MA16020 Exam 3 Study Guide: Cal II

Differential Equations

- **Growth & Decay:** $y' = ky \Rightarrow y = Ce^{kt}$ where k is a constant.

- **Separation of Variables:** Solve the differential equations of the type

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

The idea is to try to get terms w/ y on one side and x-terms on the other. Integrate and solve for y.

- **First-Order Linear Differential Equations:** Are equations of the form $a(t)y' + b(t)y = c(t)$

How to solve:

① Using simple algebra, rewrite your equation to be

$$y' + P(t)y = Q(t)$$

② Determine $P(t)$ and $Q(t)$

③ Find integrating factor: $u(t) = \exp[\int P(t)\,dt]$

④ Plug $u(t)$ and $Q(t)$ in

$$y \cdot u(t) = \int Q(t)u(t)\,dt + C$$

⑤ Integrate the RHS of ④

⑥ Divide both sides of the equation from ⑤ by $u(t)$.

Sums / Series

- **Geometric Series:** Are of the form $\sum_{n=0}^{\infty} ar^n$
  - ↳ Converge if $|r|<1$ and $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$
  - ↳ Diverges if $|r| \geq 1$

- **Power Series:** Are of the form $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ where $|x|<1$
  - ↳ Radius of convergence is R when $|x|<R$.

  e.g. $\sum_{n=0}^{\infty} (2x)^n \Rightarrow |2x|<1$

  $|x| < 1/2 \Rightarrow R = 1/2$

- **Maclaurin Series:** Are of the form

  $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \quad \text{where } |x|<R$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \; ; \; \ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \; ; \; \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

Will be provided on the exam

Using Series to Estimate Definite Integrals

1. Convert the function into a series
2. Integrate the series (remember x is the variable)
3. Write out the number of terms to be used.
4. Substitute the bounds.

Functions of Several Variables

Domain: All points (x,y) in the xy-plane for which $f(x,y)$ is defined

Range: All values that the function $f(x,y)$ produces

Techniques for Finding the Domain

- Given $\sqrt{?} \Rightarrow ? \geq 0$
- Given $\ln(?) \Rightarrow ? > 0$
- Given $\frac{1}{?} \Rightarrow ? \neq 0$
- Given $\frac{1}{\sqrt{?}} \Rightarrow ? > 0$

Level Curves: $f(x,y) = k$ where k is a constant.

Descriptions of this curves can be found on the next page.

Descriptions of Curves

USEFUL DEFINITIONS

1. Point at the origin $\Rightarrow$ (0,0)

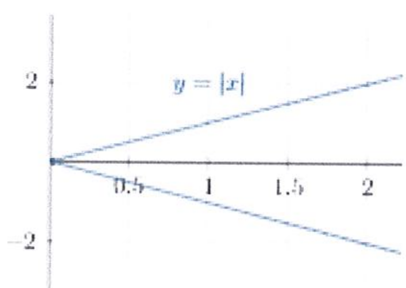
2. Lines $\Rightarrow$ $y = mx + b$ where m is the slope and b is the y-intercept

3. Parabolas $\Rightarrow$ $y = a(x-h)^2 + k$ where (h, k) is the vertex of the parabola

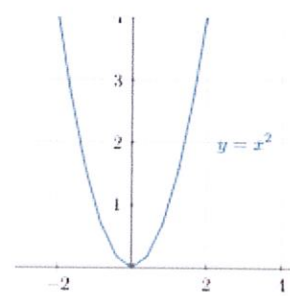
4. Exponential Functions
   a. Increasing $\Rightarrow$ example $y = e^x$
   b. Decreasing $\Rightarrow$ example $y = e^{-x}$

5. Logarithmic Functions
   a. Increasing $\Rightarrow$ example $y = \ln x$
   b. Decreasing $\Rightarrow$ example $y = -\ln x$

6. Rational Functions are functions of the form: $y = \frac{p(x)}{q(x)}$
   a. x-axis symmetry
      $\Rightarrow$ $f(x) = -f(x)$



   b. y-axis symmetry
      $\Rightarrow$ $f(x) = f(-x)$



7. Circles $\Rightarrow$ $(x-h)^2 + (y-k)^2 = r^2$ where r is radius and (h, k) is the center

8. Ellipses $\Rightarrow$ $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where (h, k) is the center

9. Hyperbolas $\Rightarrow$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ where (h, k) is the center

To find the foci for 8 and 9, we use the equation $c^2 = a^2 + b^2$, and solve for c.