MA 16020 LESSON 18: VOLUME BY REVOLUTION – SHELL METHOD (SUPPEMENTAL HOMEWORK)

Formulas:

- **Rotating around y-axis:**

$$V = 2\pi \int_a^b x \cdot (Top - Bottom)\, dx$$

- **Rotating around y = #**
  - **If $a \geq \#$, then**

$$V = 2\pi \int_a^b (x - \#) \times (Top - Bottom)\ dx$$

  - **If b $\leq$ #, then**

$$V = 2\pi \int_a^b (\# - x) \times (Top - Bottom)\ dx$$

- **Rotating around x-axis:**

$$V = 2\pi \int_c^d y \cdot (Right - Left)\, dy$$

- **Rotating around x = #**
  - **If $a \geq \#$, then**

$$V = 2\pi \int_a^b (y - \#) \times (Right - Left)\ dy$$

  - **If b $\leq$ #, then**

$$V = 2\pi \int_a^b (\# - y) \times (Right - Left)\ dy$$

MA16020 Exam 3 Study Guide: Cal II

Differential Equations

- Growth & Decay: $y' = ky \Rightarrow y = Ce^{kt}$ where k is a constant.

- Separation of Variables: Solve the differential equations of the type

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

The idea is to try to get terms w/ y on one side and x-terms on the other. Integrate and solve for y.

- First-Order Linear Differential Equations: Are equations of the form $a(t)y' + b(t)y = c(t)$

How to solve:

1. Using simple algebra, rewrite your equation to be
$$y' + P(t)y = Q(t)$$
2. Determine $P(t)$ and $Q(t)$
3. Find integrating factor: $u(t) = \exp[\int P(t)\,dt]$
4. Plug $u(t)$ and $Q(t)$ in
$$y \cdot u(t) = \int Q(t)u(t)\,dt + C$$
5. Integrate the RHS of 4
6. Divide both sides of the equation from 5 by $u(t)$.

Sums / Series

- Geometric Series: Are of the form $\sum_{n=0}^{\infty} ar^n$

  ↳ Converge if $|r|<1$ and $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$

  ↳ Diverges if $|r| \geq 1$

- Power Series: Are of the form $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ where $|x|<1$

  ↳ Radius of convergence is R when $|x|<R$.

  e.g. $\sum_{n=0}^{\infty} (2x)^n \Rightarrow |2x|<1$

  $|x| < 1/2 \Rightarrow R = 1/2$

- Maclaurin Series: Are of the form

  $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ where $|x|<R$

$$\left[\begin{array}{l} e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \; ; \; \ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \\ \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \; ; \; \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \end{array}\right]$$

Will be provided on the exam

Using Series to Estimate Definite Integrals

1. Convert the function into a series
2. Integrate the series (remember x is the variable)
3. Write out the number of terms to be used.
4. Substitute the bounds.

Functions of Several Variables

Domain: All points (x,y) in the xy-plane for which $f(x,y)$ is defined

Range: All values that the function $f(x,y)$ produces

Techniques for Finding the Domain

- Given $\sqrt{?} \Rightarrow ? \geq 0$
- Given $\ln(?) \Rightarrow ? > 0$
- Given $\frac{1}{?} \Rightarrow ? \neq 0$
- Given $\frac{1}{\sqrt{?}} \Rightarrow ? > 0$

Level Curves: $f(x,y) = k$ where k is a constant.

Descriptions of this curves can be found on the next page.

Partial & Higher Order Partial Derivatives

$f_x \Rightarrow$ Find the derivative w/ respect to x and treat y as a constant.

$f_y \Rightarrow$ Find the derivative w/ respect to y and treat x as a constant.

$f_{xx} = \frac{d}{dx}(f_x) \quad f_{xy} = \frac{d}{dy}(f_x) \quad f_{yy} = \frac{d}{dy}(f_y)$