Please show all your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Solutions

Name:\_

1. Find the derivative of  $S(t) = 4t^3 \tan(t) - \sqrt{t}$   $S(+) = \frac{1}{3} + \frac{1}{4} \ln(t) - \frac{1}{3} + \frac{1}{4} \ln(t) - \frac{1}{4} \ln(t) + \frac{1}{4} \ln(t) +$ 

$$5'(+) = 12t^2 + tan(+) + 4t^3 + tan(+) - \frac{1}{2}t^{-1/2}$$

$$\frac{dS}{dt} = \frac{12t^2 \tan(t) + 4t^3 \sec^2(t)}{-1/2 + -1/2}$$

2. Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position, s(t), at time t.

$$v(t) = -4t + 2,$$
  $s(0) = 3$ 

 $5(t) = \int v(t) dt$ = 5(-4t+2) dt=  $-4t^{2} + 2t + C$ =  $-2t^{2} + 2t + C$ 

When 
$$J(0) = 3$$
  
 $C = 3$ 

$$s(t) = \begin{bmatrix} -2 + ^2 + 2 + + 3 \end{bmatrix}$$

3. Evaluate the definite integral

$$\int_{0}^{\pi/6} (3\cos(x) - 6) \, dx$$

$$= (3\sin(x) - 6x)$$

$$= 3\sin(x) - 6(x) - 6(x)$$

$$= 3\sin(x) - 6(x) - 6(x)$$

$$=\frac{3}{2}$$
  $\text{T}$ 

$$\int_0^{\pi/6} (3\cos(x) - 6) \, dx = \frac{3}{2} - \pi$$

4. Evaluate the definite integral

$$= (3e^{x} + 2x) \int_{0}^{4} (3e^{x} + 2) dx$$

$$= (3e^{x} + 2x) \int_{0}^{4} (3e^{x} + 2) dx$$

$$= 3e^{4} + 2(4) - (3e^{0} + 2(0))$$

$$= 3e^{4} + 8 - 3$$

$$\int_0^4 (3e^x + 2) \, dx = \frac{3e^4 + 5}{}$$

5. A faucet is turned on at 9:00 am and water starts to flow into a tank at the rate of

$$r(t) = 6\sqrt{t}$$

where t is time in hours after 9:00 am and the rate r(t) is in cubic feet per hour.

(a) How much water, in cubic feet, flows into the tank from 10:00 am to 1:00 pm?

$$|0.00am = |hr| = 546 + 124$$
  
 $|0.00pm = |4hr| = 6.2 + 3/2 | 4$   
 $= 4 + 3/2 | 4$   
 $= 22$ 

Answer:

(b) How many hours after 9:00 am will there be 121 cubic feet of water in the tank?

Solve 
$$5 + 6 + \sqrt{2} H = |2|$$
  
 $+ 3/2 = |2|$   
 $+ |2| = |2|$   
 $+ |2| = |3|$   
 $+ = (|2|)^{2/3}$ 

(12/4)

Answer:

6. During a snowstorm, the rate, in inches per hour, at which the snow falls on a certian town is modeled by the function

$$R'(t) = -\cos(t) - 1.2t + 4$$

where t is measured in hours and  $0 \le t \le 4$ . Based on the model, what is the total amount of snow, in inches, that fell on town from t = 0 to t = 4? Round to one decimal place.

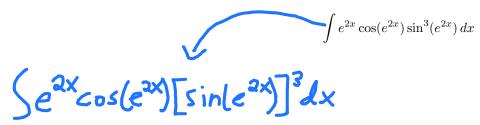
$$\int_{0}^{4} R'(t)dt = \int_{0}^{4} (-20)(t) - 1.2t + 4)dt$$

$$= (-\sin(t) - 1.2t^{2} + 4t) \int_{0}^{4} (-\sin(t) - 0.6t^{2} + 4t) \int_{0}^{4} (-\cos(t) - \cos(t) - \cos(t) - \cos(t) \int_{0}^{4} (-\cos(t) - \cos(t) - \cos(t) + 3t) \int_{0}^{4} (-\cos(t) - \cos(t) - 3t) \int_{0}^{4} (-\cos(t) - 3t) \int_{0}^{4} ($$

7.16

- 7. Which derivative rule is undone by integration by substitution?
  - (A) Power Rule
  - (B) Quotient Rule
  - (C) Product Rule
  - (D) Chain Rule
  - (E) Constant Rule
  - (F) None of these

- 8. Which derivative rule is undone by integration by parts?
  - (A) Power Rule
  - (B) Quotient Rule
  - (C) Product Rule
  - (D) Chain Rule
  - (E) Constant Rule
  - (F) None of these
- 9. What would be the best substitution to make the solve the given integral?



u = Sin(e<sup>2×</sup>)

Always check du is in the solve the given integral? integral

10. What would be the best substitution to make the solve the given integral?

$$\int \sec^2(5x)e^{\tan(5x)} dx$$

11. What would be the best substitution to make the solve the given integral?

$$\int \tan(5x)\sec(5x)e^{\sec(5x)}\,dx$$

u = Sec(5x)

Always check du is in the solve the given integral?

integral

12. What would be the best substitution to make the solve the given integral?

$$\int e^y \csc(e^y + 1) \cot(e^y + 1) \, dy$$

Always check du is in integral

13. Find the area under the curve  $y = 14e^{7x}$  for  $0 \le x \le 4$ .

 $A = \int_{0}^{4} 14e^{7x} dx \frac{u = 7x}{du = 7dx} \int 2e^{u} du$ 

 $= 2e^{4} = 2e^{7} \times \left[ \frac{4}{6} \right]_{0}^{4}$ 

Area = 28-2

14. Evaluate the definite integral.

$$\int_{0}^{2} (5e^{2x} + 8) dx$$

$$\int_{0}^{2} (5e^{2x} + 8) dx$$

$$= \frac{5}{2}e^{2x} \int_{0}^{2} + \frac{2}{3}e^{2x} \int_{0}^{2} + \frac{2}{3}e$$

$$\int_{0}^{2} (5e^{2x} + 8) dx = \frac{5}{3} e^{4} + \frac{27}{2}$$

15. Evaluate the indefinite integral.

$$\frac{U=5\times^{2}}{du=10\times dx}$$

$$\frac{18x\cos(5x^{2})dx}{du} = \int_{5}^{9} \cos(u) du$$

$$= \frac{9}{5}\sin(u) + C$$

$$= \frac{9}{5}\sin(5x^{2}) + C$$

$$\int 18x \cos(5x^2) \, dx = \frac{9 \sin(5x^2) + C}{5 \sin(5x^2)}$$

$$\frac{U = -x^{4}}{du = -4x^{3}dx} \leq 9x^{3}e^{u} \frac{du}{-4x^{3}} = -\frac{9}{4} \leq e^{u}du$$

$$\frac{du}{-4x^{3}} = dx$$

$$= -\frac{9}{4}e^{u} = -\frac{9}{4}e^{-x^{4}} + c$$

$$\int 9x^3 e^{-x^4} dx = \frac{-4 e^{-X^4} + 4}{4}$$

17. Evaluate the indefinite integral

$$\int x(x^2+4)^3 dx$$

$$\frac{u = x^{2} + 4}{du = 2 \times dx} \begin{cases} x u^{3} du = \frac{1}{2} (u^{3} du = \frac{1}{2} \cdot \frac{u^{4} + c}{4}) \\ du = 2 \times dx \end{cases} = \frac{1}{2} (x^{2} + 4)^{4} + c$$

$$\frac{du}{dx} = dx$$

$$\int x(x^2+4)^3 dx = \frac{1}{2} (x^2+4)^4 + C$$

18. After an oil spill, a company uses oil-eating bacteria to help clean up. It is estimated that t hours after being placed in the spill, the bacteria will eat the oil at a rate of

$$L'(t) = \sqrt{3t+2}$$
 gallows per hour.

How many gallons of oil will the bacteria eat in the first 4 hours? Round to 4 decimal places.

i.e. 
$$\int_{0}^{4} (3t+2)^{3}dt \frac{u=3++2}{du=3dt} \int_{0}^{4} u^{2}du$$
  

$$= \frac{1}{3} \cdot \frac{2}{3}u^{3/2} = \frac{2}{9}(3++2)^{3/2}$$

$$= \frac{1}{3} \cdot \frac{2}{3}u^{3/2} = \frac{2}{9}(3++2)^{3/2}$$

$$= \frac{1}{3} \cdot \frac{2}{3}u^{3/2} = \frac{2}{9}(3++2)^{3/2}$$

19. It is estimated that t-days into a semester, the average amount of sleep a college math student gets  $\frac{-4t}{e^{t^2}} = -4te^{-+2}$ per day S(t) changes at a rate of

hours per day. When the semester begins, math students sleep an average of 8.2 hours per day. What is S(t), 2 days into the semester?

$$\int \int -4te^{-t^2}dt \frac{u=-t^2}{du=-2t}dt \int -4te^{u} \frac{du}{du}$$

$$\frac{du}{du}=dt$$

$$= \int 2e^{u}du = 2e^{u} + C$$

$$= 2e^{-t^{2}} + C$$

$$35(+)=2e^{-+^3}+6.2$$

$$S(2) = 2e^{-4} + 6.2$$

20. A biologist determines that, t hours after a bacterial colony was established, the population of bacteria in the colony is changing at a rate given by

$$P'(t) = \frac{5e^t}{1 + e^t}$$

million bacteria per hour,  $0 \le t \le 5$ .

If the bacterial colony started with a population of 1 million, how many bacteria, in millions are present in the colony after the 5-hour experiment?

② 
$$P(0)=|$$
 Find  $C$ .  
 $|=5|n|+e^{0}+C$   
 $|=5|n|+1+C$   
 $|=5|n|+C$   
 $|-5|n|=C$ 

(3) 
$$P(+) = 5 \ln |1 + e^{+}| + |-5| n^{2}$$
  
 $P(5) = 5 \ln |1 + e^{5}| + |-5| n^{2}$   
 $22.57$ 

21. Evaluate the definite integral.

$$U = \frac{2x}{4u = 2dx}$$

$$\int_{0}^{\pi/4} 3\sin(2x) dx$$

$$du = \frac{3}{2} \int \sin(u) du = -\frac{3}{2} \cos(u)$$

$$du = \frac{3}{2} \cos(2x)$$

$$= -\frac{3}{2} \cos(2x)$$

$$= -\frac{3}{2} \cos(2x)$$

$$= -\frac{3}{2} \cos(2x)$$

$$\int_0^{\pi/4} 3\sin(2x) \, dx =$$

22. Evaluate the indefinite integral.

$$\frac{1}{\sqrt{1 + 4}} = \frac{1}{\sqrt{1 + 4}}$$

$$= \frac{1}{\sqrt{1 + 4}$$

$$\int (x+4)\sqrt{x^2+8x} \, dx = \frac{\int (x^2+8x)^{3/2} + C}{\int (x+4)\sqrt{x^2+8x} \, dx}$$

23. Evaluate the definite integral.

$$u = \sqrt{x} + 1$$

$$u = x'' + 1$$

$$du = \frac{1}{2} x^{-1/2} dx$$

$$du = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} dx$$

$$= \ln |\sqrt{x}| + 1 - \ln |\sqrt{x}|$$

$$2\sqrt{x} du = dx$$

$$= \ln (4)$$

$$\int_0^9 \frac{dx}{2\sqrt{x}(\sqrt{x}+1)} = \ln(4)$$

24. Evaluate the indefinite integral.

$$\int \frac{u = x + 2}{du = 4x} \int x \int u du$$

$$\int \frac{x}{x} = \frac{1}{4x} \int x \int u du$$

$$= \int \frac{x}{x} = \frac{1}{4x} \int \frac{x}{x} = \frac{1}{4$$

$$\int x\sqrt{x+2}\,dx = \frac{\frac{2}{5}(x+2)}{-4/3(x+2)^{3/3}} + 6$$

25. A tree is transplanted and after t years is growing at a rate

$$r'(t) = 1 + \frac{1}{(t+1)^2}$$
 meters per year.

After 2 years it has reached a height of 5 meters. How tall was the tree when it was originally transplanted? Round to one decimal place.

So 
$$r(t) = t - \frac{1}{11} + \frac{11}{3}$$
  
 $r(0) = 0 - 1 + \frac{10}{3}$   
 $= 7/3 \approx 2.3$ 

26. The marginal revenue from the sale of x units of a particular product is estimated to be  $R'(x) = 50 + 350xe^{-x^2}$  dollars per unit, and where R(x) is revenue in dollars. What revenue should be expected from the sale of 100 units? Assume that R(0) = 0.

$$R(x) = \int 50 + 350xe^{-x^{2}} dx$$

$$= \int 50dx + \int 350xe^{-x^{2}} dx$$

$$u = -x^{2}$$

$$du = -2xdx$$

$$\frac{du}{du} = dx$$

$$= \int 50dx - 175 \int e^{u} du$$

$$= \int 50x - 175e^{-x^{2}} + C$$

$$R(0) = 0$$

$$0 = 0 - 175 + C$$

$$c = 175$$

$$R(x) = \int 50x - 175e^{-x^{2}} + 175$$

$$R(100) \approx \int 175$$

$$\int \int 175$$

R(100) =

27. Evaluate the indefinite integral

$$\frac{\int \frac{\ln(7x)}{x} dx}{du = \frac{1}{7x} \cdot 7 dx}$$

$$\int \frac{\ln(7x)}{x} dx$$

$$\int \frac{\ln(7x)}{x} dx = \frac{\left( \ln(7x) \right)^2}{2} + C$$

28. Evaluate

Rewrite 
$$\int_{1}^{1} \frac{dx}{x} dx$$

$$\frac{|y| + |y|}{|x|} = \frac{|y| + |y|}{|x|} = \frac{|y|}{|x|} = \frac{|y|}{|x|$$

$$\int_{1}^{e} \frac{\ln(x^{4})}{x} dx = \square$$

29. Evaluate

$$\frac{|u| = |n(x)|}{du = \frac{1}{x} dx}$$

$$= \frac{1}{|n(x)|}$$

$$= \frac{1}{x} dx$$

$$= \frac{1}{|n(x)|}$$

30. Evaluate the definite integral.

So. Evaluate the definite integral.

$$\int_{0}^{\pi/2} (x-1) \sin(x) dx$$

$$\frac{1}{\sqrt{2}} = -(x-1) \cos(x) = -(x$$

$$\int_0^{\pi/2} (x-1)\sin(x) \, dx = \underline{\hspace{1cm}}$$

$$\int 3x \ln(x^7) \, dx$$

Rewrite 
$$\int 3 \times (7 \ln x) dx = \int 2 \ln x \ln x dx$$

$$\frac{u = 2 \ln x}{du = 2 \ln x} \frac{dv = x dx}{dv = x dx} \frac{dv - \int x dx}{dv - \int x dx}$$

$$= \frac{2 \ln x^2 \ln x}{2} - \int \frac{x^2}{2} \frac{2 \ln x}{x} dx$$

$$= \frac{2 \ln x^2 \ln x}{2} - \frac{2 \ln x}{2} \frac{x^2}{2} + C$$

$$= \frac{2 \ln x^2 \ln x}{2} - \frac{2 \ln x^2}{2} + C$$

$$= \frac{2 \ln x^2 \ln x}{2} - \frac{2 \ln x^2}{2} + C$$

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$$= \frac{2 \ln x^2 \ln x}{2} - \frac{2 \ln x^2}{2} + C$$

$$\int x^3 \ln(2x) \, dx$$

$$\frac{u=\ln(2x)}{du=\frac{1}{2}\cdot2dx} \xrightarrow{dv=\frac{x^3dx}{4}} uv-\int vdu=\frac{x^4\ln(2x)}{4}-\int \frac{x^4\cdot\frac{1}{2}dx}{4}$$

$$=\frac{x^4\ln(2x)}{4}-\frac{1}{4}\int x^3dx$$

$$=\frac{x^4\ln(2x)}{4}-\frac{1}{4}\cdot\frac{x^4}{4}+c$$

$$\int x^{3} \ln(2x) dx = \frac{\frac{1}{2} \ln(2x) - \frac{1}{2} + \frac{1}{2} \ln(2x)}{\frac{1}{2} + \frac{1}{2} + \frac{$$

33. Evaluate the indefinite integral

$$\frac{u = \ln(x)}{du = \frac{1}{x} dx} \frac{dv = \frac{x^{3}dx}{dx}}{v = \frac{2}{3}x^{3/2}} = \frac{2}{3}x^{3/2} \ln(x) - \frac{2}{3}x^{3/2} \cdot \frac{1}{x} dx$$

$$= \frac{2}{3}x^{3/2} \ln(x) - \frac{2}{3}x^{3/2} + C$$

$$\int \sqrt{x} \ln(x) dx = \frac{1/5}{-4/4} \times \frac{1}{5}$$

34. Evaluate the definite integral.

$$\int_0^3 xe^{3x} dx$$

$$\frac{1}{du} = \frac{1}{4} = \frac{1}{4} = \frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{du} = \frac{1}{4} = \frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4} = \frac{1}{4}$$

35. The population of pink elephants in Dumbo's dreams, in hundreds, t years after the year 1980 is given by

$$P(t) = \frac{e^{5t}}{1 + e^{5t}}$$

What is the average population during the decade between 1980 and 2000?

i.e. 
$$\frac{1}{2000-1980} \int_{0}^{20} \frac{e^{5t}}{1+e^{5t}} dt \frac{u=1+e^{5t}}{du=5e^{5t}dt} \frac{1}{20} \left( \frac{e^{5t}}{u} \cdot \frac{du}{5e^{5t}} \right) \frac{du}{du=5e^{5t}dt} = \frac{1}{20} \left( \frac{e^{5t}}{u} \cdot \frac{du}{5e^{5t}} \right)$$

$$= \frac{1}{100} \int \frac{du}{u} = \frac{1}{100} \ln \ln u$$

$$= \frac{1}{100} \ln \ln e^{-51} \int_{0}^{20} du$$

$$\propto 0.9931$$

0.9931 hundreds or 993

36. Evaluate the indefinite integral.

$$\int 20x\sin(2x)\,dx$$

$$\frac{u=20\times}{du=20dx} \frac{dv=\sin(2x)dx}{v=-\cos(2x)} uv-\int vdu$$

$$= \frac{20}{2} \times \cos(2x) + \int_{\frac{\pi}{2}}^{20} (+\cos(2x)) dx$$

$$\int 20x\sin(2x)\,dx = \underline{\hspace{1cm}}$$

37. The velocity of a cyclist during an hour-long race is given by the function

$$v(t) = 166te^{-2.2t}$$
 mi/hr,  $0 \le t \le 1$ 

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?

① 
$$\int 166+e^{-2.2+}dt$$
 $u=166t$ 
 $dv=e^{-2.2+}dt$ 
 $v=e^{-2.2+}dt$ 
 $v=e^{-2.2+}$ 

38. After t days, the growth of a plant is measured by the function  $2000te^{-20t}$  inches per day. What is the change in the height of the plant (in inches) after the first 14 days?

$$\frac{u=2000+}{du=2000d+} \frac{dv=e^{-20t}dt}{v=e^{-20t}} uv-\int vdu$$

$$= 2000+\left(\frac{e^{-20t}}{-20}\right)+\int \left(\frac{e^{-20t}}{+20}\right) 2000dt$$

$$= -100 + e^{-20t} + 100 \int e^{-20t}dt$$

$$= -100 + e^{-20t} + 100 \left(\frac{e^{-20t}}{-20}\right)$$

$$= \left(-100 + e^{-20t} - 5 e^{-20t}\right) \left[\frac{14}{0}\right]$$

