Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name: Solutions

1. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1-\cos x}\,dx$$

$1-\cos x = 0$

$1 = \cos x$

$x = 0, \pi, 2\pi$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

2. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \tan(x)\,dx$$

$\tan x = \frac{\sin x}{\cos x}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

3. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \cos(x)\,dx \rightarrow$$ $\cos(x)$ is defined everywhere.

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

Bonus do this question w/ all trig functions

4. Evaluate the following integral;

$$\int_1^\infty \frac{5}{\sqrt{x}} dx$$

$$\int_1^\infty \frac{5}{\sqrt{x}} dx = \lim_{N\to\infty} \int_1^N 5x^{-1/2} dx = \lim_{N\to\infty} \left(5 \cdot 2x^{1/2}\right)\Big]_1^N$$

$$= \lim_{N\to\infty} \left(10(N)^{1/2} - 10\right) = \infty$$

$$\int_1^\infty \frac{5}{\sqrt{x}} dx = \boxed{\infty}$$

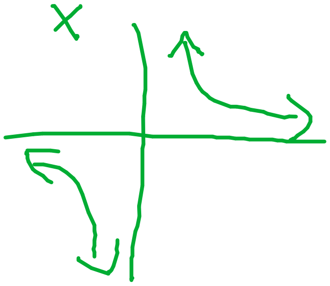
5. Evaluate the following integral;

$$\int_1^\infty \frac{3}{x^2} dx$$

$$\int_1^\infty \frac{3}{x^2} dx = \lim_{N\to\infty} \int_1^N 3x^{-2} dx = \lim_{N\to\infty} \left(\frac{3x^{-1}}{-1}\right)\Big]_1^N$$

$$= \lim_{N\to\infty} \left(-\frac{3}{x}\right)\Big]_1^N = \lim_{N\to\infty} \left(-\frac{3}{N} + 3\right)$$

$y = \frac{1}{x}$



$$\int_1^\infty \frac{3}{x^2} dx = \boxed{3}$$

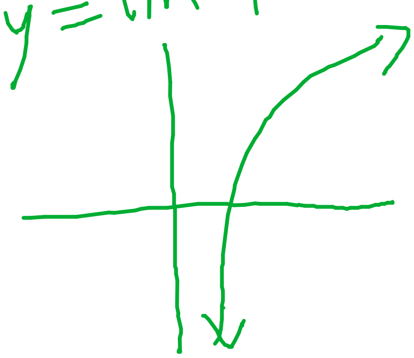
6. Evaluate the following integral;

$$\int_1^\infty \frac{10}{x} dx$$

$$\int_1^\infty \frac{10}{x} dx = \lim_{N\to\infty} \int_1^N \frac{10}{x} dx = \lim_{N\to\infty} \left(10 \ln|x|\right)\Big]_1^N$$

$$= \lim_{N\to\infty} \left(10 \ln|N| - 0\right)$$

$y = \ln|x|$



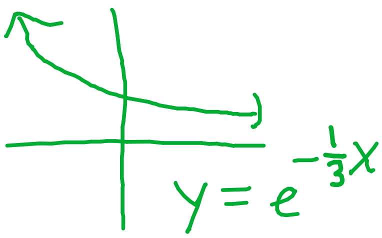
$$\int_1^\infty \frac{10}{x} dx = \boxed{\infty}$$

7. Evaluate the following integral;

$$\int_0^\infty 7e^{-\frac{1}{3}x}dx$$

$$\int_0^\infty 7e^{-\frac{1}{3}x}dx = \lim_{N\to\infty}\int_0^N 7e^{-\frac{1}{3}x}dx = \lim_{N\to\infty}\left(7\frac{e^{-\frac{1}{3}x}}{-1/3}\right)\Bigg]_0^N$$

$$= \lim_{N\to\infty}\left(-21e^{-\frac{1}{3}x}\right)\Bigg]_0^N = \lim_{N\to\infty}\left(\underbrace{\cancel{-21e^{-\frac{1}{3}N}}}_{\to 0}+21\right)$$



$y = e^{-\frac{1}{3}x}$

$$\int_0^\infty 7e^{-\frac{1}{3}x}dx = \boxed{21}$$

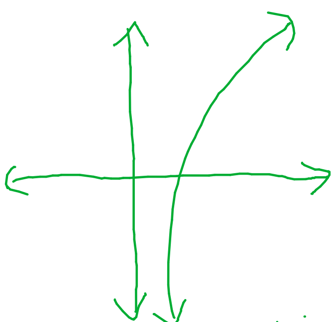
8. Evaluate the definite integral

$$\int_2^\infty \frac{dx}{5x+2}$$

$$\lim_{N\to\infty}\int_2^N \frac{dx}{5x+2} \qquad u=5x+2,\quad du=5dx,\quad \frac{du}{5}=dx$$

$$\lim_{N\to\infty}\int \frac{1}{5}\frac{du}{u} = \lim_{N\to\infty}\frac{1}{5}\ln|u| = \lim_{N\to\infty}\frac{1}{5}\ln|5x+2|\Bigg]_2^N$$

$$= \lim_{N\to\infty}\left(\frac{1}{5}\ln|5N+2| - \frac{1}{5}\ln|12|\right) = \infty$$



ln-function

$$\int_2^\infty \frac{dx}{5x+2} = \boxed{\infty}$$

9. Set up the integral that computes the **AREA** shown to the right with respect to x.

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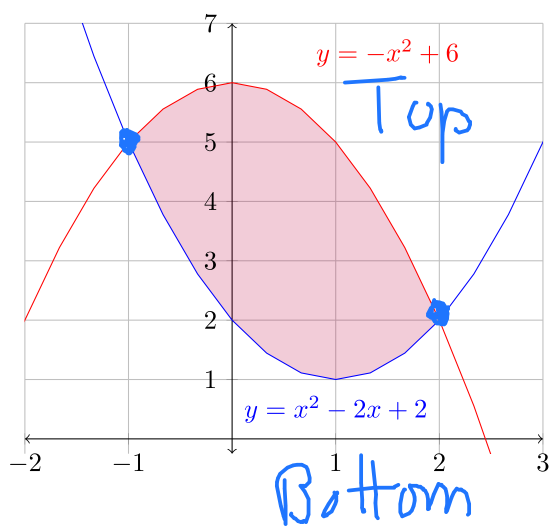

$$\boxed{\int_{-1}^{2}(-x^2+6)-(x^2-2x+2)\,dx}$$

Area = ____________________

10. Set up the integral that computes the **AREA** shown to the right with respect to y. $\Rightarrow dy$

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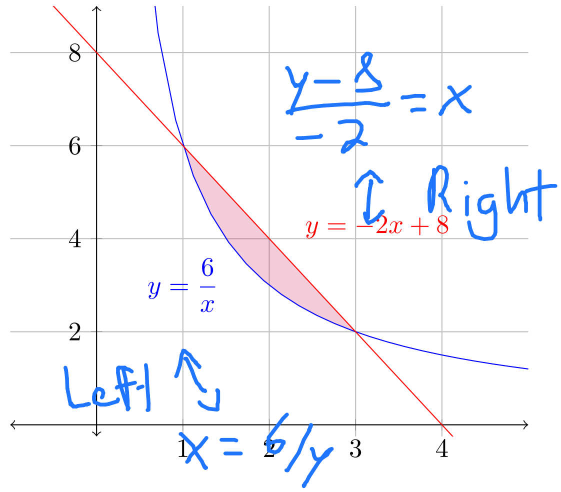


Area $= \boxed{\int_2^6 \left(\frac{y-8}{-2}\right) - \frac{6}{y}\, dy}$

11. Set up the integral that computes the **AREA** with respect to x of the region bounded by $\Rightarrow dx$ problem

$$y = \frac{2}{x} \quad \text{and} \quad y = -x + 3$$

Bounds:

$$\frac{2}{x} = -x + 3$$

$$2 = -x^2 + 3x$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

Test Pt: $x = 1.5$

$y = \frac{2}{x} \Rightarrow y = \frac{2}{1.5} = \frac{4}{3} \approx 1.33 \to$ Bottom

$y = -x + 3 \Rightarrow y = -1.5 + 3 = 1.5 \to$ Top

Area $= \boxed{\int_1^2 \left(-x + 3 - \frac{2}{x}\right) dx}$

12. Find the area of the region bounded by $y = 6x^2$ and $y = 12x$.

Bounds: $6x^2 = 12x$
$6x^2 - 12x = 0$
$6x(x-2) = 0$
$x = 0, 2$

Test pt: $x = 1$
$y = 6x^2 \longrightarrow y = 6 \rightarrow$ Bottom
$y = 12x \longrightarrow y = 12 \rightarrow$ Top

$$A = \int_0^2 (12x - 6x^2)\,dx$$
$$= \left(\frac{12x^2}{2} - \frac{6x^3}{3}\right)\Big]_0^2$$
$$= \left(6x^2 - 2x^3\right)\Big]_0^2$$
$$= 8$$

Area = $\boxed{8}$

13. Find the area of the region bounded by $y = 6x - x^2$ and $y = 2x^2$.

Bounds:
$6x - x^2 = 2x^2$
$6x - 3x^2 = 0$
$3x(2-x) = 0$
$x = 0, 2$

Test pt: $x = 1$
$y = 6x - x^2 \Rightarrow y = 5 \rightarrow$ Top
$y = 2x^2 \Rightarrow y = 2 \rightarrow$ Bottom

$$A = \int_0^2 \left[(6x - x^2) - 2x^2\right] dx$$
$$= \int_0^2 (6x - 3x^2)\,dx$$
$$= \left(3x^2 - x^3\right)\Big]_0^2 = 4$$

Area = $\boxed{4}$

14. Calculate the **AREA** of the region bounded by the following curves.

$$x = 100 - y^2 \quad \text{and} \quad x = 2y^2 - 8$$

Bounds:

$100 - y^2 = 2y^2 - 8$

$108 = 3y^2$

$36 = y^2$

$y = \pm 6$

Test Pt: $y = 0$

$X = 100 - y^2 \rightarrow x = 0 \rightarrow$ Right

$X = 2y^2 - 8 \rightarrow x = -8 \rightarrow$ Left

$A = \int_{-6}^{6} (100 - y^2) - (2y^2 - 8)\, dy$

$= \int_{-6}^{6} (108 - 3y^2)\, dy$

$= (108y - y^3)\Big]_{-6}^{6}$

$= 864$

Area $=$ $\boxed{864}$

15. After t hours studying, one student is working $Q_1(t) = 25 + 9t - t^2$ problems per hour, and a second student is working on $Q_2(t) = 5 - t + t^2$ problems per hour. How many more problems will the first student have done than the second student after 10 hours?

$\int_0^{10} Q_1(t) - Q_2(t)\, dt$

$= \int_0^{10} (25 + 9t - t^2) - (5 - t + t^2)\, dt$

$= \int_0^{10} (20 + 10t - 2t^2)\, dt$

$= \left(20t + 5t^2 - \frac{2}{3}t^3\right)\Big]_0^{10}$

$= \frac{100}{3}$

Answer: $\boxed{100/3}$

16. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x + 8, \quad \text{and} \quad y = (x-4)^2$$

about the x-axis

Bounds:

$x+8 = (x-4)^2$

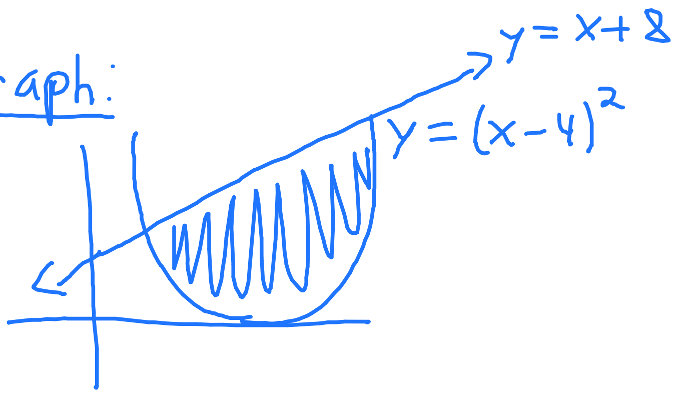
$x+8 = x^2 - 8x + 16$

$0 = x^2 - 9x + 8$

$0 = (x-8)(x-1)$

$x = 1, 8$

Graph:

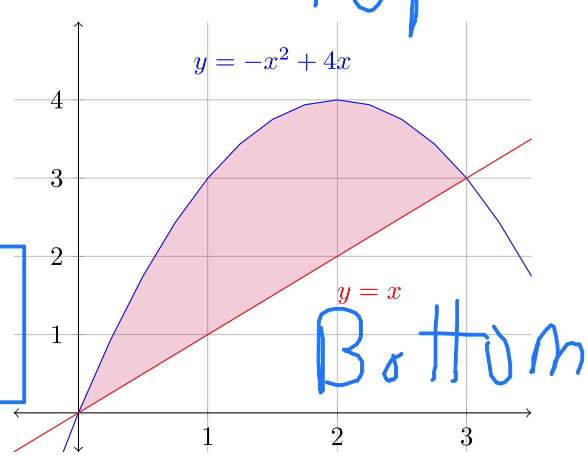


Volume = $\boxed{\pi \int_1^8 \left[(x+8)^2 - (x-4)^4 \right] dx}$

17. Let R be the region shown below. Set up the integral that computes the **VOLUME** as R is rotated around the x-axis.

DON'T COMPUTE IT!!!

Top



$\boxed{\pi \int_0^3 \left[(-x^2+4x)^2 - (x)^2 \right] dx}$

Volume = ________________________

18. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{16 - x}, \quad y = 0 \quad \text{and} \quad x = 0$$

about the y-axis $\Rightarrow dy$ problem

Disk

$y = \sqrt{16 - x}$

$y^2 = 16 - x$

$x = 16 - y^2$

Bounds: Given $y = 0$

Plug $x = 0$ into $y = \sqrt{16 - x}$

$y = \sqrt{16 - x}$

$y = \sqrt{16}$

$y = 4$

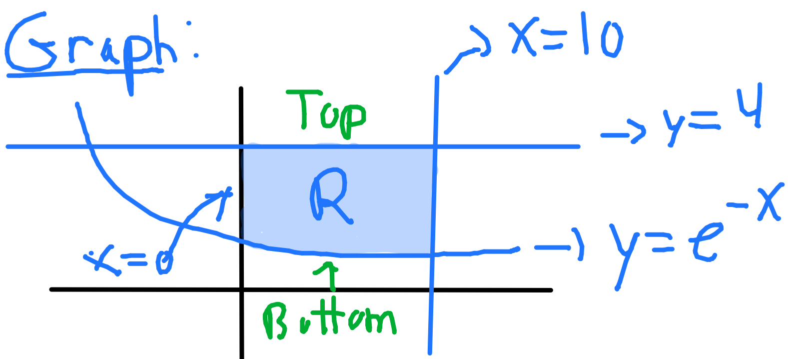
Volume = $\boxed{\pi \int_0^4 (16 - y^2)^2 \, dy}$

19. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = e^{-x}, \quad y = 4 \quad x = 0 \quad \text{and} \quad x = 10$$

about the x-axis $\rightarrow dx$

Graph:



$$V = \pi \int_0^2 \left[4^2 - (e^{-x})^2 \right] dx$$

Volume = $\boxed{\pi \int_0^2 (16 - e^{-2x}) \, dx}$

20. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 0 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis

Disk

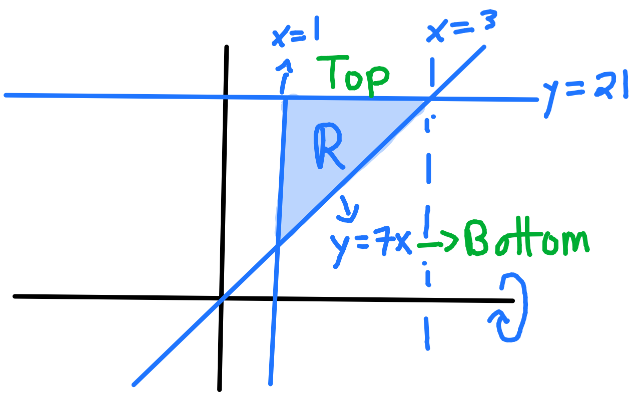
$$V = \pi \int_1^3 (7x)^2 \, dx$$
$$= \pi \int_1^3 49x^2 \, dx$$
$$= \pi \left(\frac{49x^3}{3} \right) \Big]_1^3$$
$$= \frac{49\pi}{3} (3^3 - 1)$$
$$= \boxed{\frac{1274}{3}\pi}$$

Volume = $\boxed{\frac{1274}{3}\pi}$

21. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 21 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis $\longrightarrow dx$



Washer

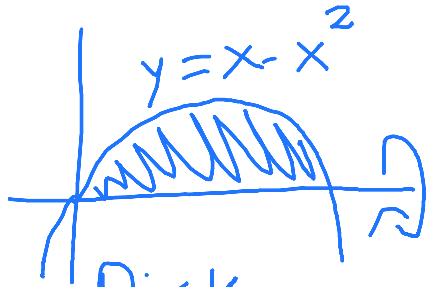
$$V = \pi \int_1^3 \left[21^2 - (7x)^2 \right] dx$$
$$= \pi \int_1^3 (441 - 49x^2) \, dx$$
$$= \pi \left(441x - \frac{49x^3}{3} \right) \Big]_1^3$$
$$= \frac{1274}{3}\pi$$

Volume = $\boxed{\frac{1274\pi}{3}}$

22. Find the **VOLUME** of the region bounded by

$$y = x - x^2, \quad \text{and} \quad y = 0$$

around the x-axis



Disk

Bounds:

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$

$V = \pi \int_0^1 (x - x^2)^2 \, dx$

$= \pi \int_0^1 (x^2 - 2x^3 + x^4) \, dx$

$= \pi \left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \Big]_0^1$

$= \frac{\pi}{30}$

Volume $=$ $\boxed{\pi/30}$

23. Find the **VOLUME** of the solid generate by revolving the given region about the x-axis: $\longrightarrow dx$

$$y = 8\sqrt{x}, \quad y = 0, \quad x = 3, \quad x = 6$$

$V = \pi \int_3^6 (8\sqrt{x})^2 \, dx$

$= \pi \int_3^6 64x \, dx$

$= \pi \, \frac{64x^2}{2} \Big]_3^6$

$= \pi \, 32x^2 \Big]_3^6$

$= 864\pi$

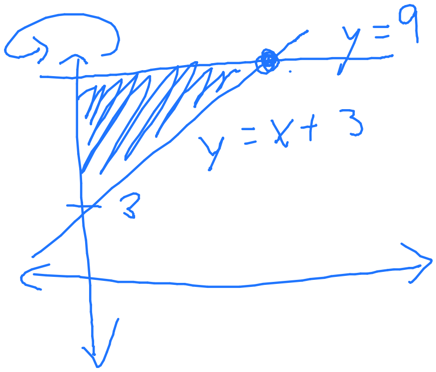
Volume $=$ $\boxed{864\pi}$

24. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 3, \quad x = 0, \quad y = 9$$

$x = y - 3$

around the y-axis $\rightarrow$ dy problem.



$$V = \pi \int_3^9 (y-3)^2\, dy$$
$$= \pi \int_3^9 (y^2 - 6y + 9)\, dy$$
$$= \pi \left(\frac{y^3}{3} - 3y^2 + 9y \right) \Big]_3^9$$

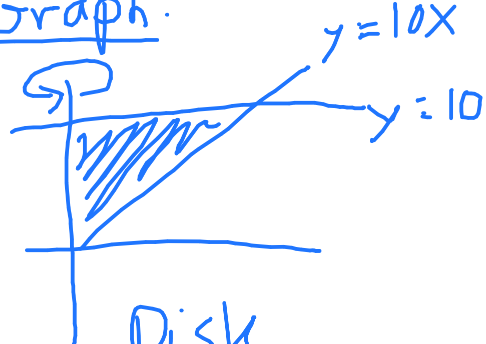
Volume $=$ $\boxed{72\pi}$

25. Find the **VOLUME** of the region bounded by

$$y = 10x, \quad x = 0, \quad y = 10$$

around the y-axis

Graph:



Disk

But y-axis $\Rightarrow$ dy problem

$y = 10x$

$\frac{y}{10} = x$

$$V = \pi \int_0^{10} \left(\frac{y}{10}\right)^2 dy$$
$$= \pi \int_0^{10} \frac{y^2}{100}\, dy$$
$$= \frac{\pi}{100} \left(\frac{y^3}{3} \right) \Big]_0^{10}$$
$$= \frac{10\pi}{3}$$

Volume $=$ $\boxed{10\pi/3}$

26. Find the **VOLUME** of the region bounded by

$$x + 3y = 9, \quad x = 0, \quad y = 0$$

around the y-axis

$x + 3y = 9$

$3y = -x + 9$

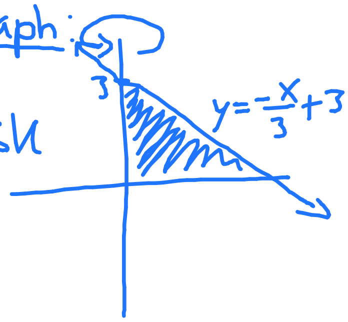
$y = \frac{-x}{3} + 3$

But y-axis $\Rightarrow dy$

So $x + 3y = 9$

$x = 9 - 3y$

Graph:



Disk

$$V = \pi \int_0^3 (9-3y)^2 \, dy$$

$$= \pi \int_0^3 (81 - 54y + 9y^2) \, dy$$

$$= \pi \left(81y - 27y^2 + 3y^3\right)\Big]_0^3$$

$$= 81\pi$$

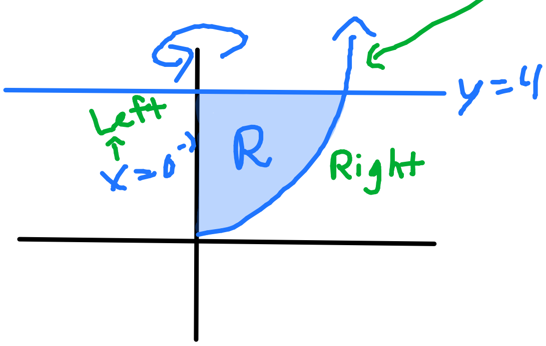
Volume = $\boxed{81\pi}$

27. Find the **VOLUME** of the region bounded by

$$y = 4x^2, \quad x = 0, \quad y = 4$$

around the y-axis → dy

$x = \sqrt{\frac{y}{4}}$



Disk

$$V = \pi \int_0^4 \left(\sqrt{\frac{y}{4}}\right)^2 dy$$

$$= \pi \int_0^4 \frac{y}{4} \, dy$$

$$= \frac{\pi}{4} \cdot \frac{y^2}{2} \Big]_0^4$$

$$= \frac{\pi}{8} \cdot 16$$

$$= 2\pi$$

Volume = $\boxed{2\pi}$

28. Let R be the region shown to the right. Set up the integral that computes the **VOLUME** as R is rotated around the line $x = 4$. $\Rightarrow$ dy problem

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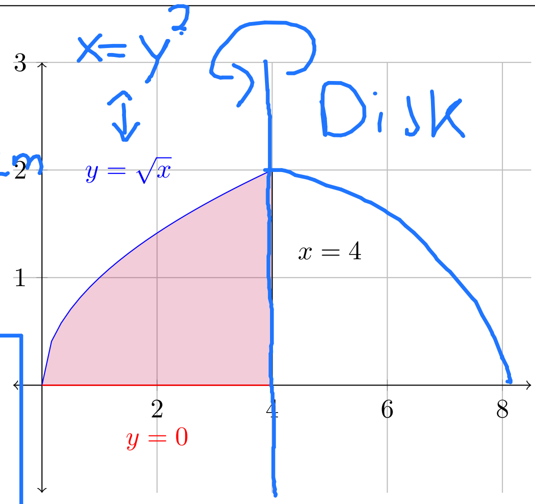


Volume $= \boxed{\pi \int_0^2 (y^2 - 4)^2 \, dy}$

29. Set up the integral needed to find the volume of the solid obtained when the region bounded by

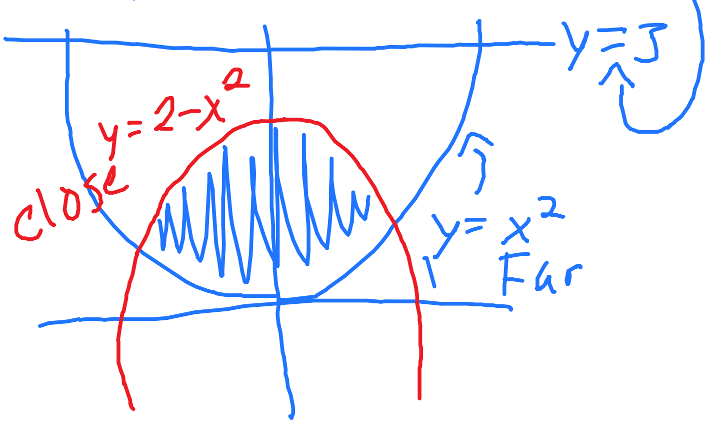
$$y = 2 - x^2 \quad \text{and} \quad y = x^2$$

is rotated about the line $y = 3$.

$y = 3 \Rightarrow dx$ problem

Graph

Washer



Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

$$\boxed{\pi \int_{-1}^{1} (2 - x^2 - 3)^2 - (x^2 - 3)^2 \, dx}$$

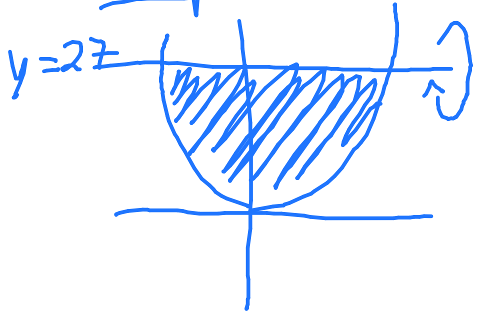
Volume $=$ ________________

30. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Graph Disk



$y = 27 \Rightarrow dx$ problem

Bounds: Given $x = 0$

$27 = 3x^2$

$9 = x^2 \rightarrow x = 3$

$$V = \pi \int_0^3 (3x^2 - 27)^2 \, dx$$

$$= \pi \int_0^3 (9x^4 - 162x^2 + 729) \, dx$$

$$= \pi \left(\frac{9x^5}{5} - 54x^3 + 729x \right) \Big]_0^3$$

$$= 11664.4\pi$$

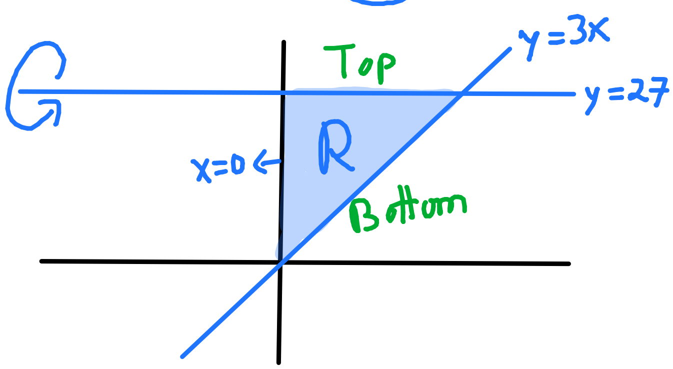
Volume $=$ $\boxed{\frac{83322}{5}\pi}$

31. Find the **VOLUME** of the region bounded by

$$y = 3x, \quad x = 0, \quad y = 27$$

Disk

around the line $y = 27 \rightarrow dx$



Bound: $3x = 27$

$x = 9$

$$V = \pi \int_0^9 (3x - 27)^2 \, dx$$

$$= \pi \int_0^9 (9x^2 - 162x + 729) \, dx$$

$$= \pi \left(\frac{9x^3}{3} - \frac{162x^2}{2} + 729x \right) \Big]_0^9$$

$$= \pi \left(3x^3 - 81x^2 + 729x \right) \Big]_0^9$$

Volume $=$ $\boxed{2187\pi}$

32. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$x = 2y - y^2, \quad \text{and} \quad x = 0$$

about the x-axis. → dy

+

Bounds: $0 = 2y - y^2$

$0 = y(2-y)$

$y = 0, 2$

$$V = 2\pi \int_0^2 y(2y - y^2)\,dy$$

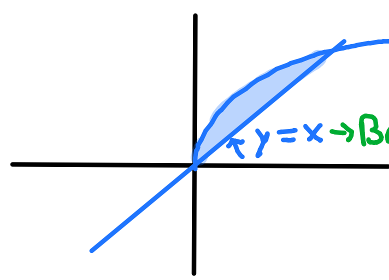
Volume = $\boxed{2\pi \int_0^2 y(2y - y^2)\,dy}$

33. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{x}, \quad \text{and} \quad y = x$$

about the y-axis. → dx

+



Bounds: $\sqrt{x} = x$

$(\sqrt{x})^2 = x^2$

$x = x^2$

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$

$$V = 2\pi \int_0^1 x(\sqrt{x} - x)\,dx$$

Volume = $\boxed{V = 2\pi \int_0^1 x(\sqrt{x} - x)\,dx}$

34. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 2 - x^2, \quad \text{and} \quad y = x^2$$

about the y-axis. $\to$ + $\to dx$

$$V = 2\pi \int_{-1}^{1} x(2 - x^2 - x^2)\,dx$$

Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

Test Pt: $x = 0$

$y = 2 - x^2 \longrightarrow y = 2 \to$ Top

$y = x^2 \longrightarrow y = 0 \to$ Bottom

Volume $= \boxed{2\pi \int_{-1}^{1} x(2 - 2x^2)\,dx}$

35. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = x, \quad \text{and } y = x^2$$

about the line $x = -2$. $\to$ + $\to dx$

Since $x = -2$ is smaller than the bounds,

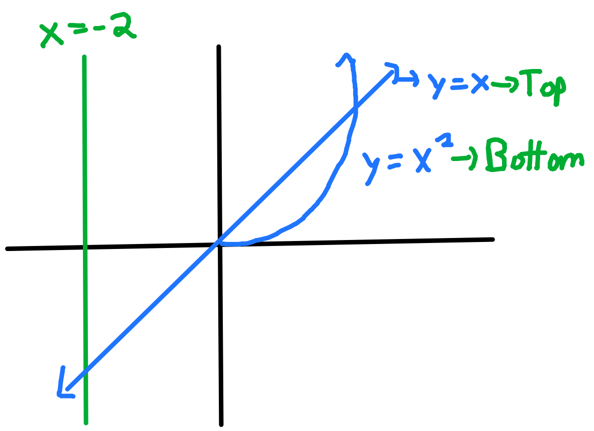
$$V = 2\pi \int_0^1 (x - (-2))\left[x - x^2\right]dx$$

Bounds: $x = x^2$

$x - x^2 = 0$

$x(1 - x) = 0$

$x = 0, 1$



Volume $= \boxed{2\pi \int_0^1 (x+2)(x - x^2)\,dx}$

36. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

+ $\to dx$

$$y = 7x^2, \quad y = 0 \text{ and } \quad x = 2$$

about the line $x = 3$

$$V = 2\pi \int_0^2 (\_\_\_\_)(7x^2)\,dx$$

Since $x=3$ is larger than the bounds,

$$V = 2\pi \int_0^2 (3-x)(7x^2)\,dx$$

Volume = $\boxed{2\pi \int_0^2 (3-x)(7x^2)\,dx}$

37. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

+ $\to dy$

$$x = y^2 + 1, \text{ and } \quad x = 2$$

about the line $y = -2$

Bounds: $y^2+1=2$

$y^2 = 1$

$y = \pm 1$

Test Pt: $y = 0$

$x = y^2+1 \to x = 1 \to$ Left

$x = 2 \to x = 2 \to$ Right

Since $y=-2$ is smaller than the bounds

$$V = 2\pi \int_{-1}^{1} (y-(-2))(2-(y^2+1))\,dy$$

Volume = $\boxed{2\pi \int_{-1}^{1} (y+2)(2-(y^2+1))\,dy}$