Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name:________________________________

1. The rate of change of the population $n(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate k in terms of minutes. Round to four decimal places.

$k=$ ________________________

2. Let y denote the mass of a radioactive substance at time t. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time t in hours does half the original mass remain? Round your answer to 3 decimal places.

$t \quad =$ ________________________

3. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \quad \text{where} \quad y = 10 \quad \text{when} \quad x = 2$$

Find the value of the integration constant, C.

$C =$ ____________________

4. Calculate the constant of integration, C, for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \qquad y(1) = 2$$

$C=$ ____________________

5. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

$y \quad = \quad$ ______________________________

6. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

$y \quad = \quad$ ______________________________

7. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

$y \quad = \quad$ ________________________________

8. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

$y =$ ________________________________

9. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$y =$ ______________________

10. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$y =$ ______________________

11. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$y =$ ______________________

12. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$y =$ ______________________

13. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \text{ and } y(0) = 1$$

$y =$ ______________________________

14. What is the **integrating factor** of the following differential equation?

$$x^8 y' - 14x^7 y = 32e^{7x}$$

$u(x) =$ ______________________________

15. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right) y = 10 \ln(x)$$

$u(x) =$ ______________________

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16. What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2 + x)y = (x+1)e^{x^2}$$

$u(x) =$ ______________________

17. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) =$ ____________________

18. Solve the initial value problem.

$$\frac{dy}{dx} + 13y = 16e^{-13x} \text{ with } y(0) = 10$$

$y =$ ____________________

19. Solve the initial value problem.

$$x^4 y' + 4x^3 \cdot y = 10x^9 \text{ with } y(1) = 23$$

$y =$ ______________________________

20. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \dots$$

Answer:______________________________

(b) Use the sum from (a) and compute the sum.

Answer:______________________________

21. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \underline{\hspace{6cm}}$$

22. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^{n}$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^{n} =$ ______________________

23. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ ______________________

24. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \underline{\hspace{10cm}}$$

25. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \underline{\hspace{10cm}}$$

26. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty}\left(\frac{3^{n-1}}{4^n}+\frac{(-1)^{n+1}}{9^n}\right)$$

Answer:____

27. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

$R =$ ____

28. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

$R =$ ____________________

29. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$\dfrac{3}{1+2x} =$ ____________________

$R =$ ____________________

30. Express $f(x) = \dfrac{3x}{10+2x}$ as a power series and determine it's radius of converge.

$\dfrac{3x}{10+2x} =$ ________________________________

$R =$ ________________________________

31. Express $f(x) = \dfrac{5x}{3+2x^2}$ as a power series and determine it's radius of converge.

$\dfrac{5x}{3+2x^2} =$ ________________________________

$R =$ ________________________________

32. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$\int \sin(x^{3/2})\,dx =$ ______________________________

33. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx$$

$\int_0^{0.11} \frac{1}{1+x^4}\,dx \approx$ ______________________________

34. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5+x^6}\,dx$$

$\int_0^{0.24} \frac{x}{5+x^6}\,dx \approx$ ________________________________

35. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\,dx$$

$\int_0^{0.23} e^{-x^2}\,dx \approx$ ________________________________

36. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ ______________________

37. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1 + x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$\ln(1.56) \approx$ ______________________

38. Find the domain of

$$f(x, y) = \frac{\sqrt{x + y - 1}}{\ln(y - 11) - 9}$$

Domain = ______________________________

39. Find the domain of

$$f(x, y) = \frac{\ln(x^2 - y + 3)}{\sqrt{x - 6}}$$

Domain = ______________________________

40. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \;\; C = \ln(36)$$

(a) Parabola with vertices at $(0, 0)$
(b) Circle with center at $(0, \ln(36))$ and radius 6
(c) Parabola with vertices at $(0, \ln(36))$
(d) Circle with center at $(0, 0)$ and radius 6
(e) Increasing Logarithm Function

41. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

42. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

43. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas