# Lesson 2: Review of Integration

**Indefinite Integration:** $\int f(x)\,dx = F(x) + C$ where $C$ is a constant

**Basic Integration Rules**

- $\int 0\,dx = C$
- $\int k\,dx = kx + C$
- $\int k f(x)\,dx = k \int f(x)\,dx$
- $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\boxed{\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \quad \text{for } n \neq -1}$ ← Power Rule
- $\int \cos x\,dx = \sin x + C$
- $\int \sin x\,dx = -\cos x + C$
- $\int \sec^2 x\,dx = \tan x + C$
- $\int \csc^2 x\,dx = -\cot x + C$
- $\int \sec x \tan x\,dx = \sec x + C$
- $\int \csc x \cot x\,dx = -\csc x + C$
- $\int e^x\,dx = e^x + C$
- $\int \dfrac{1}{x}\,dx = \ln|x| + C$

Recall you can check your answer by taking the derivative of it and seeing if it matches the original function.

Example 1: Evaluate the following

ⓐ $\int(6\sec^2 x - 5e^x)\,dx = 6\int \sec^2 x\,dx - 5\int e^x\,dx$

$= 6\tan x - 5e^x + C$

ⓑ $\int(x^2 + 2\sqrt{x})\,dx = \int(x^2 + 2x^{1/2})\,dx$

$= \frac{x^3}{3} + \frac{2\cdot 2}{3}x^{3/2} + C$

$= \frac{x^3}{3} + \frac{4}{3}x^{3/2} + C$

ⓒ $\int\left(\frac{3}{x} + \sqrt[3]{x^2}\right)dx = 3\int \frac{1}{x}\,dx + \int x^{2/3}\,dx$

$= 3\ln|x| + \frac{3}{5}x^{5/3} + C$

Differential Equations

Example 2: Solve the differential equation $y' = 3x$.

Recall $y' = \frac{dy}{dx}$. So

$\int y'\,dx = \int 3x\,dx$

$\int \frac{dy}{\cancel{dx}}\,\cancel{dx} = \int 3x\,dx$

$\int dy = \int 3x\,dx$

$y = \frac{3}{2}x^2 + C \Rightarrow$ This is called the general solution.

What if we are given an initial condition (such as $y(0) = 2$)?

Example 3: Solve the initial value problem (IVP) $y' = 3x$ with $y(0) = 2$.

From Ex 2, $y = \frac{3}{2}x^2 + C$.

Using $y(0) = 2$, we can find $C$.

$2 = \frac{3}{2}(0)^2 + C \Rightarrow C = 2$

Hence $y = \frac{3}{2}x^2 + 2 \Rightarrow$ This is can a particular solution.

Definite Integrals

A definite integral looks like $\int_a^b f(x)\,dx$. Remember $+C$ isn't necessary for definite integrals.

Fundamental Theorem of Calculus (FTC)

$$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$$

where $F$ is the antiderivative of $f$.

Example 4: Evaluate the following

ⓐ $\int_0^{\pi/4} \sec^2 x\,dx = \tan x\Big]_0^{\pi/4}$

$$= \tan\left(\frac{\pi}{4}\right) - \tan(0)$$

$$= 1 - 0$$

$$= 1$$

ⓑ $\int_1^4 \frac{x^2 + x}{\sqrt{x}}\,dx = \int_1^4 \left(\frac{x^2}{x^{1/2}} + \frac{x}{x^{1/2}}\right) dx$

$$= \int_1^4 \left(x^{3/2} + x^{1/2}\right) dx$$

$$= \left(\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2}\right)\Big]_1^4$$

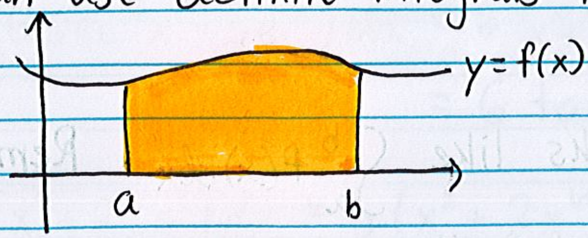
$$= \left(\frac{2}{5}(4)^{5/2} + \frac{2}{3}(4)^{3/2}\right) - \left(\frac{2}{5}(1)^{5/2} + \frac{2}{3}(1)^{3/2}\right)$$

$$= \frac{2}{5}\cdot 2^5 + \frac{2}{3}\cdot 2^3 - \frac{2}{5} - \frac{2}{3}$$

$$= \frac{256}{15}$$

# Area under a Curve

We can use definite integrals to find area under a curve

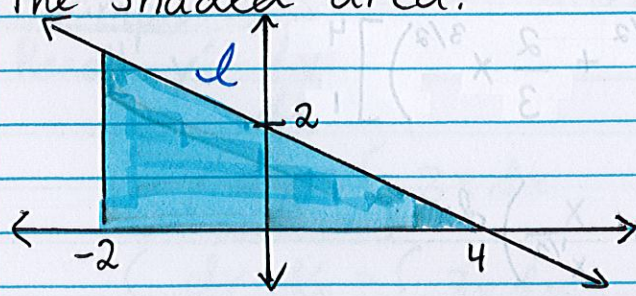



Bounded by
$y=0$, $y=f(x)$
$x=a$, $x=b$

**Example 5:** Find the area of the region bounded by
$y=2x+1$ ; $y=0$ ; $x=1$ ; $x=3$

$$\int_1^3 (2x+1)\,dx = \left(\frac{2x^2}{2}+x\right)\Big]_1^3$$
$$= (x^2+x)\Big]_1^3$$
$$= (3^2+3)-(1^2+1) = 10$$

**Example 6:** Write the definite integral that represents the shaded area.




We can see the bounds of the integral will be -2 to 4. So,

$$\int_{-2}^{4} \boxed{\phantom{xx}}\,dx$$

Now we need to determine the equation of $l$. Note from the graph we have 2 points on $l$, which are $(0,2)$ and $(4,0)$. So the slope of $l$ is

$$m = \frac{0-2}{4-0} = \frac{-2}{4} = \frac{-1}{2}$$

Note we are also given the y-intercept of $l$, $(0,2)$. So

$$l = \frac{-1}{2}x+2$$

Hence the definite integral is

$$\int_{-2}^{4}\left(\frac{-1}{2}x+2\right)dx$$