

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: _____

1. For the following function $f(x, y)$, evaluate $f_y(-2, -3)$.

$$f(x, y) = 8x^4y^5 + 3x^3 - 12y^2$$

$$f_y(-2, -3) = \underline{\hspace{10em}}$$

2. For the following function $f(x, y)$, evaluate $f_x(x, y)$.

$$f(x, y) = -7 \cos(x^7y^8)$$

$$f_x(x, y) = \underline{\hspace{10em}}$$

3. Compute $f_x(6, 5)$ when

$$f(x, y) = \frac{(6x - 6y)^2}{\sqrt{y^2 - 1}}$$

$$f_x(6, 5) = \underline{\hspace{10cm}}$$

4. Find the first order partial derivatives of

$$f(x, y) = \frac{3x^2y^3}{(y - 1)^2}$$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

5. Find the first order partial derivatives of $f(x, y) = (xy - 1)^2$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

6. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

7. Find the first order partial derivatives of $f(x, y) = y \cos(x^2y)$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

8. Given the function $f(x, y) = 4x^5 \tan(3y)$, compute $f_{xy}(2, \pi/3)$

$$f_{xy}(2, \pi/3) = \underline{\hspace{10cm}}$$

9. Given the function $f(x, y) = 3y^4 \sin(x)$, compute $f_{xy}(\pi, 3)$

$$f_{xy}(\pi, 3) = \underline{\hspace{10cm}}$$

10. A function $f(x, y)$ has 2 critical points. The partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 8x - 16y \quad \text{and} \quad f_y(x, y) = 8y^2 - 16x$$

One of the critical points is $(0, 0)$. Find the second critical point of $f(x, y)$.

$$(a, b) = \underline{\hspace{10cm}}$$

11. Find the second order partial derivatives of

$$f(x, y) = x^2 y \ln(7x)$$

$$f_{xx}(x, y) = \underline{\hspace{10cm}}$$

$$f_{xy}(x, y) = \underline{\hspace{10cm}}$$

$$f_{yy}(x, y) = \underline{\hspace{10cm}}$$

12. Find the discriminant of

$$f(x, y) = e^x \sin(y)$$

Simplify your answer. Note: $\sin^2(y) + \cos^2(y) = 1$.

$$D(x, y) = \underline{\hspace{10cm}}$$

13. Using the information in the table below, classify the critical points for the function $g(x, y)$.

(a, b)	$g_{xx}(a, b)$	$g_{yy}(a, b)$	$g_{xy}(a, b)$
(4, 5)	0	4	-2
(5, -10)	5	-10	6
(10, 10)	-4	-6	-4
(7, 9)	5	7	4
(4, 8)	2	2	2

(4,5) is _____

(5,-10) is _____

(10,10) is _____

(7,9) is _____

(4,8) is _____

14. Classify the critical points of the function $f(x, y)$ given the partial derivatives:

$$f_x(x, y) = x - y \quad f_y(x, y) = y^3 - x$$

- (a) Two saddle points and one local minimum
- (b) Two saddle points and one local maximum
- (c) One saddle point, one local maximum, and one local minimum
- (d) Three saddle points
- (e) Two local minimums and one saddle point

Answer: _____

15. The critical points for a function $f(x, y)$ are (1,1) and (2,4). Given that the partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 7x - 3y \quad f_y(x, y) = 4x^2 - 6y$$

Classify each critical point as a maximum, minimum, or saddle point.

(1,1) is _____

(2,4) is _____

16. Find all local maximum and minimum points of

$$f(x, y) = 4x^2 - xy + 8y^2 - 46x - 26y + 11$$

Local max at _____

Local min at _____

17. Fleet feet stores two most sold running shoes brands are Aesics and Brookes. The total venue from selling x pairs of Aesics and y pairs of Brookes is given by

$$R(x, y) = -10x^2 - 16y^2 - 4xy + 84 + 204y$$

where x and y are in **thousands of units**. Determine the number of Brookes shoes to be sold to maximize the revenue.

The # of Brookes shoes sold is _____

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18. Find the point(s) (x, y) where the function $f(x, y) = 3x^2 + 4xy + 6x - 15$ attains maximal value, subject to the constraint $x + y = 10$.

$(x,y) =$ _____

19. Find the minimum of the function using LaGrange Multipliers of the function $f(x, y) = 2x^2 + 4y^2$ subject to the constraint $x^2 + y^2 = 1$.

Minimum Value = _____

20. Find the minimum value of the function $f(x, y) = 2x^2y - 3y^2$ subject to the constraint $x^2 + 2y = 1$.

Minimum Value = _____

21. Locate and classify the points that maximize and minimize the function $f(x, y) = 5x^2 + 10y$ subject to the constraint $5x^2 + 5y^2 = 5$.

Minimum Value occurs at _____

Maximum Value occurs at _____

22. Find the maximum value of the function $f(x, y) = 8x - 11y^2$ subject to the constraint $x^2 + 11y^2 = 25$.

Max value is _____

23. We are baking a tasty treat where customer satisfaction is given by $S(x, y) = 6x^{3/2}y$. Here, x and y are the amount of sugar and spice respectively. If the sugar and spice we use must satisfy $9x + y = 4$, what is the maximum customer satisfaction we can achieve? (Note: the function is defined only for $x \geq 0$ and $y \geq 0$.) Round your answer to 2 decimal places.

Maximum Value = _____

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24. A customer has \$280 to spend on two items, Item A, which costs \$2 per unit, and Item B, which costs \$5 per unit. If the enjoyment of each item by the customer is given by $f(A, B) = 100AB^3$, how many of each unit should be purchase to maximize the enjoyment of the customer?

Units of A: _____

Units of B: _____

25. Evaluate the following double integral.

$$\int_0^2 \int_0^3 (x + y) dy dx$$

$$\int_0^2 \int_0^3 (x + y) dy dx = \underline{\hspace{2cm}}$$

26. Evaluate the double integral

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x) dy dx$$

$$\int_0^{\pi/3} \int_0^2 25y^4 \sec^2(x) dy dx = \underline{\hspace{2cm}}$$

27. Evaluate the double integral

$$\int_0^{\pi/2} \int_0^1 12x^3 \sin(y) dx dy$$

$$\int_0^1 \int_0^{\pi/2} 12x^3 \sin(y) dx dy = \underline{\hspace{2cm}}$$

28. Evaluate the double integral

$$\int_0^4 \int_2^y (y+x) dx dy$$

$$\int_0^4 \int_2^y (y+x) dx dy = \underline{\hspace{4cm}}$$

29. Evaluate the double integral

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2} dy dx$$

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2} dy dx = \underline{\hspace{4cm}}$$

30. Compute the following definite integral.

$$\int_0^7 \int_1^x 36x \, dy \, dx$$

$$\int_0^7 \int_1^x 36x \, dy \, dx = \underline{\hspace{10em}}$$

31. Find the bounds for the integral $\iint_R 5e^x \sin(y) \, dA$ where R is a triangle with vertices $(0,0)$, $(1,2)$, and $(0,2)$.

DON'T COMPUTE!!!

Answer: _____

32. Switch the order of integration on the follow integral

$$\int_0^6 \int_{x^2}^{36} f(x, y) dy dx$$

Answer: _____

33. Switch the order of integration on the follow integral

$$\int_0^1 \int_{10y}^{10} f(x, y) dx dy$$

Answer: _____

34. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2} dy dx$$

(Hint: Change the order of integration)

$$\int_0^2 \int_x^2 4e^{y^2} dy dx = \underline{\hspace{10em}}$$

35. Evaluate the double integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx dy$$

Round your answer to 2 decimal places.

(Hint: Change the order of integration)

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx dy = \underline{\hspace{10em}}$$