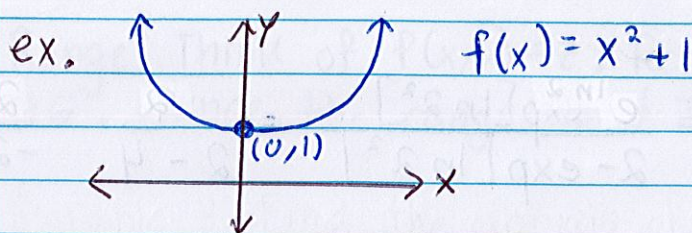


Lesson 21: Intro to Functions of Several Variables

Single Variable Functions

- Can be written as $y = f(x)$
i.e. y is a function of x
- Takes as an input a # and produces as an output a #
- Graph: a curve in the xy -plane
i.e. 2-D

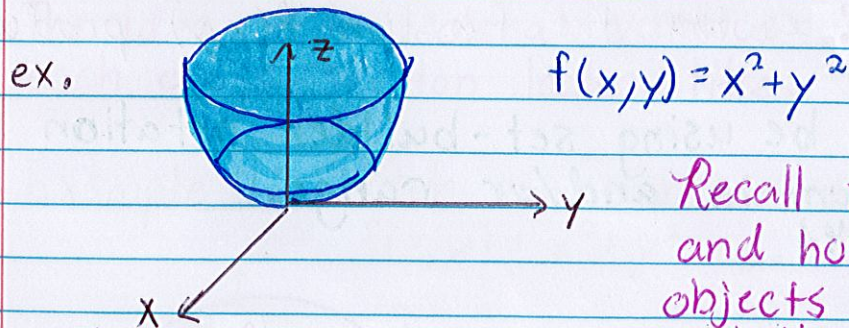


Multivariable Functions

Definition: A multivariable function has 2 or more variables as inputs.

In this class, we are limiting ourselves to just 2 variables. So

- Can be written as $z = f(x, y)$
i.e. z is a function of x and y
- Takes as an input a pair of # or a point and produces as an output a #
- Graph: A surface in xyz -space
i.e. 3-D



Recall the idea of cross sections and how stacking multiple 2-D objects became one 3-D object. Well this is a bunch of circles with increasing radii stacked on top of another.

Example 1: Compute the indicated functional value

$$(a) f(x,y) = \frac{3x+2y}{2x+3y} ; f(-4,6)$$

$$f(-4,6) = \frac{3(-4)+2(6)}{2(-4)+3(6)} = \frac{-12+12}{-8+18} = \frac{0}{10} = 0$$

$$(b) f(x,y) = \frac{e^{xy}}{2-e^{xy}} ; f(\ln 2, 2) \text{ and } f(1, \ln 1)$$

$$f(\ln 2, 2) = \frac{e^{2 \ln 2}}{2 - e^{2 \ln 2}} = \frac{\exp[\ln 2^2]}{2 - \exp[\ln 2^2]} = \frac{4}{2-4} = \frac{4}{-2} = -2$$

$$f(1, \ln 1) = \frac{e^{\ln 1}}{2 - e^{\ln 1}} = \frac{1}{2-1} = \frac{1}{1} = 1$$

Domain and Range for Multivariable Functions

Just as for functions of one variable, we can find the domain and range of functions of two variables in a similar fashion. **If you need a review of domain and range of one variable functions, look at the Algebra Review pdf for Lesson 18.**

The main difference for two variable functions is

Domain: All points (x,y) in the xy -plane for which $f(x,y)$ is defined

Range: All values that the function $f(x,y)$ produces

Notation: We will be using set-builder notation for denoting the domain and/or range.

i.e. $\{x \mid x \geq 4\}$

Example 2: Describe the domain and range of the function
 $f(x,y) = \sqrt{x^2 - y}$

Domain: Recall domain of $\sqrt{\quad}$ is ≥ 0 . So

$$x^2 - y \geq 0$$

$$x^2 \geq y$$

Hence the domain is $\{(x,y) \mid x^2 \geq y\}$. Note this domain consists parabolas.

Range: Think of $f(x,y) = z$. Recall that the range of $\sqrt{\quad}$ is ≥ 0 . Hence the range is $\{z \mid z \geq 0\}$

Example 3: Find the domain of $f(x,y) = \frac{\sqrt{x-18}}{\ln(y-9)-3}$

To find the domain, we need to play with 3 portions of the function.

$$\bullet \sqrt{x-18} \Rightarrow x-18 \geq 0$$
$$x \geq 18$$

$$\bullet \ln(y-9) \Rightarrow y-9 > 0$$
$$y > 9$$

$$\bullet \ln(y-9)-3 \neq 0$$
$$\ln(y-9) \neq 3$$

$$y-9 \neq e^3$$

$$y \neq e^3 + 9$$

Hence the domain is $\{(x,y) \mid x \geq 18, y > 9, y \neq e^3 + 9\}$

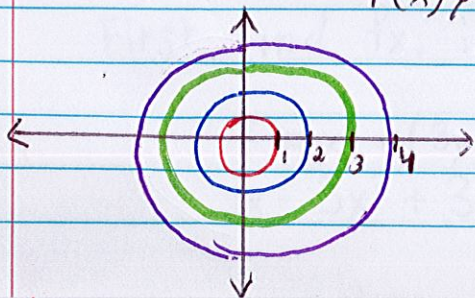
Level Curves

Definition: The level curves of a function of two variables are the curves with equations $f(x,y) = C$ where C is a constant (in the range of f).

The idea is to look at various values of z and see what each cross-section looks like.

Example 4: Sketch the indicated level curves

$$f(x,y) = x^2 + y^2, \quad C = 1, 4, 9, 16$$



$$x^2 + y^2 = 1 = 1^2$$

$$x^2 + y^2 = 4 = 2^2$$

$$x^2 + y^2 = 9 = 3^2$$

$$x^2 + y^2 = 16 = 4^2$$

Example 6: What do the level curves for $f(x,y) = 11\sqrt{y+6x^2}$ look like?

Let $f(x,y) = C$. So $C = 11\sqrt{y+6x^2}$. Now solve for y .

$$\frac{C}{11} = \sqrt{y+6x^2}$$

$$C = 11\sqrt{y+6x^2}$$

$$C^2 = 121(y+6x^2)$$

$$C = 11\sqrt{y+6x^2}$$

$$y = -6x^2 + \frac{C^2}{121}$$

So the level curves are in the shapes of parabolas.

Example 5: What does the level curve for $f(x,y) = \ln(x^2+y^2)$ at $C = \ln(36)$ look like?

Let $f(x,y) = C$. So

$$\ln(x^2+y^2) = \ln(36)$$

$$x^2+y^2 = 36$$

$$x^2+y^2 = 6^2$$

So the level curve is a circle with center at the origin and with a radius of 6.

