Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name:

1. For the following function $f(x, y)$, evaluate $f_y(-2, -3)$.

$$f(x, y) = 8x^4y^5 + 3x^3 - 12y^2$$

$$f_y = 40x^4y^4 + 0 - 24y$$
$$= 40x^4y^4 - 24y$$

$$f_y(-2,-3) = 40(-2)^4(-3)^4 - 24(-3)$$

$f_y(-2, -3) =$ $\boxed{51912}$

2. For the following function $f(x, y)$, evaluate $f_x(x, y)$.

$$f(x, y) = -7\cos(x^7y^8)$$

$$f_x = -7(-\sin(x^7y^8))\frac{d}{dx}(x^7y^8)$$
$$= 7\sin(x^7y^8)[7x^6y^8]$$

$f_x(x, y) =$ $\boxed{49x^6y^8\sin(x^7y^8)}$

3. Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y^2-1}}$$

$$f_x(x,y) = \frac{d}{dx}\left(\frac{(6x-6y)^2}{\sqrt{y^2-1}}\right)$$

$$= \frac{1}{\sqrt{y^2-1}}\frac{d}{dx}\left((6x-6y)^2\right)$$

$$= \frac{1}{\sqrt{y^2-1}}\cdot 2(6x-6y)\frac{d}{dx}(6x+6y)$$

$$= \frac{1}{\sqrt{y^2-1}}\cdot 2(6x-6y)\cdot 6$$

$$= \frac{72x-72y}{\sqrt{y^2-1}}$$

$f_x(6,5)=$ $\boxed{72/\sqrt{24}}$

4. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$$f(x,y) = 3x^2\cdot\frac{y^3}{(y-1)^2}$$

$$f_x(x,y) = \frac{d}{dx}\left(3x^2\cdot\frac{y^3}{(y-1)^2}\right) = \frac{y^3}{(y-1)^2}\cdot\frac{d}{dx}\left(3x^2\right) = \frac{y^3}{(y-1)^2}\cdot 6x$$

$$f_y(x,y) = \frac{d}{dy}\left(3x^2\cdot\frac{y^3}{(y-1)^2}\right) = 3x^2\frac{d}{dy}\left(\frac{y^3}{(y-1)^2}\right) = 3x^2\left(\frac{3y^2(y-1)^2 - y^3\cdot 2(y-1)}{(y-1)^4}\right)$$

$$= 3x^2\left(\frac{\cancel{(y-1)}\left[3y^2(y-1)-2y^3\right]}{(y-1)^{\cancel{4}\,3}}\right) = \frac{3x^2(3y^3-3y^2-2y^3)}{(y-1)^3}$$

$$= \frac{3x^2(y^3-3y^2)}{(y-1)^3}$$

$f_x(x,y) =$ $\boxed{6xy^3/(y-1)^2}$

$f_y(x,y) =$ $\boxed{\dfrac{3x^2(y^3-3y^2)}{(y-1)^3}}$

5. Find the first order partial derivatives of $f(x, y) = (xy-1)^2$

$$f_x(x,y) = \frac{d}{dx}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dx}(xy-1)$$
$$= 2(xy-1)\,y$$
$$= 2xy^2 - 2y$$

$$f_y(x,y) = \frac{d}{dy}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dy}(xy-1)$$
$$= 2(xy-1)\,x$$
$$= 2x^2y - 2x$$

$f_x(x, y) =$ $\boxed{2xy^2 - 2y}$

$f_y(x, y) =$ $\boxed{2x^2y - 2x}$

6. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$$f_x(x,y) = \frac{d}{dx}(x)\,e^{x^2+xy+y^2} + x\frac{d}{dx}\left(e^{x^2+xy+y^2}\right)$$
$$= e^{x^2+xy+y^2} + x\left(e^{x^2+xy+y^2}\right)(2x+y)$$
$$= (1+2x^2+xy)e^{x^2+xy+y^2}$$

$$f_y(x,y) = x\frac{d}{dy}\left(e^{x^2+xy+y^2}\right) = x\left(e^{x^2+xy+y^2}\right)(x+2y)$$
$$= (x^2+2xy)e^{x^2+xy+y^2}$$

$f_x(x, y) =$ $\boxed{(1+2x^2+xy)e^{x^2+xy+y^2}}$

$f_y(x, y) =$ $\boxed{(x^2+2xy)e^{x^2+xy+y^2}}$

7. Find the first order partial derivatives of $f(x,y) = y\cos(x^2y)$

$$f_x(x,y) = y\frac{d}{dx}\left(\cos(x^2y)\right) = y(-\sin(x^2y))\frac{d}{dx}(x^2y) = -y\sin(x^2y)[2xy]$$
$$= -2xy^2\sin(x^2y)$$

$$f_y(x,y) = \frac{d}{dy}(y)\cos(x^2y) + y\frac{d}{dy}\left(\cos(x^2y)\right)$$
$$= \cos(x^2y) + y(-\sin(x^2y))\frac{d}{dy}(x^2y)$$
$$= \cos(x^2y) - y\sin(x^2y)[x^2]$$
$$= \cos(x^2y) - x^2y\sin(x^2y)$$

$f_x(x,y) =$ $-2xy^2\sin(x^2y)$

$f_y(x,y) =$ $\cos(x^2y) - x^2y\sin(x^2y)$

8. Given the function $f(x,y) = 4x^5\tan(3y)$, compute $f_{xy}(2,\pi/3)$

$$f_x(x,y) = \frac{d}{dx}\left(4x^5\tan(3y)\right) = \tan(3y)\cdot\frac{d}{dx}(4x^5)$$
$$= \tan(3y)\cdot(20x^4)$$

$$f_{xy}(x,y) = \frac{d}{dy}(f_x(x,y)) = \frac{d}{dy}\left(\tan(3y)\cdot(20x^4)\right) = 20x^4\frac{d}{dy}(\tan(3y))$$
$$= 20x^4\cdot\sec^2(3y)\cdot 3$$
$$= 60x^4\sec^2(3y)$$

$$f_{xy}(2,\pi/3) = 60(2)^4\sec^2(3\pi/3)$$
$$= 60(16)\sec^2(\pi)$$
$$= 960$$

$f_{xy}(2,\pi/3) =$ $\boxed{960}$

9. Given the function $f(x, y) = 3y^4 \sin(x)$, compute $f_{xy}(\pi, 3)$

$f_x = 3y^4 \cos(x)$

$f_{xy} = \frac{d}{dy}(f_x) = 12y^3 \cos(x)$

$f_{xy}(\pi, 3) = 12(3)^3 \cos(\pi)$

$= -108$

$f_{xy}(\pi, 3) =$ $\boxed{-108}$

10. A function $f(x, y)$ has 2 critical points. The partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 8x - 16y \quad \text{and} \quad f_y(x, y) = 8y^2 - 16x$$

One of the critical points is $(0, 0)$. Find the second critical point of $f(x, y)$.

$$\begin{cases} 8x - 16y = 0 & (1) \\ 8y^2 - 16x = 0 & (2) \end{cases}$$

Solve (1) for x.

$8x = 16y$

$x = 2y$

Plug $x = 2y$ into (2).

$8y^2 - 16(2y) = 0$

$8y^2 - 32y = 0$

$8y(y - 4) = 0$

$y = 0, 4$

Plug $y = 0, 4$ into $x = 2y$.

$y = 0 \rightarrow x = 0 \rightarrow (0,0)$

$y = 4 \rightarrow x = 8 \rightarrow (8,4)$

$(a, b) =$ $\boxed{(8, 4)}$

11. Find the second order partial derivatives of

$$f(x,y) = x^2 y \ln(7x)$$

$$f(x,y) = \left(x^2 \ln(7x)\right) y$$

$$f_x(x,y) = \frac{d}{dx}\left(\left(x^2 \ln(7x)\right)\cdot y\right) = y\frac{d}{dx}\left(x^2 \ln(7x)\right)$$

$$= y\left(2x\ln(7x) + x^2 \frac{1}{7x}\cdot 7\right) = y(2x\ln(7x) + x)$$

$$f_{xx}(x,y) = \frac{d}{dx}\left(y\left(2x\ln(7x)+x\right)\right) = y\frac{d}{dx}\left(2x\ln(7x)+x\right)$$

$$= y\left(2\ln(7x) + 2x\cdot\frac{1}{7x}\cdot 7 + 1\right) = y\left(2\ln(7x) + 2 + 1\right.$$

$$= y\left(2\ln(7x)+3\right)$$

$$f_{xy}(x,y) = \frac{d}{dy}\left(y\left(2x\ln(7x)+x\right)\right) = \left(2x\ln(7x)+x\right)\frac{d}{dy}(y)$$

$$= 2x\ln(7x) + x$$

$$f_y(x,y) = \frac{d}{dy}\left(\left(x^2\ln(7x)\right)\cdot y\right) = \left(x^2\ln(7x)\right)\frac{d}{dy}(y) = x^2\ln(7x)$$

$$f_{yy}(x,y) = \frac{d}{dy}\left(x^2\ln(7x)\right) = 0$$

$f_{xx}(x,y) =$ $(2\ln(7x)+3)y$

$f_{xy}(x,y) =$ $2x\ln(7x)+x$

$f_{yy}(x,y) =$ 0

12. Find the discriminant of

$$f(x,y) = e^x \sin(y)$$

Simplify your answer. Note: $\sin^2(y) + \cos^2(y) = 1$.

$f_x(x,y) = e^x \sin(y)$

$f_{xx}(x,y) = e^x \sin(y)$

$f_{xy}(x,y) = e^x \cos(y)$

$f_y(x,y) = e^x \cos(y)$

$f_{yy}(x,y) = -e^x \sin(y)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= (e^x \sin(y))(-e^x \sin(y)) - (e^x \cos(y))^2$

$= -e^{2x} \sin^2(y) - e^{2x} \cos^2(y)$

$= -e^{2x}(\sin^2(y) + \cos^2(y))$

$= -e^{2x}(1)$

$D(x,y) =$ $\boxed{-e^{2x}}$

13. Using the information in the table below, classify the critical points for the function $g(x,y)$.

(a,b)	$g_{xx}(a,b)$	$g_{yy}(a,b)$	$g_{xy}(a,b)$
$(4,5)$	0	4	-2
$(5,-10)$	5	-10	6
$(10,10)$	-4	-6	-4
$(7,9)$	5	7	4
$(4,8)$	2	2	2

$D(4,5) = (0)(4) - (-2)^2 = -4 < 0 \rightarrow$ saddle pt

$D(5,-10) = (5)(-10) - 6^2 = -86 < 0 \rightarrow$ saddle pt

$D(10,10) = (-4)(-6) - (-4)^2 = 8 > 0 \rightarrow$ relative

$g_{xx} = -4 < 0 \rightarrow$ max

$D(7,9) = (5)(7) - (4)^2 = 19 > 0 \rightarrow$ relative

$g_{xx} = 5 > 0 \rightarrow$ min

$D(4,8) = (2)(2) - 2^2 = 0$

$\downarrow$

Inconclusive

(4,5) is saddle pt

(5,-10) is saddle pt

(10,10) is relative max

(7,9) is relative min

(4,8) is inconclusive

14. Classify the critical points of the function $f(x, y)$ given the partial derivatives:

$$f_x(x, y) = x - y \qquad f_y(x, y) = y^3 - x$$

(a) Two saddle points and one local minimum

(b) Two saddle points and one local maximum

(c) One saddle point, one local maximum, and one local minimum

(d) Three saddle points

(e) Two local minimums and one saddle point

$f_x = 0$
$x - y = 0$
$x = y$

$f_y = 0$
$y^3 - x = 0$
$y^3 = x$

$\begin{cases} x = y \\ y^3 = x \end{cases} \Rightarrow y = y^3$
$y - y^3 = 0$
$y(1 - y^2) = 0$
$y = 0, \pm 1$

$f_x = x - y$ $\quad f_y = y^3 - x$
$f_{xx} = 1$ $\quad f_{yy} = 3y^2$
$f_{xy} = -1$

$D = f_{xx} f_{yy} - (f_{xy})^2$
$= (1)(3y^2) - (-1)^2$
$= 3y^2 - 1$

Note we don't need to find the x-values b/c D which we found on the left only has y's.

When $y = 0$, $D = -1 < 0 \to$ saddle

When $y = -1$, $D = 2 > 0 \to$ rel extrema
When $y = +1$, $D = 2 > 0 \to$ rel extrema
Check $f_{xx} = 1 > 0$ $\to$ rel mins @ $y = \pm 1$

Answer: ________________

15. The critical points for a function $f(x, y)$ are (1,1) and (2,4). Given that the partial derivatives of $f(x, y)$ are

$$f_x(x, y) = 7x - 3y \qquad f_y(x, y) = 4x^2 - 6y$$

Classify each critical point as a maximum, minimum, or saddle point.

$f_x = 7x - 3y$ $\quad f_y = 4x^2 - 6y$
$f_{xx} = 7$ $\quad f_{yy} = -6$
$f_{xy} = -3$

$D = f_{xx} f_{yy} - (f_{xy})^2$
$= (7)(-6) - (-3)^2$
$= -42 - 9 = -51$

Since $D < 0$ always, both pts are saddle pts.

(1,1) is saddle pt

(2,4) is saddle pt

16. Find all local maximum and minimum points of

$$f(x,y) = 4x^2 - xy + 8y^2 - 46x - 26y + 11$$

$$\begin{cases} f_x = 8x - y - 46 = 0 & (1) \\ f_y = -x + 16y - 26 = 0 & (2) \end{cases}$$

Multiply (2) by 8. Then add

$$\begin{aligned} \cancel{8x} - y + 46 &= 0 \\ -\cancel{8x} + 128y - 208 &= 0 \\ \hline 127y - 162 &= 0 \end{aligned}$$

$$y = \frac{162}{127}$$

Plug $y = \frac{162}{127}$ into (1)

$$8x - \frac{162}{127} - 46 = 0$$

$$x = \frac{1501}{254}$$

Critical Pt

$$\left(\frac{1501}{254}, \frac{162}{127}\right)$$

$$\left.\begin{aligned} f_{xx} &= 8 \\ f_{xy} &= -1 \\ f_{yy} &= 16 \end{aligned}\right\} \to D = f_{xx}f_{yy} - (f_{xy})^2$$

$$= 8(16) - (-1)^2 > 0$$

and $f_{xx} = 8 > 0$

For all pts. So we have only rel min

Local max at $\boxed{\text{None}}$

Local min at $\boxed{\left(\frac{1501}{254}, \frac{162}{127}\right)}$

17. Fleet feet stores two most sold running shoes brands are Aesics and Brookes. The total venue from selling x pairs of Aesics and y pairs of Brookes is given by

$$R(x,y) = -10x^2 - 16y^2 - 4xy + 84 + 204y$$

where x and y are in **thousands of units**. Determine the number of Brookes shoes to be sold to maximize the revenue.

First find the critical pts.

$$\begin{cases} R_x = -20x - 4y = 0 & (1) \\ R_y = -32y - 4x + 204 = 0 & (2) \end{cases}$$

Divide (1) and (2) by -4.

$$\begin{cases} 5x + y = 0 & (1) \\ x + 8y - 51 = 0 & (2) \end{cases}$$

$$\Rightarrow \begin{cases} 5x + y = 0 & (1) \\ x + 8y = 51 & (2) \end{cases}$$

Multiply (2) by 5.

$$\Rightarrow \begin{cases} 5x + y = 0 & (1) \\ 5x + 40y = 255 & (2) \end{cases}$$

Subtract (1) and (2)

$$-39y = -255$$

$$y \approx 6.5$$

$$\Rightarrow y = 7$$

The # of Brookes shoes sold is $\boxed{7000}$

18. Find the point(s) (x, y) where the function $f(x, y) = 3x^2 + 4xy + 6x - 15$ attains maximal value, subject to the constraint $x + y = 10$.

$f = 3x^2 + 4xy + 6x - 15$

$f_x = 6x + 4y + 6$

$f_y = 4x$

$g = x + y = 10$

$g_x = 1$

$g_y = 1$

System
$$\begin{cases} 6x + 4y + 6 = \lambda & (1) \\ 4x = \lambda & (2) \\ x + y = 10 & (3) \end{cases}$$

Set (1) = (2)

$6x + 4y + 6 = 4x$

$2x + 4y + 6 = 0$

$2x = -4y - 6$

$x = -2y - 3$

Plug $x = -2y - 3$ into (3)

$x + y = 10$

$-2y - 3 + y = 10$

$-y - 3 = 10$

$-y = 13$

$y = -13$

Plug $y = -13$ into $x = -2y - 3$.

$x = -2(-13) - 3$

$= 26 - 3$

$= 23$

(x,y) = $\boxed{(23, -13)}$

19. Find the minimum of the function using LaGrange Multipliers of the function $f(x, y) = 2x^2 + 4y^2$ subject to the constraint $x^2 + y^2 = 1$.

$f = 2x^2 + 4y^2$

$f_x = 4x$

$f_y = 8y$

$g = x^2 + y^2 = 1$

$g_x = 2x$

$g_y = 2y$

System:
$$\begin{cases} 4x = 2x\lambda & (1) \\ 8y = 2y\lambda & (2) \\ x^2 + y^2 = 1 & (3) \end{cases}$$

Solve (1).

$4x = 2x\lambda$

$4x - 2x\lambda = 0$

$2x(1 - \lambda) = 0$

$x = 0, \lambda = 1$

Plug $x = 0$ into (3)

$0^2 + y^2 = 1$

$y = \pm 1$

Pts: $(0, 1), (0, -1)$

Plug $\lambda = 1$ into (2)

$8y = 2y$

only true when $y = 0$

Plug $y = 0$ into (3)

$x^2 + 0^2 = 1$

$x = \pm 1$

Pts: $(1, 0), (-1, 0)$

Now plug the pts into $f(x, y) = 2x^2 + 4y^2$

$f(0, 1) = 4$

$f(0, -1) = 4$

$\left.\begin{matrix} f(1, 0) = 2 \\ f(-1, 0) = 2 \end{matrix}\right\} \rightarrow$ Min

Minimum Value = $\boxed{2}$

20. Find the minimum value of the function $f(x,y) = 2x^2y - 3y^2$ subject to the constraint $x^2 + 2y = 1$.

$f = 2x^2y - 3y^2$ $g = x^2 + 2y = 1$

$f_x = 4xy$ $g_x = 2x$

$f_y = 2x^2 - 6y$ $g_y = 2$

System
$$\begin{cases} 4xy = 2x\lambda & (1) \\ 2x^2 - 6y = 2\lambda & (2) \\ x^2 + 2y = 1 & (3) \end{cases}$$

Solve (1)

$4xy - 2x\lambda = 0$

$2x(2y - \lambda) = 0$

$x = 0, \ \lambda = 2y$

Plug $x=0$ into (3)

$0^2 + 2y = 1$

$y = 1/2$

Pts: $(0, 1/2)$

Plug $\lambda = 2y$ into (2)

$2x^2 - 6y = 2(2y)$

$2x^2 - 6y = 4y$

$2x^2 = 10y$

$x^2 = 5y$

Plug $x^2 = 5y$ into (3)

$5y + 2y = 1$

$7y = 1$

$y = 1/7$

Plug $y = 1/7$ into $x^2 = 5y$

$x^2 = \frac{5}{7}$

$x = \pm\sqrt{\frac{5}{7}}$

Pts: $\left(\sqrt{\frac{5}{7}}, \frac{1}{7}\right), \left(-\sqrt{\frac{5}{7}}, \frac{1}{7}\right)$

Test for Min

$f(0, 1/2) = -3/4$

$f\left(\pm\sqrt{\frac{5}{7}}, \frac{1}{7}\right) = \frac{1}{7}$

Minimum Value = $\boxed{-3/4}$

21. Locate and classify the points that maximize and minimize the function $f(x,y) = 5x^2 + 10y$ subject to the constraint $5x^2 + 5y^2 = 5$.

$f = 5x^2 + 10y$ $g = 5x^2 + 5y^2 = 5$

$f_x = 10x$ $g_x = 10x$

$f_y = 10$ $g_y = 10y$

System:
$$\begin{cases} 10x = 10x\lambda & (1) \\ 10 = 10y\lambda & (2) \\ 5x^2 + 5y^2 = 5 & (3) \end{cases}$$

Solve (1)

$10x - 10x\lambda = 0$

$10x(1 - \lambda) = 0$

$x = 0, \ \lambda = 1$

Plug $x = 0$ into (3)

$5y^2 = 5$

$y^2 = 1$

$y = \pm 1$

Pts: $(0,1)(0,-1)$

Plug $\lambda = 1$ into (2)

$10 = 10y$

$y = 1$

Plug $y = 1$ into (3)

$5x^2 + 5 = 5$

$5x^2 = 0$

$x = 0$

Pt: $(0,1)$ again

Test w/ $f(x,y)$

$f(0,-1) = -10$

$f(0,1) = 10$

Minimum Value occurs at $\boxed{-10}$

Maximum Value occurs at $\boxed{10}$

22. Find the maximum value of the function $f(x,y) = 8x - 11y^2$ subject to the constraint $x^2 + 11y^2 = 25$.

$f_x = 8$ $g_x = 2x$

$f_y = -22y$ $g_y = 22y$

$\begin{cases} 8 = 2x\lambda & ① \\ -22y = 22y\lambda & ② \\ x^2 + 11y^2 = 25 & ③ \end{cases}$

Solve ①

$-22y = 22y\lambda$

$0 = 22y\lambda + 22y$

$0 = 22y(\lambda + 1)$

$y = 0, \lambda = -1$

Plug $y=0$ into ③

$x^2 + 0 = 25$

$x = \pm 5$

Critical Pt: $(5,0), (-5,0)$

Plug $\lambda = -1$ into ①

$8 = -2x$

$x = -4$

Plug $x = -4$ into ③

$16 + 11y^2 = 25$

$11y^2 = 9$

$y^2 = \frac{9}{11}$

$y = \pm\sqrt{\frac{9}{11}}$

Critical Pt: $\left(-4, \sqrt{\frac{9}{11}}\right)$, $\left(-4, -\sqrt{\frac{9}{11}}\right)$

$f(5,0) = 40 \rightarrow$ max

$f(-5,0) = -40$

$f\left(-4, \sqrt{\frac{9}{11}}\right) = -49$

$f\left(-4, -\sqrt{\frac{9}{11}}\right) = -49$

$\frac{9}{11}$

Max value is $\boxed{40}$

23. We are baking a tasty treat where customer satisfaction is given by $S(x,y) = 6x^{3/2}y$. Here, x and y are the amount of sugar and spice respectively. If the sugar and spice we use must satisfy $9x + y = 4$, what is the maximum customer satisfaction we can achieve? (Note: the function is defined only for $x \geq 0$ and $y \geq 0$.) Round your answer to 2 decimal places.

$S = 6x^{3/2}y$

$S_x = 9x^{1/2}y$

$S_y = 6x^{3/2}$

$g = 9x + y = 4$

$g_x = 9$

$g_y = 1$

System: $\begin{cases} 9x^{1/2}y = 9\lambda & ① \\ 6x^{3/2} = \lambda & ② \\ 9x^2 + y^2 = 4 & ③ \end{cases}$

Plug ② in ①

$\cancel{9}x^{1/2}y = \cancel{9}(6x^{3/2})$

$x^{1/2}y = 6x^{3/2}$

$x^{1/2}y - 6x^{3/2} = 0$

$x^{1/2}(y - 6x) = 0$

$x = 0, y = 6x$

Plug $x=0$ into ③

$0 + y = 4$

Pt: $(0,4)$

Plug $y = 6x$ into

$9x + 6x = 4$

$15x = 4$

$x = \frac{4}{15}$

Plug $x = \frac{4}{15}$ into $y = 6x$

$y = \frac{8}{5}$

Pt: $\left(\frac{4}{15}, \frac{8}{5}\right)$

Test for max

$S(0,4) = 0$

$S\left(\frac{4}{15}, \frac{8}{5}\right) \approx 1.32$

max

Maximum Value = $\boxed{1.32}$

24. A customer has \$280 to spend on two items, Item A, which costs \$2 per unit, and Item B, which costs \$5 per unit. If the enjoyment of each item by the customer is given by $f(A, B) = 100AB^3$, how many of each unit should be purchase to maximize the enjoyment of the customer?

$f = 100AB^3$

$g = 2A + 5B = 280$

$f_A = 100B^3 \qquad g_A = 2$

$f_B = 300AB^2 \qquad g_B = 5$

$$\begin{cases} 100B^3 = 2\lambda & (1) \\ 300AB^2 = 5\lambda & (2) \\ 2A + 5B = 280 & (3) \end{cases}$$

Simplify (1) and (2)

$$\begin{cases} 50B^3 = \lambda & (1) \\ 60AB^2 = \lambda & (2) \\ 2A + 5B = 280 & (3) \end{cases}$$

Set (1) = (2)

$50B^3 = 60AB^2$

$50B^3 - 60AB^2 = 0$

$10B^2(5B - 6A) = 0$

$B = 0, \ B = \frac{6A}{5}$

Plug $B = 0$ into (3)

$2A + 0 = 280$

$A = 140$

Plug $B = \frac{6A}{5}$ into (3)

$2A + 5\left(\frac{6A}{5}\right) = 280$

$2A + 6A = 280$

$8A = 280$

$A = 35$

So $B = \frac{6}{5} \cdot 35 = 42$

$f(140, 0) = 0$

$f(35, 42) = 259308000$

Units of A: $\boxed{35}$

Units of B: $\boxed{42}$

25. Evaluate the following double integral.

$$\int_0^2 \int_0^3 (x + y)\, dy\, dx$$

$$= \int_0^2 \left(xy + \frac{y^2}{2} \right) \Big]_0^3 dx$$

$$= \int_0^2 \left(3x + \frac{9}{2} \right) dx$$

$$= \left(\frac{3x^2}{2} + \frac{9}{2}x \right) \Big]_0^2$$

$$= 15$$

$$\int_0^2 \int_0^3 (x + y)\, dy\, dx = \boxed{15}$$

26. Evaluate the double integral

$$\int_0^{\pi/3}\int_0^2 25y^4\sec^2(x)\,dy\,dx$$

$$\int_0^{\pi/3}\sec^2(x)\left(\int_0^2 25y^4\,dy\right)dx$$

$$=\int_0^{\pi/3}\sec^2(x)\left(5y^5\Big]_0^2\right)dx$$

$$=\int_0^{\pi/3}\sec^2(x)\,(20)\,dx$$

$$=20\int_0^{\pi/3}\sec^2(x)\,dx$$

$$=20\tan x\Big]_0^{\pi/3}$$

$$=20\sqrt{3}$$

$$\int_0^{\pi/3}\int_0^2 25y^4\sec^2(x)\,dy\,dx = \boxed{20\sqrt{3}}$$

27. Evaluate the double integral

$$\int_0^{\pi/2}\int_0^1 12x^3\sin(y)\,dx\,dy$$

$$=\int_0^{\pi/2}\sin(y)\left(\int_0^1 12x^3\,dx\right)dy$$

$$=\int_0^{\pi/2}\sin(y)\left(3x^4\Big]_0^1\right)dy$$

$$=\int_0^{\pi/2}\sin(y)\,(3)\,dy$$

$$=3\int_0^{\pi/2}\sin(y)\,dy$$

$$=-3\cos(y)\Big]_0^{\pi/2}$$

$$=-3\cos\left(\tfrac{\pi}{2}\right)-\left(-3\cos(0)\right)$$

$$=0-(-3)$$

$$=3$$

$$\int_0^1\int_0^{\pi/2} 12x^3\sin(y)\,dx\,dy = \boxed{3}$$

28. Evaluate the double integral

$$\int_0^4 \int_2^y (y+x)\,dx\,dy$$

$$\int_{y=0}^{y=4}\int_{x=2}^{x=y} (y+x)\,dx\,dy$$

$$= \int_{y=0}^{y=4} \left(\left(xy + \frac{x^2}{2} \right) \Big]_{x=2}^{x=y} \right) dy$$

$$= \int_{y=0}^{y=4} \left(y^2 + \frac{y^2}{2} - (2y+2) \right) dy$$

$$= \int_{y=0}^{y=4} \left(\frac{3}{2} y^2 - 2y - 2 \right) dy$$

$$= \left(\frac{3}{2} \cdot \frac{y^3}{3} - \frac{2y^2}{2} - 2y \right) \Big]_{y=0}^{y=4}$$

$$= \left(\frac{y^3}{2} - y^2 - 2y \right) \Big]_{y=0}^{y=4}$$

$$= 8$$

$$\int_0^4 \int_2^y (y+x)\,dx\,dy = \boxed{8}$$

29. Evaluate the double integral

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\,dy\,dx$$

$$= \int_{x=1}^{x=2}\int_{y=1}^{y=x^2} x y^{-2}\,dy\,dx$$

$$= \int_{x=1}^{x=2} x \left(\int_{y=1}^{y=x^2} y^{-2}\,dy \right) dx$$

$$= \int_{x=1}^{x=2} x \left(-y^{-1} \Big]_{y=1}^{y=x^2} \right) dx$$

$$= \int_{x=1}^{x=2} x \left(-\frac{1}{y} \Big]_{y=1}^{y=x^2} \right) dx$$

$$= \int_{x=1}^{x=2} x \left(-\frac{1}{x^2} + 1 \right) dx$$

$$= \int_1^2 \left(x - \frac{1}{x} \right) dx$$

$$= \left(\frac{x^2}{2} - \ln(x) \right) \Big]_1^2$$

$$= (2 - \ln(2)) - \left(\frac{1}{2} - 0 \right)$$

$$= \frac{3}{2} - \ln(2)$$

$$\int_1^2 \int_1^{x^2} \frac{x}{y^2}\,dy\,dx = \boxed{\frac{3}{2} - \ln(2)}$$

30. Compute the following definite integral.

$$\int_0^7 \int_1^x 36x \, dy \, dx$$

$$= \int_0^7 36x \left(\int_1^x dy \right) dx$$

$$= \int_0^7 36x \, (y) \Big]_1^x \, dx$$

$$= \int_0^7 36x [x-1] \, dx$$

$$= \int_0^7 (36x^2 - 36x) \, dx$$

$$= \left(\frac{36x^3}{3} - \frac{36x^2}{2} \right) \Big]_0^7$$

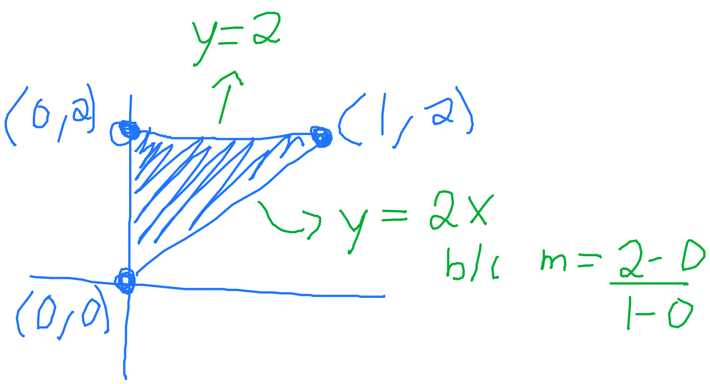
$$= (12x^3 - 18x^2) \Big]_0^7$$

$$= 3234$$

$$\int_0^7 \int_1^x 36x \, dy \, dx = \boxed{3234}$$

31. Find the bounds for the integral $\iint_R 5e^x \sin(y) \, dA$ where R is a triangle with vertices (0,0), (1,2), and (0,2).

DON"T COMPUTE!!!



Hence

$$\int_0^1 \int_{2x}^2 5e^x \sin(y) \, dy \, dx$$

Answer: $\boxed{\int_0^1 \int_{2x}^2 5e^x \sin(y) \, dy \, dx}$

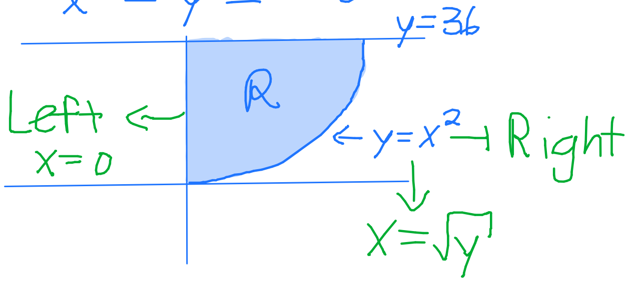
32. Switch the order of integration on the follow integral

$$\int_0^6 \int_{x^2}^{36} f(x,y)\,dy\,dx$$

The bounds tell me

$0 \leq x \leq 6$

$x^2 \leq y \leq 36$



So $0 \leq x \leq \sqrt{y}$

what does y range from?

$0 \leq y \leq 36$

Answer: $\boxed{\int_0^{36} \int_0^{\sqrt{y}} f(x,y)\,dx\,dy}$

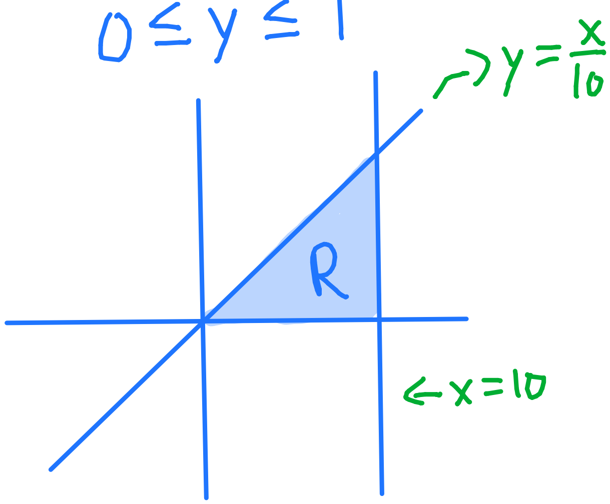
33. Switch the order of integration on the follow integral

$$\int_0^1 \int_{10y}^{10} f(x,y)\,dx\,dy$$

The bounds tell me

$y = \frac{x}{10} \leftarrow 10y \leq x \leq 10$

$0 \leq y \leq 1$



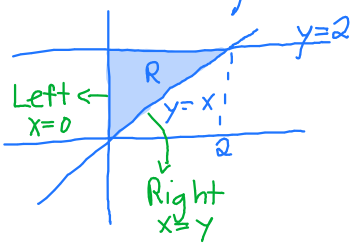
Answer: $\boxed{\int_0^1 \int_0^{x/10} f(x,y)\,dy\,dx}$

34. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx$$

(Hint: Change the order of integration)

Bounds: $0 \le x \le 2$
$x \le y \le 2$



So $0 \le y \le 2$
$0 \le x \le y$

$$\int_0^2 \int_x^2 4e^{y^2} dy\, dx$$

$$= \int_{y=0}^{y=2} \int_{x=0}^{x=y} 4e^{y^2} dx\, dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2} \left(\int_{x=0}^{x=y} dx \right) dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2} \left(x \Big]_{x=0}^{x=y} \right) dy$$

$$= \int_{y=0}^{y=2} 4ye^{y^2} dy$$

$$\frac{u = y^2}{du = 2y\,dy} \int 2e^u du$$

$$= 2e^u.$$

$$= 2e^{y^2} \Big]_{y=0}^{y=2}$$

$$= 2e^4 - 2$$

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx = \boxed{2e^4 - 2}$$

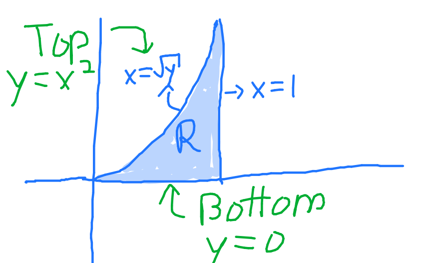
35. Evaluate the double integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy$$

Round your answer to 2 decimal places.

(Hint: Change the order of integration)

Bounds: $0 \le y \le 1$
$\sqrt{y} \le x \le 1$



New Bounds: $0 \le y \le x^2$
$0 \le x \le 1$

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx\, dy$$

$$= \int_{x=0}^{x=1} \int_{y=0}^{y=x^2} \sin(x^3) dy\, dx$$

$$= \int_{x=0}^{x=1} \sin(x^3) \left(\int_{y=0}^{y=x^2} dy \right) dx$$

$$= \int_{x=0}^{x=1} \sin(x^3) \left(y \Big]_{y=0}^{y=x^2} \right) dx$$

$$= \int_{x=0}^{x=1} \sin(x^3) \cdot x^2 dx$$

$$\frac{u = x^3}{du = 3x^2 dx} \int \frac{1}{3} \sin(u) du$$

$$= -\frac{1}{3} \cos(u)$$

$$= -\frac{1}{3} \cos(x^3) \Big]_{x=0}^{x=1}$$

$$\approx 0.15$$

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy = \boxed{0.15}$$