Please show all your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name:
Solutions

1. Find the derivative of $S(t)=4 t^{3} \tan (t)-\sqrt{t}$

$$
\begin{aligned}
& S(t)=\frac{4 t^{3}-\tan (t)-t^{1 / 2}}{\text { Product Rule }} \\
& S^{\prime}(t)=12 t^{2} \tan (t)+4 t^{3} \sec ^{2}(t)-\frac{1}{2} t^{-1 / 2}
\end{aligned}
$$

$$
\frac{d s}{d t}=\begin{aligned}
& 12 t^{2} \tan (t)+4 t^{3} \sec ^{2}(t) \\
& -1 / 2 t^{-1 / 2}
\end{aligned}
$$

2. Given the velocity and initial position of a body moving along a coordinate line at time $t$, find the body's position, $s(t)$, at time $t$.

$$
\begin{array}{rlrl}
S(t) & =\int v(t) d t & & \\
& =\int(-4 t+2) d t & & 3=0+0+0+c \\
& & =-\frac{4 t^{2}}{2}+2 t+c & \\
& & \\
& =-2 t^{2}+2 t+c & &
\end{array}
$$

3. Evaluate the definite integral

$$
\int_{0}^{\pi / 6}(3 \cos (x)-6) d x
$$

$=(3 \sin (x)-6 x)]_{0}^{\pi / 6}$
$=3 \sin \left(\frac{\pi}{6}\right)-6\left(\frac{\pi}{6}\right)-(3 \sin (0)-6(0))$
$=\frac{3}{2}-\pi$

$$
\int_{0}^{\pi / 6}(3 \cos (x)-6) d x=
$$


4. Evaluate the definite integral

5. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$
r(t)=6 \sqrt{t}
$$

where $t$ is time in hours after 9:00 am and the rate $r(t)$ is in cubic feet per hour.
(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

$$
\begin{aligned}
& \begin{aligned}
& 10.000 \mathrm{~m} \Rightarrow 1 \mathrm{hr} \\
&1: 00 \mathrm{pm} \Rightarrow 4 \mathrm{hrs}\} \Rightarrow\left\{\int_{1}^{4} 6 t^{1 / 2} \mathrm{dt}\right. \\
&\left.=6 \frac{2}{3} t^{3 / 2}\right]_{1}^{4}
\end{aligned} \\
& \left.=4 t^{3 / 2}\right]_{1}^{4} \\
& =28
\end{aligned}
$$


(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

$$
\begin{array}{r}
\operatorname{solve}_{0} \int_{4} \operatorname{tin}^{1 / 2} d=121 \\
+3 / 2=121 \\
+3 / 2=\frac{121}{4} \\
+=\left(\frac{121}{4}\right)^{2 / 3}
\end{array}
$$



Answer: $\qquad$
6. During a snowstorm, the rate, in inches per hour, at which the snow falls on a certian town is modeled by the function

$$
R^{\prime}(t)=-\cos (t)-1.2 t+4
$$

where $t$ is measured in hours and $0 \leq t \leq 4$. Based on the model, what is the total amount of snow, in inches, that fell on town from $t=0$ to $t=4$ ? Round to one decimal place.

7. Which derivative rule is undone by integration by substitution?
(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these
8. Which derivative rule is undone by integration by parts?
(A) Power Rule
(B) Quotient Rule
(C) Product Rule
(D) Chain Rule
(E) Constant Rule
(F) None of these
9. What would be the best substitution to make the solve the given integral?

$\int e^{2 x} \cos \left(e^{2 x}\right)\left[\sin \left(e^{2 x}\right)\right]^{3} d x$


$$
\int \sec ^{2}(5 x) e^{\tan (5 x)} d x
$$


11. What would be the best substitution to make the solve the given integral?

$$
\int \tan (5 x) \sec (5 x) e^{\sec (5 x)} d x
$$

$$
\int e^{y} \csc \left(e^{y}+1\right) \cot \left(e^{y}+1\right) d y
$$


14. Evaluate the definite integral.

$$
\begin{aligned}
& \int_{0}^{2}\left(5 e^{2 x}+8\right) d x \\
& \left.=\frac{5}{2} e^{2 x}\right]_{0}^{2}+8 \\
& =\frac{5}{2}\left(e^{4}-e^{0}\right)+8( \\
& =\frac{5}{2} e^{4}-\frac{5}{2}+16 \\
& =\frac{5}{2} e^{4}-\frac{27}{2}
\end{aligned}
$$

$$
\left.\left.\begin{array}{rl}
\underbrace{2}_{u-s u b} 5 e^{2 x} d x
\end{array}+\int_{0}^{2} s d x=\frac{5}{2} e^{2 x}\right]_{0}^{2}+8 x\right]_{0}^{2},
$$

$$
\int_{0}^{2}\left(5 e^{2 x}+8\right) d x=\stackrel{\frac{5}{2} e^{4}+\frac{27}{2}}{ }
$$

15. Evaluate the indefinite integral.

$$
\begin{aligned}
\frac{u=5 x^{2}}{\frac{\int 18 \cos \left(5 x^{2}\right)^{2} d x}{d u=10 x} d x} & \int 18 x \cos (u) \frac{d u}{10 x}
\end{aligned}=\int \frac{9}{5} \cos (u) d u
$$

$$
\int_{18 s \cos \left(5 x^{2}\right) d x} \frac{9}{5} \sin \left(5 x^{2}\right)+c
$$

$$
\begin{aligned}
& \frac{u}{d u}=-x^{4} \\
& \frac{d u}{-4 x^{3}}=-d x \int x^{3} e^{u} \frac{d v}{-4 x^{3}}
\end{aligned}=-\frac{9}{4} \int e^{u} d u \quad \begin{aligned}
& \\
& \\
& \\
& =-\frac{9}{4} e^{u}=-\frac{9}{4} e^{-x^{4}}+c
\end{aligned}
$$

$$
\int_{9 x^{3} e^{3}-x^{x} d x=-\frac{9}{4} e^{-x^{4}}+c}
$$

17. Evaluate the indefinite integral

$$
\begin{aligned}
& \begin{array}{l}
\frac{u}{4}=x^{2}+4 \\
d u
\end{array}=2 x d x \\
& \frac{d u}{2 x}=d x
\end{aligned} \quad x u^{3} \frac{d u}{2 x}=\frac{1}{2} \int u^{3} d u=\frac{1}{2} \cdot \frac{u^{4}}{4}+c
$$

$$
\int_{x\left(x^{(2}+4\right)^{3} d x}=\frac{1}{8}\left(x^{2}+4\right)^{4}+c
$$

18. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that $t$ hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$
L^{\prime}(t)=\sqrt{3 t+2} \quad \text { gallows per hour }
$$

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.


19. It is estimated that $t$-days into a semester, the average amount of sleep a college math student gets per day $S(t)$ changes at a rate of

$$
\frac{-4 t}{e^{t^{2}}}=-4+e^{-+2}
$$

hours per day. When the semester begins, math students sleep an average of 8.2 hours per day. What

$$
\begin{aligned}
& \text { is } S(t), 2 \text { days into the semester? } \\
& \text { (1) } \int-4 t e^{-t^{2}} d t \frac{u=-t^{2}}{d u=-2 t d t} \int-{ }_{-T}^{2} e^{u} \frac{d u}{-2 t} \\
& \frac{d u}{-2 t}=d t \\
& \begin{aligned}
=\int 2 e^{u} d u & =2 e^{u}+c \\
& =2 e^{-t^{2}}+c
\end{aligned}
\end{aligned}
$$

(2)

$$
\begin{gathered}
S(0)=8.2 \text { Find } c . \\
8.2=2 e^{0}+c \\
8.2=2+c \\
c=6.2
\end{gathered}
$$

(3)

$$
\begin{aligned}
s(t) & =2 e^{-t^{2}}+6.2 \\
s(2) & =2 e^{-4}+6.2 \\
& \approx 6.237
\end{aligned}
$$


20. A biologist determines that, $t$ hours after a bacterial colony was established, the population of bacteria in the colony is changing at a rate given by

$$
P^{\prime}(t)=\frac{5 e^{t}}{1+e^{t}}
$$

million bacteria per hour, $0 \leq t \leq 5$.

If the bacterial colony started with a population of 1 million, how many bacteria, in millions are present in the colony after the 5 -hour experiment?
(1) $\int$

$$
\frac{5 e^{t}}{1+e^{t}}
$$

$$
\begin{aligned}
& d t \frac{u}{d u}=1+e^{+} \\
& \begin{array}{l}
\frac{d u}{e^{+}} d t \\
e d t \\
=5 \ln |u|+c \\
=5 \ln \left|1+e^{*}\right|+c
\end{array} \\
& =
\end{aligned}
$$

$$
\begin{aligned}
& =5 \ln 1 \\
& \text { ind } C \text {. }
\end{aligned}
$$

(2) $\begin{aligned} P(0) & =\mid \text { Find } C . \\ 1 & =5 \ln \left|1+e^{0}\right|+C\end{aligned}$
$1=5 \ln |1+1|+c$
$1=5 \ln 2+6$

$$
1-5 \ln 2=c
$$

(3) $P(t)=5 \ln \left|1+e^{t}\right|+1-5 \ln 2$ $P(5)=5 \ln \left|1+e^{5}\right|+\mid-5 \ln 2$ ~ 22.57
22.57
21. Evaluate the definite integral.

$$
\begin{aligned}
& \frac{u}{\frac{u}{d u}=2 x} \iint \sin (u) \frac{d u}{2}=\frac{3}{2} \int \sin (u) d u=-\frac{3}{2} \cos (u) \\
& \frac{d u}{2}=d x \\
&\left.=\frac{-3}{2} \cos (2 x)\right]_{0}^{\pi / 4} \\
&=-\frac{3}{2} \cos \left(\frac{2 \pi}{4}\right)-\left(-\frac{3}{2} \cos (u)\right)
\end{aligned}
$$

22. Evaluate the indefinite integral.

$$
\begin{aligned}
& \begin{aligned}
& \frac{u=x^{2}+\frac{8 x}{d u}}{d u=(2 x+2) d x} \\
& \begin{aligned}
d u=2(x+4) d x \\
\frac{d u}{2(x+4)}=d x
\end{aligned}=\frac{1}{2} \int u^{1 / 2} d u \\
&=\frac{1}{x} \cdot \frac{2}{3} u^{3 / 2}+c \\
&=\frac{1}{3}\left(x^{2}+8 x\right)^{3 / 2}+C
\end{aligned} \\
&
\end{aligned}
$$

23. Evaluate the definite integral.

$$
\begin{aligned}
& u=\sqrt{x}+1 \\
& u=x^{1 / 2}+1 \\
& \hline d u=\frac{1}{2} x^{-1 / 2} \\
& d u=\frac{1}{2} \cdot \frac{1}{\sqrt{x}} d x \\
& 2 \sqrt{x} d u=d x
\end{aligned}
$$

$$
\begin{aligned}
\frac{2 \sqrt{x} \mid d u}{\partial \sqrt{2} \cdot u} & =\int \frac{d x}{2 \sqrt{x}+1)} \\
& =\ln |\sqrt{\pi}+1|]_{0}^{9} \\
& =\ln |u| \\
& =\ln (4)
\end{aligned}
$$

$$
\operatorname{lig}_{\sqrt{0} \frac{d x}{\sqrt{\sqrt{(x)}}(\sqrt{\bar{x}}+1)}}=\ln (4)
$$

24. Evaluate the indefinite integral.

$$
\begin{aligned}
& \left(\begin{array}{l}
\frac{u=x+2}{\underline{u=d x}} \int x \sqrt{u} d u \\
\underline{\underline{x=u-2}} \int(u-2) u^{1 / 2} d u \\
= \\
=\frac{2}{5} u^{5 / 2}-2 u^{1 / 2} d u \\
=2 \cdot \frac{2}{3} u^{3 / 2}+c \\
\quad \int_{\sqrt{x \sqrt{x}+2 d x}} \frac{2}{5}(x+2)^{5 / 2} \\
-4 / 3(x+2)^{3 / 2}+c
\end{array}\right.
\end{aligned}
$$

25. A tree is transplanted and after $t$ years is growing at a rate

$$
r^{\prime}(t)=1+\frac{1}{(t+1)^{2}} \quad \text { meters per year. }
$$

After 2 years it has reached a height of 5 meters. How tall was the tree when it was originally transplanted? Round to one decimal place.


$$
\begin{aligned}
r(t) & =t-\frac{1}{t+1}+\frac{10}{3} \\
r(0) & =0-1+\frac{10}{3} \\
& =7 / 3 \approx 2.3
\end{aligned}
$$

26. The marginal revenue from the sale of $x$ units of a particular product is estimated to be $R^{\prime}(x)=$ $50+350 x e^{-x^{2}}$ dollars per unit, and where $R(x)$ is revenue in dollars. What revenue should be expected from the sale of 100 units? Assume that $R(0)=0$.

$R(0)=0$
$0=0-175+c$
$c=175$
$R(x)=50 x-175 e^{-x^{2}}+175$ $R(100) \approx 5175$

27. Evaluate the indefinite integral

$$
\int \frac{\ln (7 x)}{x} d x
$$

$$
\begin{aligned}
& \frac{u=\ln (7 x)}{d u=\frac{1}{7 x} \cdot 7 d x} \int u d u=\frac{u^{2}}{2}=\frac{(\ln (7 x))^{2}}{2}+C \\
& d u=\frac{1}{x} d x
\end{aligned} \int
$$

$$
\int \frac{\ln (7 x)}{x} d x=\frac{(\ln (7 x))^{2}}{2}+<
$$

28. Evaluate

$$
\int_{1}^{e} \frac{\ln \left(x^{4}\right)}{x} d x
$$

Rewrite $\left.\int_{1}^{e} \frac{4 \ln x}{x} d x \frac{u=\ln x}{d u=\frac{1}{x} d x} \int 4 u d u=\frac{4 u^{2}}{2}=2 u^{2}=2(\ln x)^{2}\right]_{1}^{e}$

$$
\begin{aligned}
& =\underbrace{2(\ln e)^{2}}_{2}-\underbrace{2(\ln 1)^{2}}_{0} \\
& =2
\end{aligned}
$$

$$
\int_{1}^{e} \frac{\ln \left(x^{4}\right)}{x} d x=2
$$

29. Evaluate

$$
\int_{e}^{4} \frac{d x}{x(\ln (x))^{2}}
$$


30. Evaluate the definite integral.

$$
\begin{aligned}
\frac{u=x-1}{d u=d x} \frac{d v=\sin (x) d x}{v=-\cos (x)} u v-\int v d u= & -(x-1) \cos x]_{0}^{\pi / 2}(x-1) \sin (x) d x \\
= & \left.-(x-1) \cos x]_{0}^{\pi / 2}+\sin (x)\right]_{0}^{\pi / 2}(-\cos x) d x \\
= & -\left(\frac{\pi}{2}-1\right) \cos \left(\frac{\pi}{2}\right)-[-(0-1) \cos (0)] \\
& +\sin \left(\frac{\pi}{2}\right)-\sin \cot \rightarrow 0 \\
= & -1+1=0
\end{aligned}
$$

$$
\int_{0}^{\pi / 2}(x-1) \sin (x) d x=\square
$$

Rewrite $\int 3 x(7 \ln (x)) d x=\int 21 x \ln x d x$ $\frac{u=21 \ln (x)}{d u=\frac{21}{x} d x} \frac{d v=x d x}{v=\frac{x^{2}}{2}} u v-\int v d u$

$$
=\frac{21 x^{2} \ln x}{2}-\int \frac{x^{2}}{2} \cdot \frac{21}{x} d x
$$

$$
=\frac{21 x^{2} \ln x}{2}-\int \frac{21}{2} x d x
$$

$$
=\frac{21 x^{2} \ln x}{2}-\frac{21}{2} \cdot \frac{x^{2}}{2}+c
$$

$$
=\frac{21 x^{2} \ln x}{2}-\frac{31 x^{2}}{4}+C_{3 x \ln \left(x^{7}\right) d x=}^{C} \frac{21 x^{2} \ln x}{2}-\frac{21 x^{2}}{4}+C
$$

32. Evaluate

$$
\begin{aligned}
& \int x^{3} \ln (2 x) d x \\
& \frac{u=\ln (2 x)}{d u=\frac{1}{2 x} \cdot 2 d x} \frac{d v=x^{3} d x}{v=\frac{x^{4}}{4}} u v-\int v d u=\frac{x^{4} \ln (2 x)}{4}-\int \frac{x^{4}}{4} \cdot \frac{1}{x} d x \\
& d u=\frac{1}{x} d x \\
& =\frac{x^{4} \ln (2 x)}{4}-\frac{1}{4} \int x^{3} d x \\
& =\frac{x^{4} \ln (2 x)}{4}-\frac{1}{4} \cdot \frac{x^{4}}{4}+c \\
& \int_{z=\min (2) x d x} \frac{x^{4}}{4} \ln (2 x)-\frac{x^{4}}{16}+<
\end{aligned}
$$

33. Evaluate the indefinite integral
$\int \sqrt{x} \ln (x) d x$

$$
\begin{aligned}
& \frac{u=\ln (x) \quad d v=x^{1 / 2} d x}{d u=\frac{1}{x} d x \quad v=\frac{2}{3} x^{3 / 2}} u v-\int v d u \\
& =\frac{2}{3} x^{3 / 2} \ln (x)-\int \frac{2}{3} x^{3 / 2} \cdot \frac{1}{x} d x \\
& =\frac{2}{3} x^{3 / 2} \ln (x)-\frac{2}{3} \int x^{1 / 2} d x \\
& =\frac{2}{3} x^{3 / 2} \ln (x)-\frac{2}{3} \cdot \frac{2}{3} x^{3 / 2}+c
\end{aligned}
$$

34. Evaluate the definite integral.

$$
\int_{\sqrt{\pi} \ln (x) d x}=\frac{1 / 3 x^{3 / 2} \ln (x)}{-4 / 9 x^{5 / 2}+c}
$$

$\int_{0}^{3} x e^{3 x} d x$

$$
\begin{aligned}
& \frac{u}{d u}=x \\
&=\frac{x}{3} e^{3 x}-\int \frac{1}{3} e^{3 x} d x \\
& v=\frac{1}{3} e^{3 x} d x \\
& 3 x-\int v d u \\
&\left.=\left(\frac{x}{3} e^{3 x}-\frac{1}{3} \cdot \frac{e^{3 x}}{3}\right)\right]_{0}^{3} \\
&=\frac{3}{3} e^{9}-\frac{1}{9} e^{9}-\left[0-\frac{1}{9}\right] \\
&=\frac{2}{9} e^{4}+\frac{1}{9}
\end{aligned}
$$

35. The population of pink elephants in Dumbo's dreams, in hundreds, $t$ years after the year 1980 is given by

$$
P(t)=\frac{e^{5 t}}{1+e^{5 t}}
$$

What is the average population during the decade between 1980 and 2000?

$$
\text { i.e. } \begin{aligned}
& \frac{1}{2000-1980} \int_{0}^{20} \frac{e^{5 t}}{1+e^{5 t}} d+\frac{u}{d u}=1+e^{5 t} \\
& \frac{d u}{5 e^{5 t}}=d t \\
&=\frac{1}{100} \int_{0}^{20} \frac{e^{5 t}}{u} \cdot \frac{d u}{5 e^{5 t}} \\
&\left.=\frac{d u}{100} \ln \left|1+e^{5 t}\right|\right]_{0}^{20}
\end{aligned}
$$

Answer:
36. Evaluate the indefinite integral.

$$
\begin{aligned}
& \frac{u=20 x}{d u=20 d x} \frac{d v=\sin (2 x) d x}{v=-\frac{\cos (2 x)}{2}} u v-\int^{\int 20 x \sin (2 x) d x} v d u \\
& =-\frac{20}{2} x \cos (2 x)+\int_{\frac{20}{2}}^{2}(+\cos (2 x)) d x \\
& =-10 x \cos (2 x)+10 \int \cos (2 x) d x \\
& =-10 x \cos (2 x)+10 \frac{\sin (2 x)}{2}+C \quad-10 x \cos (2 x)+5 \sin (2 x)+C
\end{aligned}
$$

37. The velocity of a cyclist during an hour-long race is given by the function

$$
v(t)=166 t e^{-2.2 t} \mathrm{mi} / \mathrm{hr}, \quad 0 \leq t \leq 1
$$

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?
(1) $\int 166 t e^{-2.2 t} d t$
$\frac{u=166 t}{d u=166 d t} \quad \frac{d v=e^{-2.2 t} d t}{v=\frac{e^{-2.2 t}}{-2.2}} u v-\int v d u$
$=\frac{166 t e^{-2.2 t}}{-2.2}+\int \frac{e^{-2.2 t}}{t 2.2} \cdot 166 d t$

$$
\begin{aligned}
& =-\frac{166 t e^{-2.2 t}}{2.2}+\frac{166}{2.2} \cdot \frac{e^{-2.2 t}}{-2.2}+C \\
& =\frac{-166 t e^{-2.2 t}}{2.2}-\frac{166 e^{-2.2 t}}{(2.2)^{2}}+C
\end{aligned}
$$

(2) $s(0)=0$. Find $c$.

$$
0=0-\frac{166}{(2.2)^{2}}+c \rightarrow C=\frac{166}{(2.2)^{2}}
$$

(3)

$$
\begin{aligned}
& s(t)=\frac{-166 t e^{-2.2 t}}{2.2}-\frac{166 e^{-2.2 t}}{(2.2)^{2}}+\frac{166}{(2.2)^{2}} \\
& s(1)=-\frac{166}{2.2} e^{-2.2}-\frac{166}{(2.2)^{2}} e^{-2.2}+\frac{166}{(2.2)^{2}}
\end{aligned}
$$ $\approx 22.137$

Answer:
38. After $t$ days, the growth of a plant is measured by the function $2000 t e^{-20 t}$ inches per day. What is the change in the height of the plant (in inches) after the first 14 days?
$\int_{0}^{14} 2000 t e^{-20 t} d t$

$$
\begin{aligned}
& \frac{u=2000 t}{d u=2000 d t} \frac{d v=e^{-20 t} d t}{v=\frac{e^{-20 t}}{-20}} u v-\int v d u \\
& =2000 t\left(\frac{e^{-20 t}}{-20}\right)+\int\left(\frac{e^{-20 t}}{t 20}\right) 2000 d t \\
& =-100+e^{-20 t}+100 \int e^{-20 t} d t \\
& =-100+e^{-20 t}+100\left(\frac{e^{-20 t}}{-20}\right) \\
& \left.=\left(-100+e^{-20 t}-5 e^{-20 t}\right)\right]_{0}^{14} \\
& =5
\end{aligned}
$$



