Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name:______________________________

1. **SET-UP using the Shell method**, the integral that computes the **VOLUME** of the region in quadrant I enclosed by the region defined by a triangle with vertices at (1,0), (6,0), and (6,10) about the y-axis.

(A) $\displaystyle\int_1^6 5\pi * (x^2 - x)\, dx$

(B) $\displaystyle\int_1^6 4\pi * (x^2)\, dx$

(C) $\displaystyle\int_1^6 4\pi * (x^2 - x)\, dx$

(D) $\displaystyle\int_1^6 2\pi * (x^2 - x)\, dx$

(E) $\displaystyle\int_1^6 5\pi * (x^2)\, dx$

(F) $\displaystyle\int_1^6 2\pi * (x^2)\, dx$

2. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Volume $=$ ______________________

3. Find the volume of the solid obtained by revolving the region enclosed by the following curves about the x-axis using cylindrical shells.

$$x = 4y - y^2, \quad \text{and} \quad x = 0$$

Volume $=$ ______________________

4. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 2 - x^2, \quad \text{and} \quad y = x^2$$

about the y-axis.

Volume $=$ ____________________

5. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 3\sqrt{x}, \quad \text{and} \quad y = x$$

about the $x = 12$.

Volume $=$ ____________________

6. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = x, \quad \text{and } y = x^2$$

about the line $x = -2$.

Volume $=$ ______________________

7. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 7x^2, \quad y = 0 \text{ and } \quad x = 2$$

about the line $x = 3$.

Volume $=$ ______________________

8. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$x = y^2 + 1, \text{ and } \quad x = 2$$

about the line $y = -2$.

Volume $=$ ______________________

9. The rate of change of the population $n(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate k in terms of minutes. Round to four decimal places.

$k=$ ______________________

10. Let y denote the mass of a radioactive substance at time t. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time t in hours does half the original mass remain? Round your answer to 3 decimal places.

t = ______________________________

11. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

y = ______________________________

12. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

$y \quad = \quad$ ________________________________

13. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

$y \quad = \quad$ ________________________________

14. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

$y =$ ______________________________

15. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$y =$ ______________________________

16. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2y$$

$y =$ ________________________________

17. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 16e^{2x-y}$$

$y =$ ________________________________

18. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$y =$ ______________________________

19. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$y =$ ______________________________

20. Find the particular solution to the given differential equation.

$$\frac{dy}{dx} = 3x^2(9 + y), \qquad y(0) = 5$$

$y =$ ______________________________

21. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \quad \text{where} \quad y = 10 \quad \text{when} \quad x = 2$$

Find the value of the integration constant, C.

$C =$ ____________________

22. Find the particular solution to the given differential equation if $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$y=$ ____________________

23. Calculate the constant of integration, C, for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \qquad y(1) = 2$$

$C=$ ______________________________

24. The volume of an object $V(t)$ in cubic millimeter at any time t in seconds changes according to the model

$$\frac{dV}{dt} = \cos\left(\frac{t}{10}\right),$$

where $V(0) = 5$. Find the volume of the object at $t = 3$ seconds. Round to 4 decimal places.

$V(3) =$ ______________________________

25. What is the **integrating factor** of the following differential equation?

$$2y' + \left(\frac{6}{x}\right) y = 10\ln(x)$$

$u(x) =$ ______________________

26. What is the **integrating factor** of the following differential equation?

$$y' + \left(\frac{2x+3}{x}\right) y = 10\ln(x)$$

$u(x) =$ ______________________

27. What is the **integrating factor** of the following differential equation?

$$x^8 y' - 14x^7 y = 32e^{7x}$$

$u(x) =$ ______________________

28. What is the **integrating factor** of the following differential equation?

$$(x+1)\frac{dy}{dx} - 2(x^2+x)y = (x+1)e^{x^2}$$

$u(x) =$ ______________________

29. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) =$ ____________________________________

30. What is the **integrating factor** of the following differential equation?

$$y' + \tan(x) \cdot y = \sec(x)$$

$u(x) =$ ____________________________________

31. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + (4x - 1)y = 8x - 2$$

$y =$ ______________________________

32. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + \frac{6y}{x} = x + 10$$

$y =$ ______________________________

33. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2(y+4) \text{ and } y(0) = 3$$

$y =$ ______________________

34. Solve the initial value problem.

$$x^4y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$y =$ ____________________

35. Find the general solution for the differential equation.

$$x\frac{dy}{dx} + 5y = 12x^2$$

$y =$ ______________________

36. Find the value of the constant of integration, C, for the particular solution to the following differential equation.

$$(x-5)y' + y = x^2 + 14, \qquad y(3) = 12$$

$C =$ ______________________

37. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \dots$$

Answer:______________________________

(b) Use the sum from (a) and compute the sum.

Answer:______________________________

38. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n = \underline{\qquad\qquad\qquad\qquad}$$

39. Express the repeating decimal $7.33333\ldots$ as a sum.

$7.33333\ldots =$ ____________________________

40. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ ____________________________

41. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n} \right)$$

$$\sum_{n=0}^{\infty} \left(\frac{7}{4^n} \right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

42. Compute

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n}$$

$$\sum_{n=1}^{\infty} \frac{5^{n+2}}{6^n} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

43. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} =$ ______________________________

44. Evaluate the sum of the following infinite series.

$$\sum_{n=0}^{\infty} (-1)^n \frac{5^{n+1}}{3^{2n}}$$

Answer: ______________________________

45. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \frac{4(3)^{n-1}}{5^n}$$

Answer: ____________________

46. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \left(\frac{3^{n-1}}{4^n} + \frac{(-1)^{n+1}}{9^n} \right)$$

Answer: ____________________

47. Find the radius of convergence for the power series shown below.

$$f(x) = \frac{x}{6x + 13}$$

$R =$ ______________________________

48. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

$R =$ ______________________________

49. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

$R =$ ______________________

50. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$\dfrac{3}{1+2x} =$ ______________________

$R =$ ______________________

51. Express $f(x) = \dfrac{x}{4+3x^2}$ as a power series.

$$\frac{x}{4+3x^2} = \underline{\hspace{6cm}}$$

52. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$$\int \sin(x^{3/2})\,dx = \underline{\hspace{6cm}}$$

53. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int e^{-3x}\,dx$$

$\int e^{-3x}\,dx =$ ________________________________

54. Evaluate the following indefinite integral as a series. [Leave your answer as a sum.]

$$\int 5e^{5x^3}\,dx$$

$\int 5e^{5x^3}\,dx =$ ________________________________

55. Use the first three terms of the powers series representation of the $f(x) = \dfrac{3x}{10+2x}$ to estimate $f(0.5)$. Round to 4 decimal places.

$f(0.5) \approx$ ____________________

56. Evaluate the indefinite integral as a power series.

$$\int \frac{x}{3 + 5x^2}\, dx$$

Answer:________________________________

57. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5 + x^6}\, dx$$

$\int_0^{0.24} \frac{x}{5 + x^6}\, dx \approx$ ______________________

58. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\, dx$$

$\int_0^{0.11} \frac{1}{1+x^4}\, dx \approx$ ____________________

59. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\, dx$$

$\int_0^{0.23} e^{-x^2}\, dx \approx$ ____________________

60. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ ______________________

61. Use the first five terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.44)$. Round to 5 decimal places.

$\ln(1.44) \approx$ ______________________

62. Use the first 4 terms of the Macluarin series for $f(x) = \sin(x)$ to evaluate $\sin(0.75)$. Round to 5 decimal places.

$\sin(0.75) \approx$ ____________________

63. Given $f(x, y) = 3x^3y^2 - x^2y^{1/3}$, evaluate $f(3, -8)$.

$f(3, -8)=$ ____________________

64. Find the domain of

$$f(x, y) = \frac{-5x}{\sqrt{x + 9y + 1}}$$

Domain = ____________________

65. Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

Domain = ______________________________

66. Find the domain of

$$f(x,y) = \frac{\ln(x^2-y+3)}{\sqrt{x-6}}$$

Domain = ______________________________

67. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \;\; C = \ln(36)$$

(a) Parabola with vertices at $(0, 0)$
(b) Circle with center at $(0, \ln(36))$ and radius 6
(c) Parabola with vertices at $(0, \ln(36))$
(d) Circle with center at $(0, 0)$ and radius 6
(e) Increasing Logarithm Function

68. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

69. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

70. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

71. What do the level curves for the following function look like?

$$f(x, y) = \ln(8y - 5x^2)$$

(a) logarithmic curves
(b) lines
(c) a point at the origin
(d) circles
(e) hyperbolas
(f) parabolas

72. For the following function $f(x, y)$, evaluate $f_y(-2, -3)$.

$$f(x, y) = 8x^4y^5 + 3x^3 - 12y^2$$

$f_y(-2, -3) =$ ________________________

73. Compute $f_x(6, 5)$ when

$$f(x, y) = \frac{(6x - 6y)^2}{\sqrt{y^2 - 1}}$$

$f_x(6, 5)=$ ______________________

74. Given the function $f(x, y) = \sin(x^2 y)$, evaluate $f_x\left(\frac{1}{2}, \pi\right)$.

$f_x\left(\frac{1}{2}, \pi\right) =$ ______________________

75. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$f_x(x,y) =$ ______________________

$f_y(x,y) =$ ______________________

76. Find the first order partial derivatives of

$$f(x,y) = x\sin(xy)$$

$f_x(x,y) =$ ______________________

$f_y(x,y) =$ ______________________

77. Find the first order partial derivatives of $f(x,y) = (xy-1)^2$

$f_x(x,y) =$ ______________________

$f_y(x,y) =$ ______________________

78. Find the first order partial derivatives of $f(x,y) = xe^{x^2+xy+y^2}$

$f_x(x,y) =$ ______________________

$f_y(x,y) =$ ______________________

79. Find the first order partial derivatives of $f(x, y) = -7\tan(x^7y^8)$

$f_x(x, y) =$ ____________________

$f_y(x, y) =$ ____________________

80. Find the first order partial derivatives of $f(x, y) = y\cos(x^2y)$

$f_x(x, y) =$ ____________________

$f_y(x, y) =$ ____________________

81. Find the first order partial derivatives of $f(x, y) = xe^{xy}$

$f_x(x, y) =$ ________________________________

$f_y(x, y) =$ ________________________________