Lesson 5: Integration by Parts II

- Recall from Last time, we found the formula for Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

We were also given an acronym to help in choosing what u should be.

L - Logarithmic
A - Algebraic (like polynomials)
T - Trigonometric
E - Exponential

Example 1: Evaluate

(a) $\int (t-3)e^t\,dt$ $\quad \dfrac{u = t-3 \qquad dv = e^t\,dt}{du = dt \qquad v = e^t}$ $\quad uv - \int v\,du$

$$= (t-3)e^t - \int e^t\,dt = (t-3)e^t - e^t + C$$

$$= (t-3-1)e^t + C = (t-4)e^t + C$$

(b) $\int \ln x\,dx$ $\quad \dfrac{u = \ln x \qquad dv = dx}{du = 1/x\,dx \qquad v = x}$ $\quad uv - \int v\,du$

$$= x\ln x - \int x \cdot \frac{1}{x}\,dx = x\ln x - \int dx$$

$$= x\ln x - x + C$$

Example 2: Evaluate

(a) $\int x^2 \sin x\,dx$ $\quad \dfrac{u = x^2 \qquad dv = \sin x\,dx}{du = 2x\,dx \qquad v = -\cos x}$ $\quad uv - \int v\,du$

$$= -x^2\cos x - \int (-\cos x)2x\,dx$$

$$= -x^2\cos x + \underbrace{\int 2x\cos x\,dx}$$

Note to integrate this we need to do another integration by parts

MA16020 - Lesson 5 Notes

$\int 2x\cos x\,dx$ $\quad \dfrac{u=2x}{du=2dx}$ $\quad \dfrac{dv=\cos x\,dx}{v=\sin x}$ $\quad uv-\int v\,du$

$= 2x\sin x - \int 2\sin x\,dx = 2x\sin x - (-2\cos x)$

$= 2x\sin x + 2\cos x$

$= x^2\cos x + 2x\sin x + 2\cos x + C$

(b) $\int 12x(\ln(4x))^2\,dx$ $\quad \dfrac{u=(\ln(4x))^2}{du=2(\ln(4x))\cdot\frac{1}{4x}\cdot 4\,dx}$ $\quad \dfrac{dv=12x\,dx}{v=\frac{12x^2}{2}}$

$du = \dfrac{2(\ln(4x))}{x}\,dx$ $\quad v = 6x^2$

$= 6x^2(\ln(4x))^2 - \int 6x^2\cdot\dfrac{2(\ln(4x))}{x}\,dx$

$= 6x^2(\ln(4x))^2 - \int 12x\ln(4x)\,dx$

Note to integrate this we need to do another integration by parts

$\int 12x\ln(4x)\,dx$ $\quad \dfrac{u=\ln(4x)}{du=\frac{1}{4x}\cdot 4\,dx}$ $\quad \dfrac{dv=12x\,dx}{v=\frac{12x^2}{2}}$

$du = \frac{1}{x}\,dx$ $\quad v = 6x^2$

$= 6x^2\ln(4x) - \int 6x^2\cdot\dfrac{1}{x}\,dx$

$= 6x^2\ln(4x) - \int 6x\,dx$

$= 6x^2\ln(4x) - \dfrac{6x^2}{2} = 6x^2\ln(4x) - 3x^2$

$= 6x^2(\ln(4x))^2 - (6x^2\ln(4x) - 3x^2) + C$

$= 6x^2(\ln(4x))^2 - 6x^2\ln(4x) + 6x^2 + C$

Example 3: The velocity of a car over the time period $0 \le t \le 3$ is given by the function

$$v(t) = 50te^{-t/4} \text{ miles per hour}$$

where t is time in hours. What was the distance the car traveled in the first 30 minutes?

Note t is in hours. So 30 mins $\Rightarrow$ 0.5 hrs

We need to compute

$$\int_0^{0.5} 50te^{-t/4}\,dt \qquad \frac{u = 50t}{du = 50\,dt} \qquad \frac{dv = e^{-t/4}\,dt}{v = -4e^{-t/4}} \qquad uv - \int v\,du$$

$$= 50t(-4e^{-t/4})\Big]_0^{0.5} - \int_0^{0.5} -4e^{-t/4}(50)\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200\int_0^{0.5} e^{-t/4}\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200(-4)e^{-t/4}\Big]_0^{0.5}$$

$$= -200(0.5)e^{-0.5/4} - (-200(0)e^{-0/4}) - 800e^{-0.5/4}$$

$$- (-800e^{-0/4})$$

$$= -100e^{-1/8} - 0 - 800e^{-1/8} + 800$$

$$= -900e^{-1/8} + 800$$

$$\approx 5.75$$