Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name: Solutions

1. The rate of change of the population $n(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate k in terms of minutes. Round to four decimal places.

Recall $N' = kN \to N = Ce^{kt}$

$\underline{N(0)=210}$: $210 = Ce^{k\cdot 0}$

$210 = C \to N = 210e^{kt}$

$\underline{N(5)=360}$: $360 = 210e^{k\cdot 5}$

$\frac{12}{7} = e^{5k}$

$\ln(12/7) = 5k$

$k = \boxed{\frac{1}{5}\ln\left(\frac{12}{7}\right)}$

2. Let y denote the mass of a radioactive substance at time t. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time t in hours does half the original mass remain? Round your answer to 3 decimal places.

$y' = -18y \Rightarrow y = Ce^{-18t}$

$y(0) = 20 \Rightarrow 20 = Ce^{-18(0)}$

$20 = C \quad \Rightarrow y = 20e^{-18t}$

We want solve $\frac{1}{2}(20) = y(t)$ for t.

$10 = 20e^{-18t}$

$1/2 = e^{-18t}$

$\ln(1/2) = -18t$

$\frac{\ln(1/2)}{-18} = t$

$t = \boxed{0.039}$

3. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \quad \text{where} \quad y = 10 \quad \text{when} \quad x = 2$$

Find the value of the integration constant, C.

$dy = 11x^2e^{-x^3}dx$

$\int dy = \underbrace{\int 11x^2e^{-x^3}dx}$

$u = -x^3$

$du = -3x^2dx$

$y = \int -\frac{11}{3}e^u du$

$y = -\frac{11}{3}e^{-x^3} + C$

When $y = 10$ and $x = 2$

$10 = -\frac{11}{3}e^{-2^3} + C$

$10 = -\frac{11}{3}e^{-8} + C$

$C = 10 + \frac{11}{3}e^{-8}$

$C =$ $\boxed{10 + \frac{11}{3}e^{-8}}$

4. Calculate the constant of integration, C, for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \qquad y(1) = 2$$

Rewrite: $6y\,dy = 7x^3dx$

$\int 6y\,dy = \int 7x^3dx$

$3y^2 = \frac{7}{4}x^4 + C$

Remember we want C.

$y(1) = 2$: $3(2)^2 = \frac{7}{4} + C$

$41/4 = C$

$C =$ $\boxed{41/4}$

5. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

Rewrite: $y\,dy = 3x^2 dx$

$\int y\,dy = \int 3x^2 dx$

$\frac{y^2}{2} = x^3 + c$

$y^2 = 2x^3 + c$

$y = \pm\sqrt{2x^3 + c}$

y = $\boxed{\pm\sqrt{2x^3+c}}$

6. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

Rewrite $dy = 5y\,dx$

$\frac{dy}{y} = 5dx$

$\int \frac{dy}{y} = \int 5dx$

$\ln|y| = 5x + c$

$|y| = e^{5x+c}$

$\pm y = e^c e^{5x}$

$y = \underbrace{\pm e^c}_{=c} e^{5x}$

$y = ce^{5x}$

or memorize

$\frac{dy}{dx} = ky$

$\Rightarrow y = Ce^{kx}$

y = $\boxed{Ce^{5x}}$

7. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

Rewrite: $y\,dy = -x\,dx$

$$\int y\,dy = \int -x\,dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + C$$

$$y^2 = -x^2 + C$$

$$y = \pm\sqrt{C - x^2}$$

y = $\boxed{\pm\sqrt{C-x^2}}$

8. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

Note there are 2 ways to do this problem.

(1) Separation of Variables

(2) First-Order Linear Eqn

By method 1,

$$\frac{dy}{dt} = 15y$$

$$\frac{dy}{y} = 15\,dt$$

$$\int \frac{dy}{y} = \int 15\,dt$$

$$\ln|y| = 15t + C$$

$$y = e^{15t + C}$$

$$y = e^{C} e^{15t}$$

$$y = Ce^{15t}$$

$y =$ $\boxed{Ce^{15t}}$

9. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$y\,dy = 3\,dx$

$\int y\,dy = \int 3\,dx$

$\frac{y^2}{2} = 3x + c$

$y^2 = 6x + 2c$

$y^2 = 6x + c$

$y = \pm\sqrt{6x+c}$

$y =$ $\boxed{\pm\sqrt{6x+C}}$

10. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$\frac{dy}{y} = 3x^2\,dx$

$\int \frac{dy}{y} = \int 3x^2\,dx$

$\ln|y| = x^3 + c$

$y = e^{x^3+c}$

$y = e^c e^{x^3}$

$y = Ce^{x^3}$

$y =$ $\boxed{Ce^{x^3}}$

11. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} = 8e^{-4t-y}$$

$dy = 8e^{-4t}e^{-y}\,dt$

$e^{y}\,dy = 8e^{-4t}\,dt$

$\int e^{y}\,dy = \int 8e^{-4t}\,dt$

$e^{y} = \frac{8}{-4}e^{-4t} + C$

$e^{y} = -2e^{-4t} + C$

$y = \ln(-2e^{-4t} + C)$

$y = \boxed{\ln(-2e^{-4t} + C)}$

12. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$2y\,dy = (3x+2)\,dx$

$\int 2y\,dy = \int (3x+2)\,dx$

$y^2 = \frac{3x^2}{2} + 2x + C$

So $y^2 = \frac{3x^2}{2} + 2x + 16$

$y = \pm\sqrt{\frac{3x^2}{2} + 2x + 16}$

when $y(0) = 4$

$4^2 = 0 + 0 + C$

$16 = C$

$y = \boxed{\pm\sqrt{\frac{3x^2}{2} + 2x + 16}}$

13. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$$\frac{dy}{y} = \frac{5}{6x+3}\,dx$$

$$\int \frac{dy}{y} = \int \frac{5}{6x+3}\,dx$$

$$\ln|y| = \frac{5}{6}\ln|6x+3| + C$$

$$y = \exp\left[\frac{5}{6}\ln|6x+3| + C\right]$$

$$y = e^C \exp\left[\ln|6x+3|^{5/6}\right]$$

$$y = C\cdot|6x+3|^{5/6}$$

When $y(0)=1$

$$1 = C\cdot|6(0)+3|^{5/6}$$

$$1 = C\cdot 3^{5/6}$$

$$C = 3^{-5/6}$$

$y =$ $\boxed{3^{-5/6}\cdot|6x+3|^{5/6}}$

14. What is the **integrating factor** of the following differential equation?

$$\frac{x^8y' - 14x^7y}{x^8} = \frac{32e^{7x}}{x^8}$$

$$y' + \underbrace{\left(\frac{-14}{x}\right)}_{P} y = \underbrace{\frac{32e^{7x}}{x^8}}_{Q}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$

$$= \exp\left[\int -\frac{14}{x}\,dx\right]$$

$$= \exp\left[-14\ln x\right]$$

$$= \exp\left[\ln x^{-14}\right]$$

$$= x^{-14}$$

$$= \frac{1}{x^{14}}$$

$u(x) =$ $\boxed{1/x^{14}}$

15. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right) y = 10\ln(x)$$

$$\frac{2y' + \left(\frac{6}{x}\right) y}{2} = \frac{10\ln(x)}{2}$$

$$y' + \frac{3}{x} y = 5\ln x$$

$$P(x) = \frac{3}{x} \quad Q(x) = 5\ln x$$

$$u(x) = \exp\left[\int \frac{3}{x}\,dx\right]$$

$$= \exp\left[3\ln x\right]$$

$$= \exp\left[\ln x^3\right]$$

$$= x^3$$

$u(x) =$ $\boxed{x^3}$

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16. What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = (x+1)e^{x^2}$$

$$\frac{(x+1)\frac{dy}{dx} - 2(x^2+x)y}{(x+1)} = \frac{(x+1)e^{x^2}}{(x+1)}$$

$$\frac{dy}{dx} - \frac{2x(x+1)}{(x+1)} y = e^{x^2}$$

$$\frac{dy}{dx} + (-2x)\cdot y = e^{x^2}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$

$$= \exp\left[\int -2x\,dx\right]$$

$$= \exp\left[-x^2\right]$$

$u(x) =$ $\boxed{e^{-x^2}}$

17. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) = \exp\left[\int P(x)\,dx\right]$

$= \exp\left[\int \cot x\,dx\right]$

$= \exp\left[\int \frac{\cos x}{\sin x}\,dx\right]$

$u = \sin x$

$du = \cos x\,dx$

$= \exp\left[\int \frac{du}{u}\right]$

$= \exp[\ln u]$

$u(x) = \exp[\ln \sin x]$

$= \sin x$

$u(x) =$ $\boxed{\sin x}$

18. Solve the initial value problem.

$$\frac{dy}{dx} + \underset{P}{13}y = \underset{Q}{16e^{-13x}} \quad \text{with} \quad y(0) = 10$$

$u(x) = \exp\left[\int 13\,dx\right] = \exp[13x] = e^{13x}$

$y\,u(x) = \int Q(x)\,u(x)\,dx + C$

$ye^{13x} = \int 16e^{-13x}e^{13x}\,dx + C$

$ye^{13x} = \int 16\,dx + C$

$ye^{13x} = 16x + C$

$y = 16xe^{-13x} + Ce^{-13x}$

$y(0) = 10$:

$10 = 0 + C$

$C = 10$

$y =$ $\boxed{16xe^{-13x} + 10e^{-13x}}$

19. Solve the initial value problem.

$$x^4y' + 4x^3 \cdot y = 10x^9 \text{ with } y(1) = 23$$

$$\frac{x^4y'+4x^3y}{x^4} = \frac{10x^9}{x^4}$$

$$y' + \frac{4}{x}\cdot y = 10x^5$$

$$P(x) = \frac{4}{x} \quad Q(x) = 10x^5$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int \frac{4}{x}\,dx\right]$$
$$= \exp\left[4\ln x\right]$$
$$= \exp\left[\ln x^4\right]$$
$$= x^4$$

$$y\cdot u(x) = \int Q(x)u(x)\,dx + C$$
$$y\cdot x^4 = \int 10x^5x^4\,dx + C$$
$$y\cdot x^4 = \int 10x^9\,dx + C$$
$$y\cdot x^4 = x^{10} + C$$
$$y = \frac{x^{10}}{x^4} + \frac{C}{x^4}$$
$$y = x^6 + \frac{C}{x^4}$$

$$23 = 1 + \frac{C}{1}$$
$$22 = C$$
$$y = x^6 + \frac{22}{x^4}$$

$y =$ $\boxed{x^6 + \frac{22}{x^4}}$

20. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \ldots$$

$$= 1 - \frac{6}{10} + \frac{36}{100} - \frac{216}{1000} + \ldots$$

$$= 1 - \frac{6}{10} + \left(\frac{6}{10}\right)^2 - \left(\frac{6}{10}\right)^3 + \ldots$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n$$

Answer: $\boxed{\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n}$

(b) Use the sum from (a) and compute the sum.

$$\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n = \frac{1}{1-(-6/10)} = \frac{1}{1+6/10} = \frac{1}{16/10} = \frac{10}{16} = \frac{5}{8}$$

Answer: $\boxed{5/8}$

21. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Note $r = 3/2$ and

$\left|\frac{3}{2}\right| < 1$ is false

So the sum diverges

$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n =$ $\boxed{\text{diverges}}$

22. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$$= \frac{6}{1-(-1/9)}$$

$$= \frac{6}{1+1/9}$$

$$= \frac{6}{10/9}$$

$$= 6 \cdot \frac{9}{10}$$

$$= 3 \cdot \frac{9}{5} = \frac{27}{5}$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ $\boxed{27/5}$

23. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right)$$

$$= \sum_{n=0}^{\infty} 7\left(\frac{1}{4}\right)^n$$

$$= \frac{7}{1-1/4}$$

$$= \frac{7}{3/4}$$

$$= 7 \cdot \frac{4}{3} = \frac{28}{3}$$

$\sum_{n=0}^{\infty} \left(\frac{7}{4^n}\right) =$ $\boxed{28/3}$

24. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$= \frac{5^3}{6} + \frac{5^4}{6^2} + \frac{5^5}{6^3} + \cdots$$

$$= \frac{5^3}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \cdots\right)$$

$$= \frac{125}{6} \sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n = \frac{125}{6} \cdot \frac{1}{1-5/6}$$

$$= \frac{125}{6} \cdot \frac{1}{1/6} = \frac{125}{6} \cdot \frac{6}{1} = 125$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \boxed{125}$$

25. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n}{3 \cdot 3^{2n}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3}\left(-\frac{2}{9}\right)^n$$

$$= \frac{1/3}{1-(-2/9)}$$

$$= \frac{1/3}{1+2/9}$$

$$= \frac{1/3}{11/9}$$

$$= \frac{1}{3} \cdot \frac{9}{11}$$

$$= 3/11$$

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} = \boxed{3/11}$$

26. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty}\left(\frac{3^{n-1}}{4^n}+\frac{(-1)^{n+1}}{9^n}\right)$$

$= \sum_{n=1}^{\infty}\left(\frac{3^{-1}}{1}\cdot\frac{3^n}{4^n}+\frac{(-1)^1}{1}\cdot\frac{(-1)^n}{9^n}\right)$

$= \sum_{n=1}^{\infty}\left(\frac{1}{3}\left(\frac{3}{4}\right)^n-\left(\frac{-1}{9}\right)^n\right)$

$= \frac{1}{3}\left(\frac{3}{4}\right)^1-\left(\frac{-1}{9}\right)^1$

$+\frac{1}{3}\left(\frac{3}{4}\right)^2-\left(-\frac{1}{9}\right)^2$

$+\frac{1}{3}\left(\frac{3}{4}\right)^3-\left(-\frac{1}{9}\right)^3$

$+\ldots$

$= \frac{1}{3}\left(\frac{3}{4}\right)\left[1+\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2+\ldots\right]$

$-\left(\frac{1}{-9}\right)\left[1+\left(-\frac{1}{9}\right)+\left(\frac{-1}{9}\right)^2\ldots\right]$

$= \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n+\frac{1}{9}\sum_{n=0}^{\infty}\left(\frac{-1}{9}\right)^n$

$= \frac{1}{4}\cdot\frac{1}{1-3/4}+\frac{1}{9}\cdot\frac{1}{1-(-1/9)}$

Answer: $\boxed{1.1}$

27. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

Remember $\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square}$ where $|\square| < 1$

$|-2x| < 1$

$|2x| < 1$

$2|x| < 1$

$|x| < 1/2 = R$

$R =$ $\boxed{1/2}$

28. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

Remember

$$\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square} \text{ where } |\square| < 1$$

$$|7x^2| < 1$$

$$7|x^2| < 1$$

$$|x^2| < 1/7$$

$$-1/7 < x^2 < 1/7$$

By algebra

$$x^2 < 1/7$$

$$x < \pm\sqrt{1/7}$$

$$|x| < \sqrt{1/7}$$

$R =$ $\boxed{\sqrt{1/7}}$

29. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$$\frac{3}{1+2x} = \frac{3}{1} \cdot \frac{1}{1+2x} = \frac{3}{1} \cdot \frac{1}{1-(-2x)}$$

$$\frac{1}{1-(-2x)} = \sum_{n=0}^{\infty} (-2x)^n \text{ where } |-2x| < 1$$

$$f(x) = \frac{3}{1-(-2x)} = 3\sum_{n=0}^{\infty} (-2x)^n \text{ where } 2|x| < 1$$

$$= \sum_{n=0}^{\infty} 3(-1)^n 2^n x^n \text{ where } |x| < 1/2$$

$\dfrac{3}{1+2x} =$ $\sum_{n=0}^{\infty} 3(-1)^n 2^n x^n$

$R =$ $1/2$

30. Express $f(x) = \dfrac{3x}{10+2x}$ as a power series and determine it's radius of converge.

$$\frac{3x}{10(1+\frac{2}{10}x)} = \frac{3x}{10}\cdot\frac{1}{1-(-\frac{2}{10}x)}$$

$$\frac{1}{1-(-\frac{2}{10}x)} = \sum_{n=0}^{\infty}\left(-\frac{2}{10}x\right)^n \text{ where } \left|-\frac{2}{10}x\right| < 1$$

$$\frac{2}{10}|x| < 1$$

$$|x| < \frac{10}{2}$$

$$|x| < 5$$

$$f(x) = \frac{3x}{10}\cdot\frac{1}{1-(-\frac{2}{10}x)} = \frac{3x}{10}\sum_{n=0}^{\infty}\left(-\frac{2}{10}x\right)^n$$

$$f(x) = \frac{3x}{10}\sum_{n=0}^{\infty}\frac{(-1)^n 2^n x^n}{10^n}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n 2^n \cdot 3^1 x^{n+1}}{10^{n+1}}$$

$\dfrac{3x}{10+2x} =$ $\sum_{n=0}^{\infty}\dfrac{(-1)^n 2^n \cdot 3^1 x^{n+1}}{10^{n+1}}$

$R =$ $10/2 = 5$

31. Express $f(x) = \dfrac{5x}{3+2x^2}$ as a power series and determine it's radius of converge.

$$\frac{5x}{3(1+2x^2/3)} = \frac{5x}{3}\cdot\frac{1}{1-(-(2x^2/3))}$$

$$\frac{1}{1-(-2x^2/3)} = \sum_{n=0}^{\infty}\left(-\frac{2x^2}{3}\right)^n \text{ where } \left|-\frac{2x^2}{3}\right| < 1$$

$$\frac{2}{3}|x^2| < 1$$

$$|x^2| < \frac{3}{2}$$

$$-\frac{3}{2} < x^2 < \frac{3}{2}$$

By algebra

$$x^2 < \frac{3}{2}$$

$$-\sqrt{\frac{3}{2}} < x < \sqrt{\frac{3}{2}}$$

$$|x| < \sqrt{\frac{3}{2}}$$

$$f(x) = \frac{5x}{3}\cdot\frac{1}{1-(-2x^2/3)} = \frac{5x}{3}\sum_{n=0}^{\infty}\left(\frac{-2x^2}{3}\right)^n$$

$$f(x) = \frac{5x}{3}\sum_{n=0}^{\infty}\frac{(-1)^n 2^n x^{2n}}{3^n}$$

$$f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n \cdot 5 \cdot x^{2n+1}}{3^{n+1}}$$

$\dfrac{5x}{3+2x^2} =$ $\sum_{n=0}^{\infty}\dfrac{(-1)^n 2^n \cdot 5 \cdot x^{2n+1}}{3^{n+1}}$

$R =$ $\sqrt{3/2}$

32. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\, dx$$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

$\sin(x^{3/2}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x^{3/2})^{2n+1}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}$

$\int \sin(x^{3/2})\, dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}\, dx$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{3n+3/2}\, dx$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{3n+5/2}}{3n+5/2}$

$= \frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6 \cdot (3+5/2)} + \frac{x^{17/5}}{5!\,(6+5/2)}$

$\int \sin(x^{3/2})\, dx = \boxed{\frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6 \cdot (3+5/2)} + \frac{x^{17/5}}{5!\,(6+5/2)}}$

33. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\, dx$$

$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = \sum_{n=0}^{\infty} (-x^4)^n = \sum_{n=0}^{\infty} (-1)^n x^{4n}$

$\int_0^{0.11} \frac{1}{1+x^4} dx = \int_0^{0.11} \sum_{n=0}^{\infty} (-1)^n x^{4n} dx$

$= \sum_{n=0}^{\infty} (-1)^n \int_0^{0.11} x^{4n} dx$

$= \sum_{n=0}^{\infty} \left.\frac{(-1)^n x^{4n+1}}{4n+1}\right]_0^{0.11}$

$= \left.\left(x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13}\right)\right]_0^{0.11}$

$\int_0^{0.11} \frac{1}{1+x^4}\, dx \approx \boxed{0.11000}$

34. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5+x^6}\,dx$$

$$\frac{x}{5+x^6} = \frac{x}{5}\cdot\frac{1}{(1+x^6/5)} = \frac{x}{5}\cdot\frac{1}{(1-(-x^6/5))}$$

$$\frac{1}{1-(-x^6/5)} = \sum_{n=0}^{\infty}\left(-\frac{x^6}{5}\right)^n$$

$$f(x) = \frac{x}{5}\cdot\frac{1}{(1-(x^6/5))} = \frac{x}{5}\sum_{n=0}^{\infty}\left(-\frac{x^6}{5}\right)^n$$

$$f(x) = \frac{x}{5}\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n}}{5^n} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{6n+1}}{5^{n+1}}$$

$$\int_0^{0.24}\sum_{n=0}^{\infty}\frac{(-1)^n x^{6n+1}}{5^{n+1}}\,dx$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{5^{n+1}}\int_0^{0.24} x^{6n+1}\,dx$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{5^{n+1}}\left.\frac{x^{6n+2}}{(6n+2)}\right]_0^{0.24}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{5^{n+1}}\cdot\frac{(0.24)^{6n+2}}{(6n+2)}$$

$$= \frac{1}{5}\cdot\frac{(0.24)^2}{2} - \frac{1}{5^2}\cdot\frac{(0.24)^8}{8} + \frac{1}{5^3}\cdot\frac{(0.24)^{14}}{14}$$

$\int_0^{0.24} \frac{x}{5+x^6}\,dx \approx$ $\boxed{0.0058}$

35. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\,dx$$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty}\frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}x^{2n}$$

$$\int_0^{0.23} e^{-x^2}\,dx = \int_0^{0.23}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\int_0^{0.23}x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left.\frac{x^{2n+1}}{2n+1}\right]_0^{0.23}$$

$$= \left.\left(\frac{x}{0!} - \frac{x^3}{1!(3)} + \frac{x^5}{2!(5)}\right)\right]_0^{0.23}$$

$$= \left.\left(x - \frac{x^3}{3} + \frac{x^5}{10}\right)\right]_0^{0.23}$$

$\int_0^{0.23} e^{-x^2}\,dx \approx$ $\boxed{0.226}$

36. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

$\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x^{1/2})^{2n}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$

$f(x) = 4x\cos(\sqrt{x}) = 4x \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1}$

$\int_0^{0.45} 4x\cos(\sqrt{x})dx = \int_0^{0.45} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1} dx$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\int_0^{0.45} x^{n+1} dx$

$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\frac{x^{n+2}}{n+2}\Big]_0^{0.45}$

$= \left(\frac{4x^2}{0!(2)} - \frac{4x^3}{2!(3)} + \frac{4x^4}{4!(4)} - \frac{4x^5}{6!(5)}\right)\Big]_0^{0.45}$

$= \left(2x^2 - \frac{2x^3}{3} + \frac{x^4}{6} - \frac{x^5}{900}\right)\Big]_0^{0.45}$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ $\boxed{0.35106}$

37. Use the first 3 terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.56)$. Round to 5 decimal places.

$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$

Note $1.56 = 1 + 0.56$

$\ln(1+0.56) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (0.56)^n$

$= 0.56 - \frac{(0.56)^2}{2} + \frac{(0.56)^3}{3}$

$\ln(1.56) \approx$ $\boxed{0.46174}$

38. Find the domain of

$$f(x, y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

$\sqrt{?} \to ? \geq 0$

$\sqrt{x+y-1} \to x+y-1 \geq 0$

$x+y \geq 1$

$\ln(?) \to ? > 0$

$\ln(y-11) \to y-11 > 0$

$y > 11$

$\frac{1}{?} \to ? \neq 0$

$\ln(y-11)-9 \neq 0$

$\ln(y-11) \neq 9$

$y-11 \neq e^9$

$y \neq e^9+11$

$\boxed{\{(x,y) \mid x+y \geq 1,\ y > 11,\ y \neq 11+e^9\}}$

Domain = ______________________________

39. Find the domain of

$$f(x, y) = \frac{\ln(x^2-y+3)}{\sqrt{x-6}}$$

$\ln(?) \to ? > 0$

$\ln(x^2-y+3) \to x^2-y+3 > 0$

$x^2+3 > y$

$\frac{1}{\sqrt{?}} \to ? > 0$

$\frac{1}{\sqrt{x-6}} \to x-6 > 0$

$x > 6$

$\boxed{\{(x,y) \mid x > 6,\ x^2+3 > y\}}$

Domain = ______________________________

40. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \;\; C = \ln(36)$$

$\ln(x^2+y^2)=\ln(36)$

$x^2+y^2=36$

$x^2+y^2=6^2$

(a) Parabola with vertices at $(0, 0)$

(b) Circle with center at $(0, \ln(36))$ and radius 6

(c) Parabola with vertices at $(0, \ln(36))$

(d) Circle with center at $(0, 0)$ and radius 6

(e) Increasing Logarithm Function

41. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

$\ln(y-e^{5x})=C$

$y-e^{5x}=e^C$

$y-e^{5x}=C$

$y=e^{5x}+C$

(a) Increasing exponential functions

(b) Rational Functions with x-axis symmetry

(c) Natural logarithm functions

(d) Decreasing exponential functions

(e) Rational Functions with y-axis symmetry

42. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

$\sqrt{y+4x^2}=C$

$y+4x^2=C^2$

$y+4x^2=C$

$y=-4x^2+C$

(a) Lines

(b) Parabolas

(c) Circles

(d) Point at the origin

(e) Ellipses

(f) Hyperbolas

43. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

$\cos(y+4x^2)=C$

$y+4x^2=\cos^{-1}(C)$

$y+4x^2=C$

$y=-4x^2+C$

(a) Lines

(b) Parabolas

(c) Circles

(d) Point at the origin

(e) Ellipses

(f) Hyperbolas