a "gap" between the region and the line of rotation ⇒ Washer

- x-axis ⇒ dx problem ⇒ $V = \int_a^b \pi (R^2 - r^2) dx$
- y-axis ⇒ dy problem ⇒ $V = \int_c^d \pi (R^2 - r^2) dy$
- the line $y = \#$ ⇒ dx problem ⇒ $V = \int_a^b \pi \left[(R - \#)^2 - (r - \#)^2 \right] dx$
- the line $x = \#$ ⇒ dy problem ⇒ $V = \int_c^d \pi \left[(R - \#)^2 - (r - \#)^2 \right] dy$

Where R is the farthest from the line of rotation and r is the closest to the line of rotation

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But if you find solving for x or y, in either method, is hard ⇒ Shell

- x-axis ⇒ dy problem ⇒ $V = \int_c^d 2\pi y (R - r) dy$
- y-axis ⇒ dx problem ⇒ $V = \int_a^b 2\pi x (\text{top} - \text{bottom}) dx$

MA 16020 LESSON 13: VOLUME BY REVOLUTION – SHELL METHOD (SUPPEMENTAL HOMEWORK)

Formulas:

- **Rotating around y-axis:**

$$V = 2\pi \int_a^b x \cdot (Top - Bottom)\, dx$$

- **Rotating around x-axis:**

$$V = 2\pi \int_c^d y \cdot (Right - Left)\, dy$$

- **Rotating around y = #**
  - **If $a \geq$ #, then**

$$V = 2\pi \int_a^b (x - \#) \times (Top - Bottom)\ dx$$

  - **If b $\leq$ #, then**

$$V = 2\pi \int_a^b (\# - x) \times (Top - Bottom)\ dx$$

- **Rotating around x = #**
  - **If $a \geq$ #, then**

$$V = 2\pi \int_a^b (y - \#) \times (Right - Left)\ dy$$

  - **If b $\leq$ #, then**

$$V = 2\pi \int_a^b (\# - y) \times (Right - Left)\ dy$$
