Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name: Solutions

1. Evaluate the definite integral.

$$\int_0^{\pi/2} (x-1)\sin(x)\,dx$$

$\dfrac{u = x-1}{du = dx}$ $\dfrac{dv = \sin(x)\,dx}{v = -\cos(x)}$ $uv - \int v\,du = -(x-1)\cos x\Big]_0^{\pi/2} - \int_0^{\pi/2} (-\cos x)\,dx$

$= -(x-1)\cos x\Big]_0^{\pi/2} + \sin(x)\Big]_0^{\pi/2}$

$= -\cancel{\left(\frac{\pi}{2}-1\right)\cos\left(\frac{\pi}{2}\right)}^{\,0} - \left[-(0-1)\cos(0)\right]$

$+ \sin\left(\frac{\pi}{2}\right) - \cancel{\sin(0)}^{\,0}$

$= -1 + 1 = 0$

$\int_0^{\pi/2} (x-1)\sin(x)\,dx =$ $\boxed{0}$

2. Evaluate

$$\int 3x\ln(x^7)\,dx$$

Rewrite $\int 3x(7\ln(x))\,dx = \int 21x\ln x\,dx$

$\dfrac{u = 21\ln(x)}{du = \frac{21}{x}\,dx}$ $\dfrac{dv = x\,dx}{v = \frac{x^2}{2}}$ $uv - \int v\,du$

$= \dfrac{21x^2\ln x}{2} - \int \dfrac{x^2}{2}\cdot\dfrac{21}{x}\,dx$

$= \dfrac{21x^2\ln x}{2} - \int \dfrac{21}{2}x\,dx$

$= \dfrac{21x^2\ln x}{2} - \dfrac{21}{2}\cdot\dfrac{x^2}{2} + C$

$= \dfrac{21x^2\ln x}{2} - \dfrac{21x^2}{4} + C$

$\int 3x\ln(x^7)\,dx =$ $\boxed{\dfrac{21x^2\ln x}{2} - \dfrac{21x^2}{4} + C}$

3. Evaluate

$$\int x^3 \ln(2x)\, dx$$

$\dfrac{u = \ln(2x)}{du = \frac{1}{2x} \cdot 2\,dx}$ $\dfrac{dv = x^3 dx}{v = \frac{x^4}{4}}$ $uv - \int v\,du = \dfrac{x^4 \ln(2x)}{4} - \int \dfrac{x^4}{4} \cdot \dfrac{1}{x} dx$

$du = \frac{1}{x} dx$

$= \dfrac{x^4 \ln(2x)}{4} - \dfrac{1}{4} \int x^3 dx$

$= \dfrac{x^4 \ln(2x)}{4} - \dfrac{1}{4} \cdot \dfrac{x^4}{4} + c$

$$\int x^3 \ln(2x)\, dx = \boxed{\frac{x^4}{4} \ln(2x) - \frac{x^4}{16} + c}$$

4. Evaluate the definite integral.

$$\int_0^3 5xe^{3x}\, dx$$

$\dfrac{u = x}{du = dx}$ $\dfrac{dv = e^{3x} dx}{v = \frac{1}{3} e^{3x}}$ $uv - \int v\,du$

$= \dfrac{x}{3} e^{3x} - \int \dfrac{1}{3} e^{3x} dx$

$= \left(\dfrac{x}{3} e^{3x} - \dfrac{1}{3} \cdot \dfrac{e^{3x}}{3} \right) \Big]_0^3$

$= \dfrac{3}{3} e^9 - \dfrac{1}{9} e^9 - \left[0 - \dfrac{1}{9} \right]$

$= \dfrac{2}{9} e^9 + \dfrac{1}{9}$

$$\int_0^3 5xe^{3x}\, dx = \boxed{\frac{2}{9} e^9 + \frac{1}{9}}$$

5. Evaluate the indefinite integral.

$$\int_0^{\pi/4} 5x\sin(2x)\,dx$$

$u=5x$ $\quad dv=\sin(2x)\,dx$ $\quad uv-\int v\,du$

$du=5\,dx$ $\quad v=-\frac{\cos(2x)}{2}$

$=-\frac{5x\cos(2x)}{2}-\int -\frac{5\cos(2x)}{2}\,dx$

$=-\frac{5x\cos(2x)}{2}+\frac{5}{2}\int\cos(2x)\,dx$

$=\left(-\frac{5x\cos(2x)}{2}+\frac{5}{2}\,\frac{\sin(2x)}{2}\right)\Big]_0^{\pi/4}$

$=-\frac{5(\pi/4)\cos(2\pi/4)}{2}+\frac{5\sin(2\pi/4)}{4}-\left(\frac{5(0)\cos(0)}{2}+\frac{5}{4}\sin(0)\right)$

$=5/4$

$\int_0^{\pi/4} 5x\sin(2x)\,dx =$ $\boxed{5/4}$

$u-5=2t$

6. Evaluate the indefinite integral.

$$\int 4t\sqrt{2t+5}\,dt$$

$u=2t+5$

$du=2\,dt$

$\frac{du}{2}=dt$

$\int 4t\,u^{1/2}\frac{du}{2}=\int 2t\,u^{1/2}\,du=\int(u-5)u^{1/2}\,du$

$=\int u^{3/2}-5u^{1/2}\,du$

$=\frac{2}{5}u^{5/2}-5\cdot\frac{2}{3}u^{3/2}+C$

$=\frac{2}{5}(2t+5)^{5/2}-\frac{10}{3}(2t+5)^{3/2}+C$

$\int 4t\sqrt{2t+5}\,dt =$

$\boxed{\frac{2}{5}(2t+5)^{5/2}-\frac{10}{3}(2t+5)^{3/2}+C}$

7. The velocity of a cyclist during an hour-long race is given by the function

$$v(t) = 166te^{-2.2t} \text{ mi/hr}, \qquad 0 \le t \le 1$$

Assuming the cyclist starts from rest, what is the distance in miles he traveled during the first hour of the race?

① $\int 166te^{-2.2t}\,dt$

$u = 166t \quad dv = e^{-2.2t}\,dt \quad uv - \int v\,du$

$du = 166\,dt \quad v = \dfrac{e^{-2.2t}}{-2.2}$

$= \dfrac{166t\,e^{-2.2t}}{-2.2} + \int \dfrac{e^{-2.2t}}{+2.2}\cdot 166\,dt$

$= -\dfrac{166te^{-2.2t}}{2.2} + \dfrac{166}{2.2}\cdot\dfrac{e^{-2.2t}}{-2.2} + C$

$= \dfrac{-166te^{-2.2t}}{2.2} - \dfrac{166e^{-2.2t}}{(2.2)^2} + C$

② $s(0) = 0$. Find C.

$0 = 0 - \dfrac{166}{(2.2)^2} + C \longrightarrow C = \dfrac{166}{(2.2)^2}$

③ $s(t) = \dfrac{-166te^{-2.2t}}{2.2} - \dfrac{166e^{-2.2t}}{(2.2)^2} + \dfrac{166}{(2.2)^2}$

$s(1) = -\dfrac{166}{22}e^{-2.2} - \dfrac{166}{(2.2)^2}e^{-2.2} + \dfrac{166}{(2.2)^2}$

≈ 22.137

Answer: $\boxed{22.137}$

8. After t days, the growth of a plant is measured by the function $2000te^{-20t}$ inches per day. What is the change in the height of the plant (in inches) after the first 14 days?

$$\int_0^{14} 2000te^{-20t}\,dt$$

$$\frac{u = 2000t}{du = 2000\,dt} \quad \frac{dv = e^{-20t}\,dt}{v = \frac{e^{-20t}}{-20}} \quad uv - \int v\,du$$

$$= 2000t\left(\frac{e^{-20t}}{-20}\right) + \int \left(\frac{e^{-20t}}{+20}\right) 2000\,dt$$

$$= -100\,te^{-20t} + 100 \int e^{-20t}\,dt$$

$$= -100\,te^{-20t} + 100\left(\frac{e^{-20t}}{-20}\right)$$

$$= \left(-100\,te^{-20t} - 5\,e^{-20t}\right)\Big]_0^{14}$$

$$= 5$$

Answer: $\boxed{5}$

9. A model for the ability of a child to memorize information, measured on a scale from 1 to 100, is given by

$$M(t) = 1.9t\ln(t),$$

$2 \leq t \leq 8$, where t is the child's age in years. Find the child's average memorization ability between ages 2 and 7 years. Round to threee decimal places.

Average:

$$M_{AVE}(t) = \frac{1}{7-2}\int_2^7 1.9t\ln(t)\,dt$$

$$= \int_2^7 \frac{1.9}{5} t\ln(t)\,dt$$

$$u = \frac{1.9}{5}\ln(t) \qquad dv = t\,dt \qquad uv - \int v\,du$$

$$du = \frac{1.9}{5}\cdot\frac{1}{t}\,dt \qquad v = \frac{t^2}{2}$$

$$= \frac{1.9}{5}\ln(t)\left(\frac{t^2}{2}\right) - \int \frac{1.9}{5}\left(\frac{1}{t}\right)\left(\frac{t^2}{2}\right)dt$$

$$= \frac{19}{100}t^2\ln(t) - \frac{19}{100}\int t\,dt$$

$$= \left[\frac{19}{100}t^2\ln(t) - \frac{19}{100}\left(\frac{t^2}{2}\right)\right]_2^7$$

$$\approx 13.315$$

Answer: $\boxed{13.315}$

10. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{3x+1}{x^2(x+1)^2(x^2+1)}$$

(A) $\dfrac{A}{x^2} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x^2+1}$

(B) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2} + \dfrac{E}{x^2+1}$

(C) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2} + \dfrac{Ex+F}{x^2+1}$

(D) $\dfrac{A}{x} + \dfrac{Bx+C}{x^2} + \dfrac{D}{x+1} + \dfrac{Ex+F}{(x+1)^2} + \dfrac{Gx+H}{x^2+1}$

(E) $\dfrac{A}{x} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x^2+1}$

11. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{7x-5}{x^2(x^2+9)}$$

(A) $\dfrac{A}{x} + \dfrac{B}{x} + \dfrac{Cx+D}{x^2+9}$

(B) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx+D}{x^2+9}$

(C) $\dfrac{A}{x} + \dfrac{Bx+C}{x^2} + \dfrac{Dx+E}{x^2+9}$

(D) $\dfrac{Ax+B}{x^2} + \dfrac{Cx+D}{x^2+9}$

(E) $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{x-3}$

(F) $\dfrac{Ax+B}{x^2} + \dfrac{C}{x+3} + \dfrac{D}{x-3}$

12. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{x^2+2x+3}{(x-1)^2(x-2)(x^2+4)}$$

(A) $\boxed{\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx+E}{x^2+4}}$

(B) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{D}{x^2+4}$

(C) $\dfrac{A}{x-1} + \dfrac{Bx+C}{(x-1)^2} + \dfrac{D}{x-2} + \dfrac{E}{x^2+4}$

(D) $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx}{x^2+4}$

(E) $\dfrac{A}{x-1} + \dfrac{Bx}{(x-1)^2} + \dfrac{C}{x-2} + \dfrac{Dx+E}{x^2+4}$

13. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{24}{(x^2-16)^2}$$

(A) $\dfrac{A}{x-4} + \dfrac{Bx+C}{(x-4)^2} + \dfrac{D}{x+4} + \dfrac{Ex+F}{(x+4)^2}$

(B) $\dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{x-2} + \dfrac{D}{(x-2)^2} + \dfrac{E}{x+4} + \dfrac{F}{(x+4)^2}$

(C) $\dfrac{Ax+B}{(x-4)^2} + \dfrac{Cx+D}{(x+4)^2}$

(D) $\boxed{\dfrac{A}{x-4} + \dfrac{B}{(x-4)^2} + \dfrac{C}{x+4} + \dfrac{D}{(x+4)^2}}$

(E) $\dfrac{Ax+B}{x^2-16} + \dfrac{Cx+D}{(x^2-16)^2}$

(F) $\dfrac{A}{(x^2-16)^2} + \dfrac{Bx+C}{(x^2-16)^2}$

<u>Note:</u>

$$(x^2-16)^2 = [(x-4)(x+4)]^2 = (x-4)^2(x+4)^2$$

14. Determine the partial fraction decomposition of

$$\frac{7x^2+9}{x(x^2+3)}$$

$$\frac{A}{x} + \frac{Bx+C}{x^2+3} = \frac{A(x^2+3) + x(Bx+C)}{x(x^2+3)}$$

$$= \frac{Ax^2+3A+Bx^2+Cx}{x(x^2+3)}$$

$$= \frac{(A+B)x^2+Cx+3A}{x(x^2+3)}$$

$$(A+B)x^2+Cx+3A = 7x^2+0x+9$$

$$\begin{cases} A+B=7 \\ C=0 \\ 3A=9 \to A=3 \end{cases}$$

So $B=4$

Answer: $\boxed{\frac{3}{x} + \frac{4x}{x^2+3}}$

15. Determine the partial fraction decomposition of

$$\frac{4x-11}{x^2-7x+10}$$

Factor $x^2-7x+10=(x-2)(x-5)$

$$\frac{4x-11}{(x-2)(x-5)}=\frac{A}{x-2}+\frac{B}{x-5}$$

$$=\frac{A(x-5)+B(x-2)}{(x-2)(x-5)}$$

$$=\frac{(A+B)x+(-5A-2B)}{(x-2)(x-5)}$$

So $4x-11=(A+B)x+(-5A-2B)$

$$\begin{cases} 4=A+B & (\text{i}) \\ -11=-5A-2B & (\text{ii}) \end{cases}$$

Multiply (i) by 5 and add (i) + (ii).

$$\begin{array}{rl} & 20=\cancel{5A}+5B \\ + & -11=\cancel{-5A}-2B \\ \hline & 9=3B \end{array}$$

$B=3$

Plug $B=3$ into (i)

$4=A+B$

$4=A+3$

$A=1$

Answer: $\boxed{\frac{1}{x-2}+\frac{3}{x-5}}$

16. Evaluate $\displaystyle\int \frac{5x^2+9}{x^2(x+3)}\,dx$

$$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}=\frac{Ax(x+3)+B(x+3)+Cx^2}{x^2(x+3)}$$

$$=\frac{Ax^2+3Ax+Bx+3B+Cx^2}{x^2(x+3)}$$

$$=\frac{(A+C)x^2+(3A+B)x+3B}{x^2(x+3)}$$

$$(A+C)x^2+(3A+B)x+3B=5x^2+0x+9$$

$$\begin{cases} A+C=5 \\ 3A+B=0 \\ 3B=9 \to B=3 \end{cases}$$

$$\begin{aligned} 3A+B&=0 \\ 3A+3&=0 \\ 3A&=-3 \\ A&=-1 \end{aligned} \qquad \begin{aligned} A+C&=5 \\ -1+C&=5 \\ C&=6 \end{aligned}$$

$$\int -\frac{1}{x}\,dx+\int \frac{3}{x^2}\,dx+\int \frac{6}{x+3}\,dx = -\ln|x|-\frac{3}{x}+6\ln|x+3|+c$$

$$\boxed{-\ln|x|-\frac{3}{x}+6\ln|x+3|+c}$$

$\displaystyle\int \frac{5x^2+9}{x^2(x+3)}\,dx=$ ______

17. Evaluate $\int \frac{x^2+2}{x^3+3x^2+2x}\,dx$

Factor $x^3+3x^2+2x = x(x^2+3x+2) = x(x+1)(x+2)$

So $\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x+2} = \frac{A(x+1)(x+2)+Bx(x+2)+Cx(x+1)}{x(x+1)(x+2)}$

$= \frac{A(x^2+3x+2)+B(x^2+2x)+C(x^2+x)}{x(x+1)(x+2)}$

$= \frac{(A+B+C)x^2+(3A+2B+C)x+2A}{x(x+1)(x+2)}$

So $x^2+2 = (A+B+C)x^2+(3A+2B+C)x+2A$

$1\cdot x^2+0x+2 = (A+B+C)x^2+(3A+2B+C)x+2A$

$$\begin{cases} 1 = A+B+C & \text{(i)} \\ 0 = 3A+2B+C & \text{(ii)} \\ 2 = 2A & \text{(iii)} \end{cases}$$

Solve (iii).

$2 = 2A$

$A = 1$

Plug $A=1$ into (i) and (ii).

$$\begin{cases} 1 = 1+B+C & \text{(i)} \\ 0 = 3+2B+C & \text{(ii)} \end{cases}$$

Subtract the eqns.

$$\begin{array}{r} 1 = 1+B+\cancel{C} \\ -(0 = 3+2B+\cancel{C}) \\ \hline 1 = -2-B \\ +2 \quad +2 \\ \hline 3 = -B \end{array}$$

$B = -3$

Plug $B=-3$ into (i).

$1 = 1+B+C$

$1 = 1-3+C$

$1 = -2+C$

$3 = C$

Plug $A=1$, $B=-3$, $C=3$ into decomposition.

$\frac{1}{x}+\frac{-3}{x+1}+\frac{3}{x+2}$

So $\int \frac{1}{x}\,dx + \int \frac{-3}{x+1}\,dx + \int \frac{3}{x+2}\,dx$

$$\int \frac{x^2+2}{x^3+3x^2+2x}\,dx = \boxed{\ln|x| - 3\ln|x+1| + 3\ln|x+2| + C}$$

18. Evaluate $\displaystyle\int \frac{9x^2-4x+5}{(x-1)(x^2+1)}\,dx$

	Bx	C
x	Bx^2	Cx
-1	$-Bx$	$-C$

So $\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}=\dfrac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$

$$=\frac{Ax^2+A+Bx^2-Bx+Cx-C}{(x-1)(x^2+1)}$$

$$=\frac{(A+B)x^2+(C-B)x+(A-C)}{(x-1)(x^2+1)}$$

So $\begin{cases} A+B=9 & \text{(i)} \\ C-B=-4 & \text{(ii)} \\ A-C=5 & \text{(iii)} \end{cases}$

Add (i) and (ii)

$$\begin{array}{rl} & A+\cancel{B}=9 \\ + & -\cancel{B}+C=-4 \\ \hline & A+C=5 \quad \text{(iv)} \end{array}$$

Add (iii) and (iv)

$$\begin{array}{rl} & A-\cancel{C}=5 \\ + & A+\cancel{C}=5 \\ \hline & 2A=10 \\ & A=5 \end{array}$$

Plug $A=5$ into (i)

$A+B=9$

$5+B=9$

$B=4$

Plug $A=5$ into (iii)

$A-C=5$

$5-C=5$

$C=0$

So $\dfrac{5}{x-1}+\dfrac{4x}{x^2+1}$

$$\int\frac{5}{x-1}\,dx+\underbrace{\int\frac{4x}{x^2+1}\,dx}_{\substack{u=x^2+1 \\ du=2x\,dx}}$$

$$=5\ln|x-1|+2\ln|x^2+1|+C$$

$\displaystyle\int \frac{x^2+2}{x^3+3x^2+2x}\,dx = \boxed{5\ln|x-1|+2\ln|x^2+1|+C}$

19. Evaluate $\int \frac{3x^2+3x+15}{x^3+5x^2}\,dx$

① Factor denominator

$x^3+5x^2=x^2(x+5)$

② Decomposition

$\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+5}$

③ Combine fractions

$\frac{Ax(x+5)+B(x+5)+Cx^2}{x^2(x+5)}$

$=\frac{(A+C)x^2+(5A+B)x+(5B)}{x^2(x+5)}$

④ Old numerator = New numerator

$3x^2+3x+15=(A+C)x^2+(5A+B)x+5B$

⑤ System of eqn based on ④

$$\begin{cases} 3=A+C & (i) \\ 3=5A+B & (ii) \\ 15=5B & (iii) \end{cases}$$

Solve (iii)

$15=5B \Rightarrow B=3$

Plug $B=3$ into (ii)

$3=5A+3$

$0=5A$

$A=0$

Plug $A=0$ into (i)

$3=0+C$

$C=3$

⑥ Plug $A=0, B=3, C=3$ into ②

$\frac{3}{x^2}+\frac{3}{x+5}$

$=\int \frac{3}{x^2}+\frac{3}{x+5}\,dx$

$=\int 3x^{-2}+\frac{3}{x+5}\,dx$

$=-3x^{-1}+3\ln|x+5|+C$

$=-\frac{3}{x}+3\ln|x+5|+C$

$\int \frac{3x^2+3x+15}{x^3+5x^2}\,dx = \boxed{-\frac{3}{x}+3\ln|x+5|+C}$

20. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \frac{\sin x}{1-\cos x}\,dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$1 - \cos x = 0$

$1 = \cos x$

$x = 0, \pi, 2\pi$

21. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \tan(x)\,dx$$

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

$\tan x = \frac{\sin x}{\cos x}$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

22. Determine if the following integral is proper or improper.

$$\int_0^{\pi/2} \cos(x)\,dx$$

$\to$ cos(x) is defined everywhere.

(A) It is improper because of a discontinuity at $x = \pi/6$
(B) It is improper because of a discontinuity at $x = \pi/4$
(C) It is improper because of a discontinuity at $x = \pi/3$
(D) It is improper because of a discontinuity at $x = 0$
(E) It is improper because of a discontinuity at $x = \pi/2$
(F) It is proper since it is defined on the interval $[0, \pi/2]$.

Bonus do this question w/ all trig functions

23. Which of the following integrals are diverges?

I. $\displaystyle\int_1^\infty \frac{5}{\sqrt{x}}\,dx$ II. $\displaystyle\int_1^\infty \frac{3}{x^2}\,dx$ III. $\displaystyle\int_1^\infty \frac{10}{x}\,dx$

(A) I only

(B) II only

(C) III only

(D) I and II only

$\boxed{\text{(E) I and III only}}$

(F) I, II, and III

I $\int_1^\infty 5x^{-1/2}dx = 5\cdot\frac{2}{1}x^{1/2}\Big]_1^\infty = \infty$

$\Rightarrow$ diverges

II $\int_1^\infty 3x^{-2}dx = -3x^{-1}\Big]_1^\infty = -\frac{3}{x}\Big]_1^\infty = 0+3$

III $\int_1^\infty \frac{10}{x}dx = 10\ln|x|\Big]_1^\infty = \infty$

$\Rightarrow$ diverges

24. Which of the following integrals are improper?

I. $\displaystyle\int_0^{\pi/4} \cos(x)\,dx$ II. $\displaystyle\int_0^{\pi/4} \tan(2x)\,dx$ III. $\displaystyle\int_{\pi/4}^{\pi/2} \csc(x)\,dx$ IV. $\displaystyle\int_{\pi/4}^{\pi/2} \sec\left(\frac{x}{2}\right)\,dx$

$\boxed{\text{(A) II and IV only}}$

(B) I and II only

(C) I and IV only

(D) I and III only

(E) II, III and IV only

(F) II only

I $\cos(x)$ is defined everywhere $\Rightarrow$ proper

II $\tan(2x) = \frac{\sin(2x)}{\cos(2x)} \Rightarrow \cos(2x)=0$ when $2x = \frac{\pi}{2}$

$x = \pi/4$

and $\pi/4$ is a bound $\Rightarrow$ improper

III $\csc(x) = \frac{1}{\sin(x)} \Rightarrow \sin(x)=0$ when $x = 0, \pi, 2\pi, \ldots$

which are none of the bounds $\Rightarrow$ proper

IV $\sec(x/2) = \frac{1}{\cos(x/2)} \Rightarrow \cos\left(\frac{x}{2}\right)=0$ when $\frac{x}{2} = \frac{\pi}{2}$

$x = \pi/4$

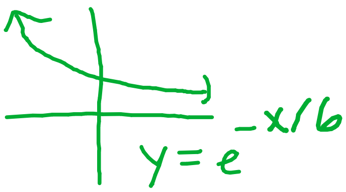
and $\pi/4$ is a bound $\Rightarrow$ improper

25. Evaluate the following integral;

$$\int_0^\infty e^{-x/6}\,dx$$

$$= \lim_{N\to\infty} \int_0^N e^{-x/6}\,dx = \lim_{N\to\infty} \left(-6e^{-x/6}\right)\Big]_0^N$$

$$= \lim_{N\to\infty} \left(-6e^{-N/6} + 6\right) = 6$$



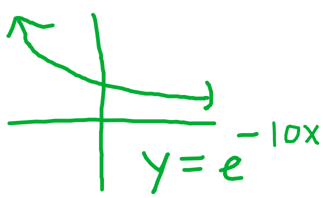
$$\int_0^\infty e^{-x/6}\,dx \quad = \quad \boxed{6}$$

26. Evaluate the following integral;

$$\int_0^\infty \frac{7}{e^{10x}}\,dx$$

$$\int_0^\infty 7e^{-10x}\,dx = \lim_{N\to\infty} \int_0^N 7e^{-10x}\,dx = \lim_{N\to\infty} \left(7\frac{e^{-10x}}{-10}\right)\Big]_0^N$$

$$= \lim_{N\to\infty} \left(\frac{7e^{-10N}}{-10} + \frac{7}{10}\right) = 0 + \frac{7}{10}$$



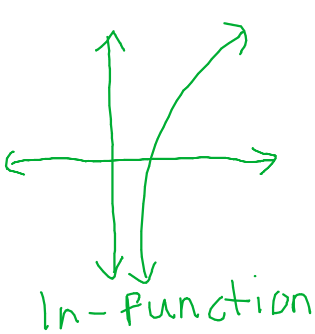
$$\int_0^\infty \frac{7}{e^{10x}}\,dx \quad = \quad \boxed{7/10}$$

27. Evaluate the definite integral

$$\int_2^\infty \frac{dx}{5x+2}$$

$$\lim_{N\to\infty} \int_2^N \frac{dx}{5x+2} \quad \frac{u = 5x+2}{du = 5\,dx}, \ \frac{du}{5} = dx \quad \lim_{N\to\infty} \int \frac{1}{5}\frac{1}{u}\,du = \lim_{N\to\infty} \frac{1}{5}\ln|u| = \lim_{N\to\infty} \frac{1}{5}\ln|5x+2|\Big]_2^N$$

$$= \lim_{N\to\infty} \left(\frac{1}{5}\ln|5N+2| - \frac{1}{5}\ln|12|\right) = \infty$$



$$\int_2^\infty \frac{dx}{5x+2} = \boxed{\infty}$$

28. Evaluate the definite integral

$$\int_4^{13} \frac{dx}{\sqrt{x-4}}$$

$= \int_4^{13} (x-4)^{-1/2}\,dx$

$= 2(x-4)^{1/2}\Big]_4^{13}$

$= 2(13-4)^{1/2} - 2(4-4)^{1/2}$

$= 6$

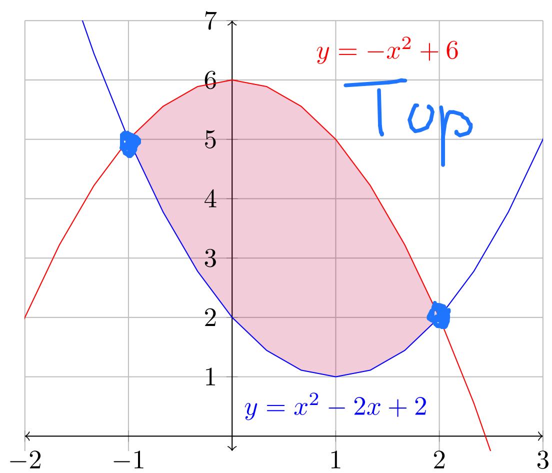
$$\int_4^{13} \frac{dx}{\sqrt{x-4}} = \boxed{6}$$

29. Set up the integral that computes the **AREA** shown to the right with respect to x.

DON'T COMPUTE IT!!!

$$\boxed{\int_{-1}^{2} (-x^2+6) - (x^2 - 2x + 2)\,dx}$$

Area = ____________________



30. Set up the integral that computes the **AREA** shown to the right with respect to y. $\to dy$

DON'T COMPUTE IT!!!

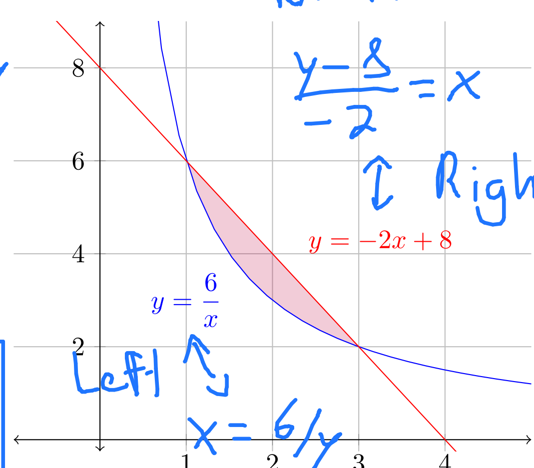
$\frac{y-8}{-2} = x$

Right

Left

$x = 6/y$

Area $= \boxed{\int_2^6 \left(\frac{y-8}{-2}\right) - \frac{6}{y}\,dy}$



31. Set up the integral that computes the **AREA** with respect to x of the region bounded by

$$y = \frac{2}{x} \quad \text{and} \quad y = -x + 3$$

$\Rightarrow$ dx problem

Bounds:

$\frac{2}{x} = -x+3$

$2 = -x^2 + 3x$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$x = 1, 2$

Test Pt: $x = 1.5$

$y = \frac{2}{x} \Rightarrow y = \frac{2}{1.5} = \frac{4}{3} \approx 1.33 \rightarrow$ Bottom

$y = -x+3 \Rightarrow y = -1.5+3 = 1.5 \rightarrow$ Top

Area $= \boxed{\int_1^2 \left(-x+3-\frac{2}{x}\right) dx}$

32. Set up the integral that computes the **AREA** with respect to x of the region bounded by

$$y = x \quad \text{and} \quad y = 7x - x^2$$

Bounds: $x = 7x - x^2$

$0 = 6x - x^2$

$0 = x(6-x)$

$x = 0, 6$

Test Pt: $x = 1$

$y = x \xrightarrow{x=1} y = 1 \rightarrow$ Small

$y = 7x - x^2 \xrightarrow{x=1} y = 6 \rightarrow$ Big

$A = \int_0^6 (7x - x^2) - x \, dx$

$= \int_0^6 (6x - x^2) dx$

$= \left(3x^2 - \frac{x^3}{3}\right)\Big]_0^6$

$= 36$

Area $= \boxed{36}$

33. Find the area of the region bounded by $y = 6x - x^2$ and $y = 2x^2$.

Bounds:

$6x - x^2 = 2x^2$

$6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0, 2$

Test Pt: $x = 1$

$y = 6x - x^2 \Rightarrow y = 5 \rightarrow$ Top

$y = 2x^2 \Rightarrow y = 2 \rightarrow$ Bottom

$$A = \int_0^2 \left[(6x - x^2) - 2x^2\right] dx$$

$$= \int_0^2 (6x - 3x^2)\, dx$$

$$= \left(3x^2 - x^3\right)\Big]_0^2 = 4$$

Area = $\boxed{4}$

34. Find the area bounded by the following curves.

$$x = y^2 + 24 \quad \text{and} \quad x = 10y$$

Bounds: $y^2 + 24 = 10y$

$y^2 - 10y + 24 = 0$

$(y - 6)(y - 4) = 0$

$y = 4, 6$

Test Pt: $y = 5$

$x = y^2 + 24 \xrightarrow{y=5} x = 49 \rightarrow$ Small

$x = 10y \xrightarrow{y=5} x = 50 \rightarrow$ Big

$$A = \int_4^6 10y - (y^2 - 24)\, dy$$

$$= \int_4^6 10y - y^2 + 24\, dy$$

$$= \left(5y^2 - \frac{y^3}{3} + 24y\right)\Big]_4^6$$

$$= \frac{292}{3}$$

Area = $\boxed{292/3}$

35. Find the area of the region bounded by $y = 2x - x^2$ and $y = x^2$.

Bounds:

$2x - x^2 = x^2$

$2x - 2x^2 = 0$

$2x(x-1) = 0$

$x = 0, 1$

$A = \int_0^1 (2x - x^2) - x^2 \, dx$

$= \int_0^1 2x - 2x^2 \, dx$

$= \left(\frac{2x^2}{2} - \frac{2x^3}{3}\right)\Big]_0^1 = \frac{1}{3}$

Test Pt: $x = 1/2$

$y = 2x - x^2 \longrightarrow y\left(\frac{1}{2}\right) = 3/4 \to$ Top

$y = x^2 \longrightarrow y\left(\frac{1}{2}\right) = 1/4 \to$ Bottom

Area = $\boxed{1/3}$

36. Calculate the **AREA** of the region bounded by the following curves.

$$x = 100 - y^2 \quad \text{and} \quad x = 2y^2 - 8$$

Bounds:

$100 - y^2 = 2y^2 - 8$

$108 = 3y^2$

$36 = y^2$

$y = \pm 6$

$A = \int_{-6}^{6} (100 - y^2) - (2y^2 - 8) \, dy$

$= \int_{-6}^{6} (108 - 3y^2) \, dy$

$= (108y - y^3)\Big]_{-6}^{6}$

$= 864$

Test Pt: $y = 0$

$x = 100 - y^2 \longrightarrow x = 0 \to$ Right

$x = 2y^2 - 8 \to x = -8 \to$ Left

Area = $\boxed{864}$

37. Calculate the **AREA** of the region bounded by the following curves.

$$y = x^3 \quad \text{and} \quad y = x^2$$

Bounds:

$x^3 = x^2$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x = 0, 1$

Test Pt: $x = \frac{1}{2}$

$y = x^3 \rightarrow y = \frac{1}{8} \rightarrow$ Bottom

$y = x^2 \rightarrow y = \frac{1}{4} \rightarrow$ Top

$A = \int_0^1 (x^2 - x^3)\,dx$

$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big]_0^1$

$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

Area = $\boxed{1/12}$

38. After t hours studying, one student is working $Q_1(t) = 25 + 9t - t^2$ problems per hour, and a second student is working on $Q_2(t) = 5 - t + t^2$ problems per hour. How many more problems will the first student have done than the second student after 10 hours?

$\int_0^{10} Q_1(t) - Q_2(t)\,dt$

$= \int_0^{10} (25 + 9t - t^2) - (5 - t + t^2)\,dt$

$= \int_0^{10} (20 + 10t - 2t^2)\,dt$

$= \left(20t + 5t^2 - \frac{2}{3}t^3\right)\Big]_0^{10}$

$= \frac{100}{3}$

Answer: $\boxed{100/3}$

39. The birthrate of a particular population is modeled by $B(t) = 1000e^{0.036t}$ people per year, and the death rate is modeled by $D(t) = 725e^{0.019t}$ people per year. How much will the population increase in the span of 10 years? $(0 \leq t \leq 20)$ Round to the nearest whole number.

$$\int_0^{10} B(t) - D(t)\,dt = \int_0^{10} 1000e^{0.036t} - 725e^{0.019t}\,dt$$

$$= \left(\frac{1000}{0.036}e^{0.036t} - \frac{725}{0.019}e^{0.019t}\right)\Bigg]_0^{10}$$

$$\approx 4052$$

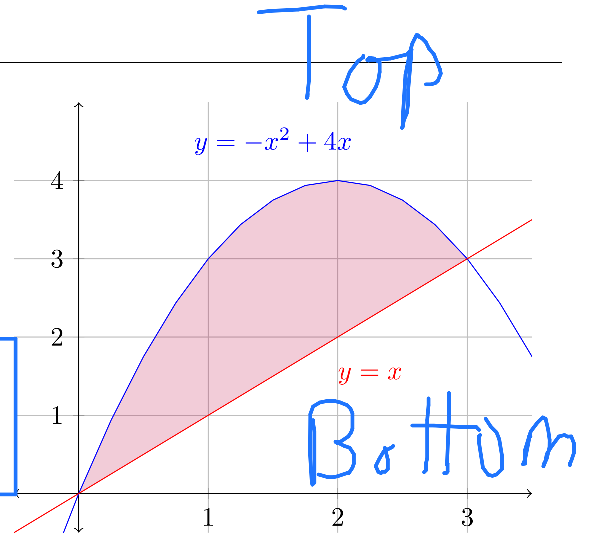
Answer: $\boxed{4052}$

40. Let R be the region shown below. Set up the integral that computes the **VOLUME** as R is rotated around the x-axis.

DON'T COMPUTE IT!!!

$$\boxed{\pi\int_0^3\left[(-x^2+4x)^2-(x)^2\right]dx}$$

Volume = ______________________



41. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = \sqrt{16-x}, \quad y = 0 \quad \text{and} \quad x = 0$$

about the y-axis $\Rightarrow$ dy problem

Disk

$y = \sqrt{16-x}$
$y^2 = 16 - x$
$x = 16 - y^2$

<u>Bounds</u>: Given $y=0$
Plug $x=0$ into $y=\sqrt{16-x}$
$y = \sqrt{16-x}$
$y = \sqrt{16}$
$y = 4$

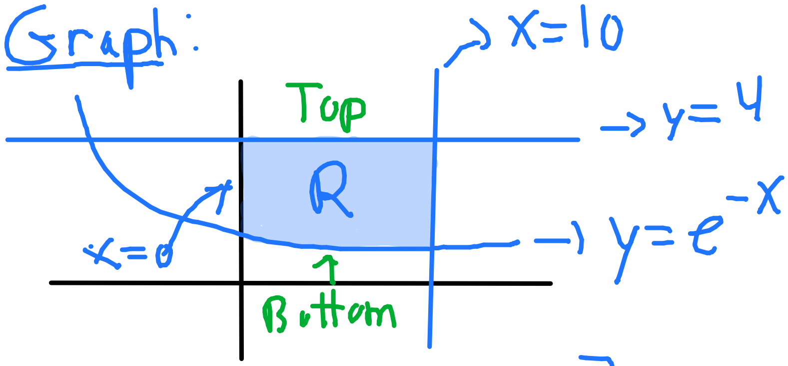
Volume = $\boxed{\pi\int_0^4 (16-y^2)^2\,dy}$

42. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = e^{-x}, \quad y = 4 \quad x = 0 \quad \text{and} \quad x = 10$$

about the x-axis $\Rightarrow dx$

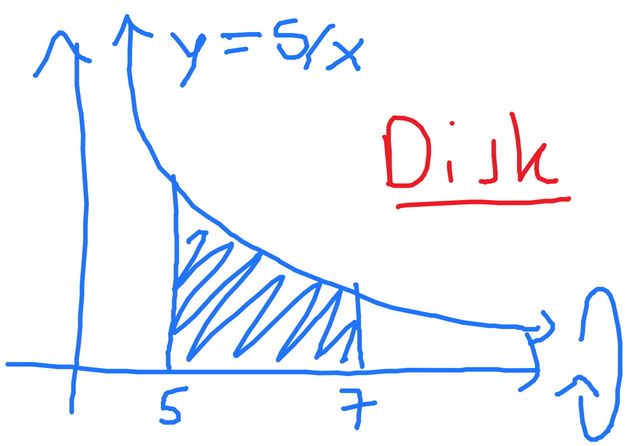
Graph:



$$V = \pi \int_0^2 \left[4^2 - (e^{-x})^2 \right] dx$$

Volume $=$ $\boxed{\pi \int_0^2 (16 - e^{-2x})\, dx}$

43. Find the volume of the solid that results by revolving the region enclosed by the curves $y = \dfrac{5}{x}$, $y = 0$, $x = 5$, and $x = 7$ about the x-axis. $\Rightarrow dx$



$$V = \pi \int_5^7 \left(\frac{5}{x}\right)^2 dx$$

$$= \pi \int_5^7 \frac{25}{x^2}\, dx$$

$$= 25\pi \int_5^7 x^{-2}\, dx$$

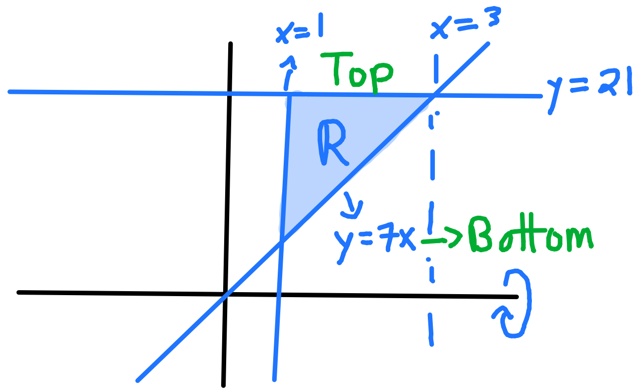
$$= 25\pi \left(-\frac{1}{x}\right)\Big]_5^7$$

Volume $=$ $\boxed{\dfrac{10\pi}{7}}$

44. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 21 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis $\to dx$



Washer

$$V = \pi\int_1^3 \left[21^2 - (7x)^2\right] dx$$

$$= \pi\int_1^3 (441 - 49x^2)\, dx$$

$$= \pi\left(441x - \frac{49x^3}{3}\right)\Big]_1^3$$

$$= \frac{1274}{3}\pi$$

Volume $=$ $\boxed{\dfrac{1274\pi}{3}}$

45. Find the **VOLUME** of the region bounded by

$$y = 7x, \quad y = 0 \quad x = 1 \quad \text{and} \quad x = 3$$

around the x-axis $\to dx$

Disk

$$V = \pi\int_1^3 (7x)^2 dx$$

$$= \pi\int_1^3 49x^2\, dx$$

$$= \pi\left(\frac{49x^3}{3}\right)\Big]_1^3$$

$$= \frac{49\pi}{3}(3^3 - 1)$$

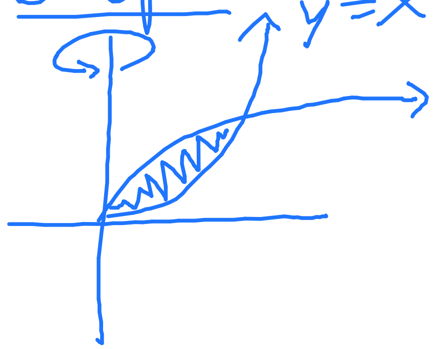
Volume $=$ $\boxed{\dfrac{1274}{3}\pi}$

46. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y = \sqrt{x}$$

about the y-axis

Graph:



Bounds:

$\sqrt{y} = y^2$

$y = y^4$

$0 = y^4 - y$

$0 = y(y^3 - 1)$

$y = 0, 1$

But y-axis ⇒ dy

Right → $y = x^2 \to x = \sqrt{y}$

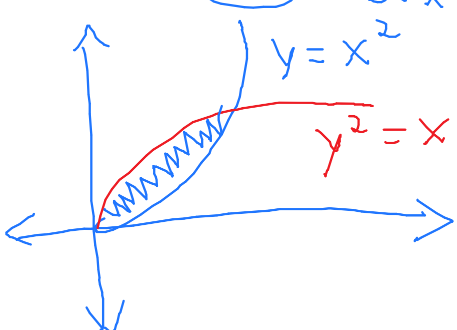
Left → $y = \sqrt{x} \to x = y^2$

Volume $= \boxed{\pi \int_0^1 \left[(\sqrt{y})^2 - (y^2)^2 \right] dy}$

47. Set up the integral that computes the **VOLUME** of the region bounded by

$$y = x^2, \quad \text{and} \quad y^2 = x$$

about the x-axis → dx problem



$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$

Bounds

$x^2 = \sqrt{x}$

$x^4 = x$

$x^4 - x = 0$

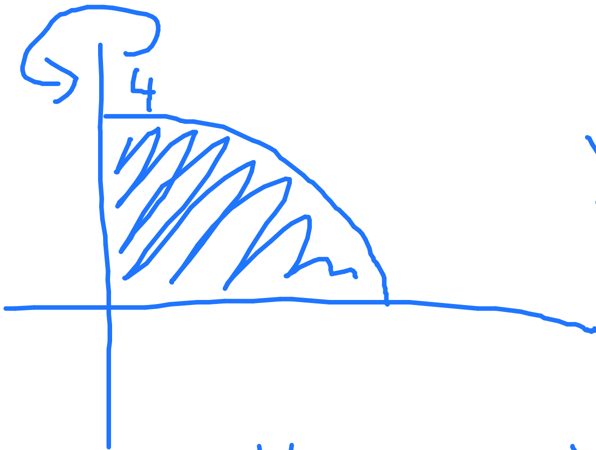
$x(x^3 - 1) = 0$

$x = 0, 1$

Volume $= \boxed{\pi \int_0^1 (x - x^4) \, dx}$

48. Set up the integral that computes the **VOLUME** of the region generated by revolving the region in Quadrant I bounded by the following curves about the y-axis using the disk/washer method. $\Rightarrow dy$

$$y = 4 - x^2, \qquad y = 0 \qquad \text{and} \qquad x = 0$$



$\Downarrow$

$$y + x^2 = 4$$

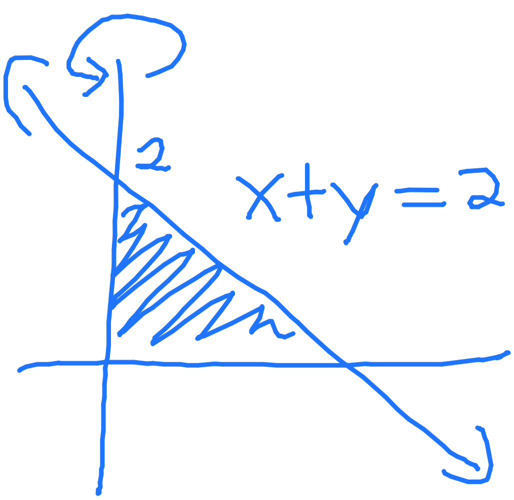
$$x^2 = 4 - y$$

$$x = \sqrt{4-y}$$

$$V = \pi \int_0^4 \left(\sqrt{4-y}\right)^2 dy$$

Volume $=$ $\boxed{\pi \int_0^4 (4-y)\, dy}$

49. Find the volume of the solid generated by revolving the region bounded by $x + y = 2$ in Quadrant I about the y-axis. $\Rightarrow dy$



$$x + y = 2 \rightarrow x = 2 - y$$

$$V = \pi \int_0^2 (2-y)^2 dy$$

$$= \pi \int_0^2 (y-2)^2 dy$$

$$= \pi \left. \frac{(y-2)^3}{3} \right]_0^2$$

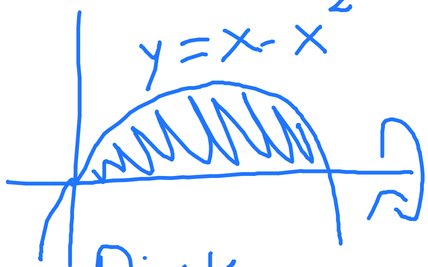
$$= \pi \left[\frac{(2-2)^3}{3} - \frac{(0-2)^3}{3} \right]$$

Volume $=$ $\boxed{8\pi/3}$

50. Find the **VOLUME** of the region bounded by

$$y = x - x^2, \quad \text{and} \quad y = 0$$

around the x-axis



Disk

Bounds:

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$

$V = \pi \int_0^1 (x - x^2)^2 \, dx$

$= \pi \int_0^1 (x^2 - 2x^3 + x^4) \, dx$

$= \pi \left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \Big]_0^1$

$= \frac{\pi}{30}$

Volume $=$ $\boxed{\pi/30}$

51. Find the **VOLUME** of the solid generate by revolving the given region about the x-axis: $\longrightarrow dx$

$$y = 8\sqrt{x}, \quad y = 0, \quad x = 3, \quad x = 6$$

$V = \pi \int_3^6 (8\sqrt{x})^2 \, dx$

$= \pi \int_3^6 64x \, dx$

$= \pi \frac{64x^2}{2} \Big]_3^6$

$= \pi \, 32x^2 \Big]_3^6$

$= 864\pi$

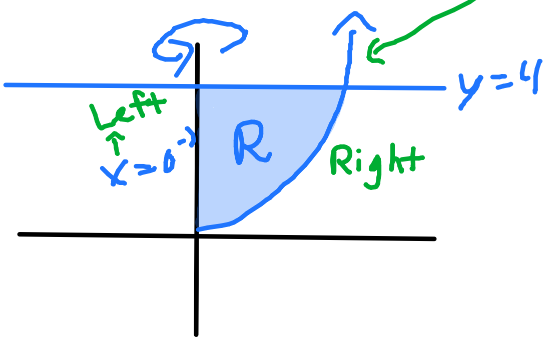
Volume $=$ $\boxed{864\pi}$

52. Find the **VOLUME** of the region bounded by

$$y = 4x^2, \quad x = 0, \quad y = 4$$

around the y-axis. → dy

$x = \sqrt{\frac{y}{4}}$



Dish

$$V = \pi \int_0^4 \left(\sqrt{\frac{y}{4}}\right)^2 dy$$
$$= \pi \int_0^4 \frac{y}{4}\, dy$$
$$= \frac{\pi}{4} \cdot \frac{y^2}{2}\Big]_0^4$$
$$= \frac{\pi}{8} \cdot 16$$
$$= 2\pi$$

Volume $=$ $\boxed{2\pi}$

53. Set up the integral that computes the **VOLUME** of the region bounded by

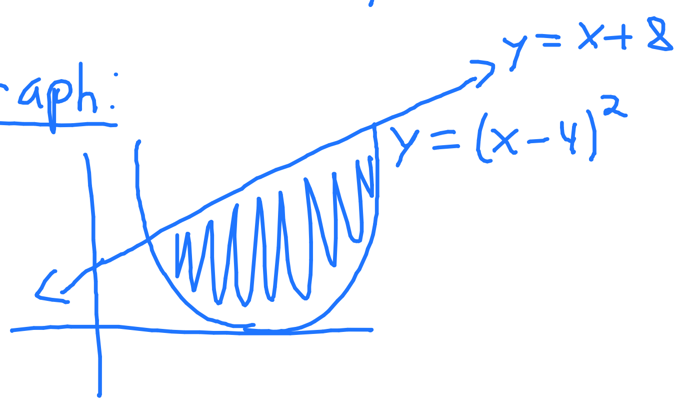
$$y = x + 8, \quad \text{and} \quad y = (x-4)^2$$

about the x-axis

Bounds:

$$x + 8 = (x-4)^2$$
$$x + 8 = x^2 - 8x + 16$$
$$0 = x^2 - 9x + 8$$
$$0 = (x-8)(x-1)$$
$$x = 1, 8$$

Graph:



Volume $=$ $\boxed{\pi \int_1^8 \left[(x+8)^2 - (x-4)^4\right] dx}$

54. Find the **VOLUME** of the region bounded by

$$y = 10x, \quad x = 0, \quad y = 10$$

around the y-axis

Graph:

Disk

But y-axis ⇒ dy problem

$y = 10x$

$\frac{y}{10} = x$

$$V = \pi \int_0^{10} \left(\frac{y}{10}\right)^2 dy$$

$$= \pi \int_0^{10} \frac{y^2}{100}\, dy$$

$$= \frac{\pi}{100}\left(\frac{y^3}{3}\right)\Big]_0^{10}$$

$$= \frac{10\pi}{3}$$

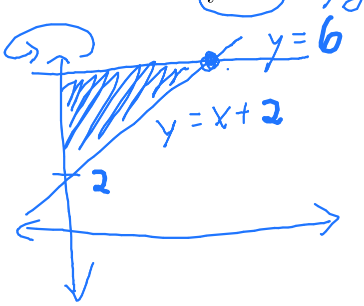
Volume = $\boxed{10\pi/3}$

55. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 2, \quad x = 0, \quad y = 6$$

$x = y - 2$

around the y-axis → dy problem.



$$V = \pi \int_2^6 (y-2)^2\, dy$$

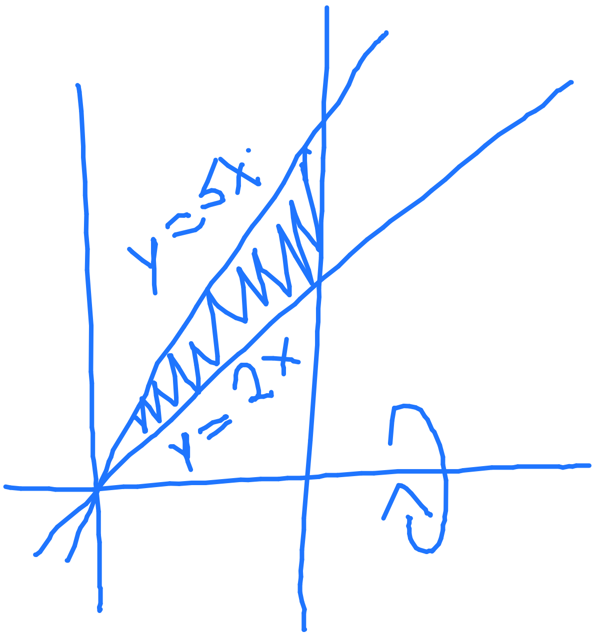
$$= \pi \int_2^6 (y^2 - 4y + 4)\, dy$$

$$= \pi\left(\frac{y^3}{3} - \frac{4y^2}{2} + 4y\right)\Big]_2^6$$

Volume = $\boxed{64\pi/3}$

56. Find the volume of the solid generated by revolving the region bounded by the following curves about the x-axis. $\Rightarrow dx$

$$y = 2x, \quad , y = 5x, \quad \text{and} x = 1$$



$$V = \pi \int_0^1 (5x)^2 - (2x)^2 \, dx$$

$$= \pi \int_0^1 25x^2 - 4x^2 \, dx$$

$$= \pi \int_0^1 21x^2 \, dx$$

$$= \pi \frac{21}{3} x^3 \Big]_0^1 = 7\pi$$

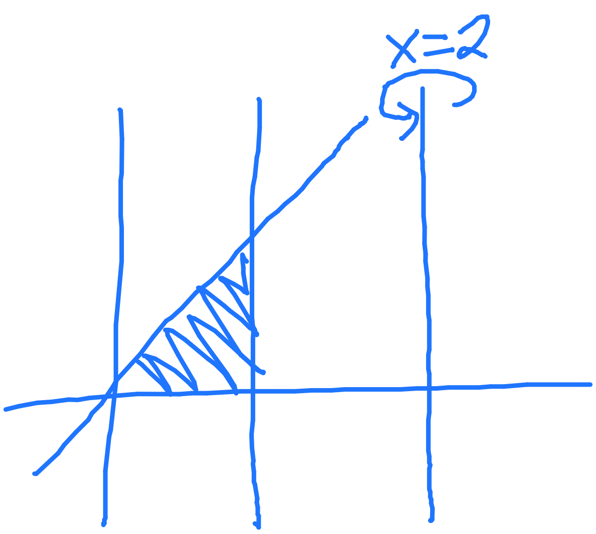
Volume = $\boxed{7\pi}$

57. Find the volume of the solid generated by revolving the region bounded by the following curves about the line $x = 2$ $\Rightarrow dx$

$$y = 2x, \quad , y = 0, \quad \text{and} \quad x = 1$$

$\Downarrow$

$x = \frac{y}{2}$



$$V = \pi \int_0^1 \left(2 - \frac{y}{2}\right)^2 - (2-1)^2 \, dy$$

$$= \pi \int_0^1 \left(4 - 2y + \frac{y^2}{2}\right) - 1 \, dy$$

$$= \pi \int_0^1 3 - 2y - \frac{y^2}{2} \, dy$$

$$= \pi \left(3y - \frac{2y^2}{3} - \frac{1}{2} \cdot \frac{y^3}{3}\right)\Big]_0^1$$

$$= \frac{13\pi}{6}$$

Volume = $\boxed{13\pi/6}$

58. Find the **VOLUME** of the region bounded by

$$x + 3y = 9, \quad x = 0, \quad y = 0$$

around the y-axis

$x + 3y = 9$

$3y = -x + 9$

$y = \frac{-x}{3} + 3$

$$V = \pi \int_0^3 (9-3y)^2\,dy$$

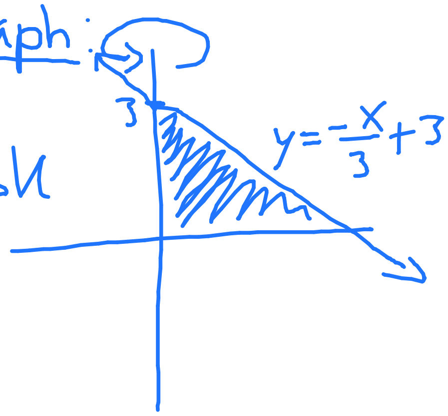
$$= \pi \int_0^3 (81 - 54y + 9y^2)\,dy$$

$$= \pi \left(81y - 27y^2 + 3y^3\right)\Big]_0^3$$

$$= 81\pi$$

Graph:

Disk



But y-axis $\Rightarrow dy$

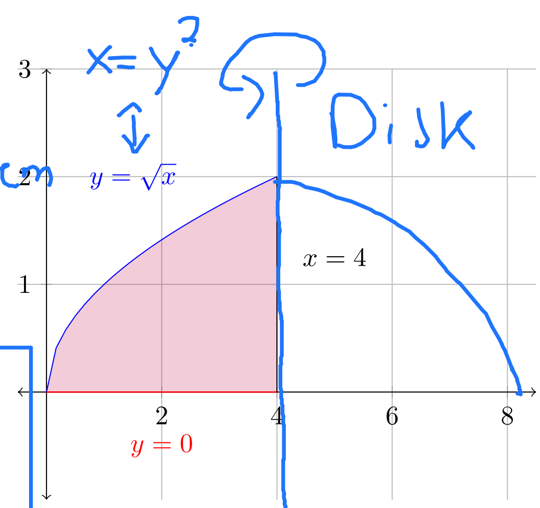
So $x + 3y = 9$

$x = 9 - 3y$

Volume $= \boxed{81\pi}$

59. Let R be the region shown to the right. Set up the integral that computes the **VOLUME** as R is rotated around the line $x = 4$. $\Rightarrow$ dy problem

DON'T COMPUTE IT!!!



Volume $= \boxed{\pi \int_0^2 (y^2 - 4)^2\,dy}$

60. **SET-UP using the washer method.** the **VOLUME** of the region bounded by

$$y = x^2, \quad y = 2x$$

around the x-axis $\to dx$

Note the bounds for all choices are the same.

(A) $\pi \int_0^2 (2x - x^2)^2 \, dx$

(B) $\boxed{\pi \int_0^2 (4x^2 - x^4) \, dx}$

(C) $\pi \int_0^2 (2x - x^2) \, dx$

(D) $\pi \int_0^2 (x^2 - 2x) \, dx$

(E) $\pi \int_0^2 (x^4 - 4x^2) \, dx$

(F) $2\pi \int_0^2 (x^3 - 2x^2) \, dx$

Test Pt: $x = 1$

$y = x^2 \to y = 1 \to$ Bottom

$y = 2x \to y = 2 \to$ Top

$V = \pi \int_0^2 (2x)^2 - (x^2)^2 \, dx$

$= \pi \int_0^2 4x^2 - x^4 \, dx$

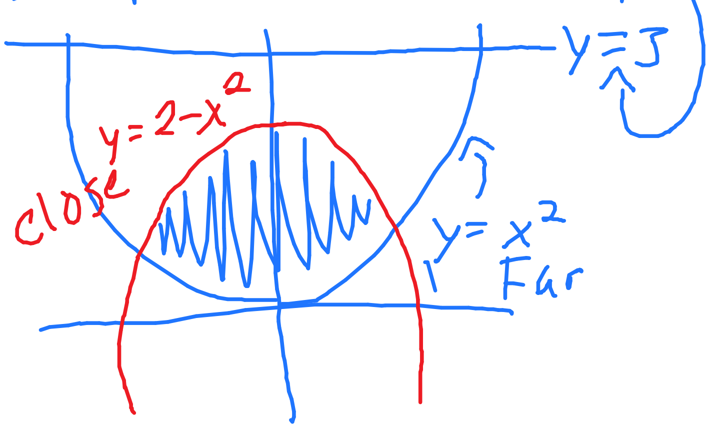
61. Set up the integral needed to find the volume of the solid obtained when the region bounded by

$$y = 2 - x^2 \quad \text{and} \quad y = x^2$$

is rotated about the line $y = 3$.

$y = 3 \Rightarrow dx$ problem

Graph Washer



Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

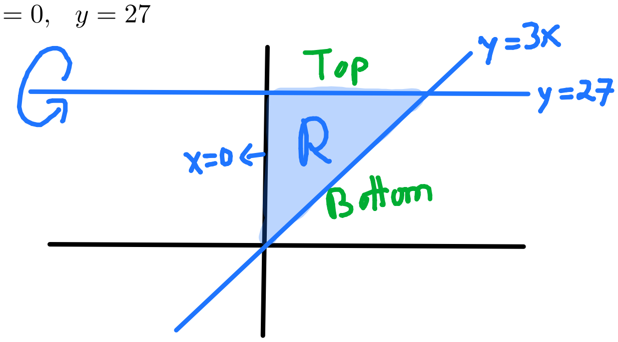
Volume $= \boxed{\pi \int_{-1}^{1} (2-x^2-3)^2 - (x^2-3)^2 \, dx}$

62. **SET-UP using the disk/washer method.** the **VOLUME** of the region bounded by

$$y = 3x, \quad x = 0, \quad y = 27$$

Disk

around the line $y = 27$ → dx



(A) $\pi \int_0^{27} (729 - 162x + 9x^2)\, dx$

(B) $\pi \int_0^{27} 9x^2\, dx$

(C) $\pi \int_0^{9} 9x^2\, dx$

(D) $\pi \int_0^{9} (9x^2 - 162x)\, dx$

(E) $\pi \int_0^{27} (729 - 9x^2)\, dx$

(F) $\boxed{\pi \int_0^{9} (729 - 162x + 9x^2)\, dx}$

Bound: $3x = 27$
$x = 9$

$$V = \pi \int_0^9 (3x - 27)^2\, dx$$
$$= \pi \int_0^9 (9x^2 - 162x + 729)\, dx$$