MATH 16020 Practice Exam 3 Tues., April 9, 2024

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

Name: Solutions

1. **SET-UP using the Shell method**, the integral that computes the **VOLUME** of the region in quadrant I enclosed by the region defined by a triangle with vertices at (1,0), (6,0), and (6,10) about the y-axis.

(A) $\int_1^6 5\pi * (x^2 - x)\, dx$

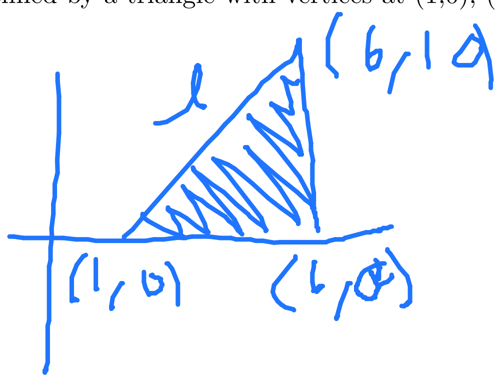
(B) $\int_1^6 4\pi * (x^2)\, dx$

(C) $\boxed{\int_1^6 4\pi * (x^2 - x)\, dx}$

(D) $\int_1^6 2\pi * (x^2 - x)\, dx$

(E) $\int_1^6 5\pi * (x^2)\, dx$

(F) $\int_1^6 2\pi * (x^2)\, dx$



$m = \frac{10-0}{6-1} = 2$

$l: y = x + b$

Find b w/ (1,0)

$0 = 2 + b \rightarrow b = -2$

$y = 2x - 2$

$$V = 2\pi \int_1^6 x(2x-2)\, dx$$

$$= \int_1^6 4\pi (x^2 - x)\, dx$$

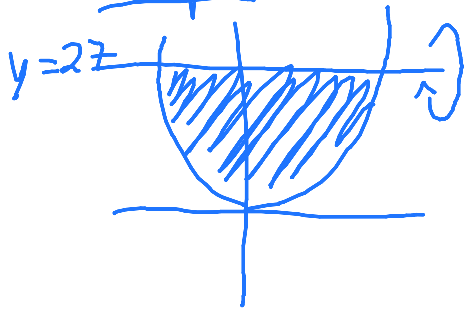
Note the bounds are all the same.

2. Find the **VOLUME** of the region bounded by

$$y = 3x^2, \quad x = 0, \quad y = 27$$

around the line $y = 27$

Graph Disk



$y = 27 \Rightarrow dx$ problem

Bounds: Given $x = 0$

$27 = 3x^2$

$9 = x^2 \rightarrow x = 3$

$$V = \pi \int_0^3 (3x^2 - 27)^2 \, dx$$

$$= \pi \int_0^3 (9x^4 - 162x^2 + 729) \, dx$$

$$= \pi \left(\frac{9x^5}{5} - 54x^3 + 729x \right) \Big]_0^3$$

$$= 11664.4\pi$$

$$\boxed{\frac{8322}{5}\pi}$$

Volume = ______________________

3. Find the volume of the solid obtained by revolving the region enclosed by the following curves about the x-axis using cylindrical shells.

$$x = 4y - y^2, \quad \text{and} \quad x = 0$$

Bounds: $0 = 4y - y^2$

$0 = y(4 - y)$

$y = 0, 4$

$$V = 2\pi \int_0^4 y(4y - y^2) \, dy$$

$$= 2\pi \int_0^4 (4y^2 - y^3) \, dy$$

$$= 2\pi \left(\frac{4y^3}{3} - \frac{y^4}{4} \right) \Big]_0^4$$

$$= \frac{128}{3}\pi$$

Volume = $\boxed{\frac{128}{3}\pi}$

4. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 2 - x^2, \quad \text{and} \quad y = x^2$$

about the y-axis. $\to dx$

$$V = 2\pi \int_{-1}^{1} x(2 - x^2 - x^2)\,dx$$

Bounds: $2 - x^2 = x^2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

Test Pt: $x = 0$

$y = 2 - x^2 \to y = 2 \to$ Top

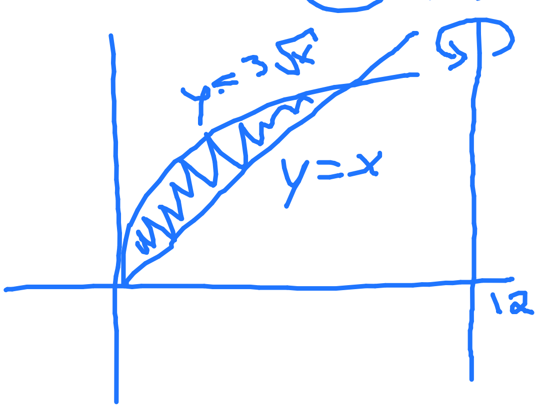
$y = x^2 \to y = 0 \to$ Bottom

Volume $=$ $\boxed{2\pi \int_{-1}^{1} x(2 - 2x^2)\,dx}$

5. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 3\sqrt{x}, \quad \text{and} \quad y = x$$

about the $x = 12$ $\to dx$



Bounds

$3\sqrt{x} = x$

$9x = x^2$

$9x - x^2 = 0$

$x(9 - x) = 0$

$x = 0, 9$

★ Note $x = 12$ is on the right of our region.

★

$$\boxed{V = 2\pi \int_{0}^{9} (12 - x)(3\sqrt{x} - x)\,dx}$$

Volume $=$ ______________________

6. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = x, \quad \text{and } y = x^2$$

about the line $x = -2$. + → dx

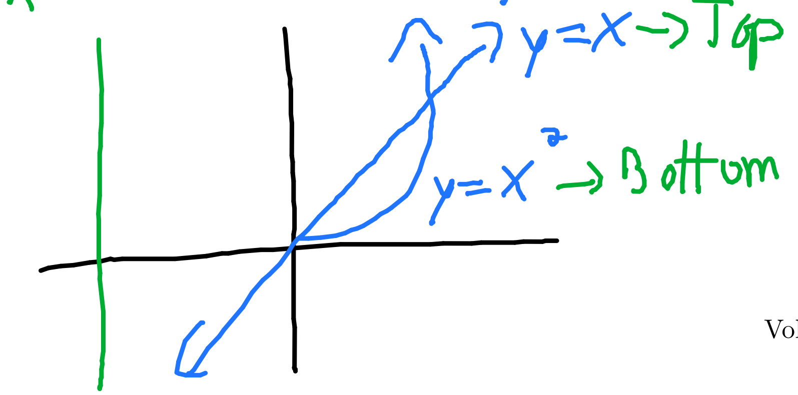
Since $x=-2$ is on the left of our region

Bounds: $x = x^2$

$x - x^2 = 0$

$x(1-x) = 0$

$x = 0, 1$



$$V = 2\pi \int_0^1 (x-(-2))[x-x^2]\,dx$$

Volume $=$ $\boxed{2\pi \int_0^1 (x+2)[x-x^2]\,dx}$

7. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$y = 7x^2, \quad y = 0 \text{ and } \quad x = 2$$

about the line $x = 3$. + → dx

$$V = 2\pi \int_0^2 (\_\_\_\_)(7x^2)\,dx$$

Since $x=3$ is larger than the bounds,

$$\boxed{V = 2\pi \int_0^2 (3-x)(7x^2)\,dx}$$

Volume $=$ ____________________

8. Using the **Shell Method**, set up the integral that computes the **VOLUME** of the region bounded by

$$x = y^2 + 1, \text{ and } \quad x = 2$$

about the line $y = -2$.

$+$ dy

Since $y=-2$ is smaller than the bounds

Bounds: $y^2+1=2$

$y^2=1$

$y=\pm 1$

$V = 2\pi \int_{-1}^{1} (y-(-2))(2-(y^2+1))\,dy$

Test Pt: $y=0$

$x=y^2+1 \rightarrow x=1 \rightarrow$ Left

$x=2 \rightarrow x=2 \rightarrow$ Right

Volume $=$ $\boxed{2\pi \int_{-1}^{1} (y+2)(2-(y^2+1))\,dy}$

9. The rate of change of the population $n(t)$ of a sample of bacteria is directly proportional to the number of bacteria present, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 210 bacteria present. If the number of bacteria after 5 hours is 360, find the growth rate k in terms of minutes. Round to four decimal places.

Recall $N' = kN \rightarrow N = Ce^{kt}$

$N(0)=210$: $210 = Ce^{k\cdot 0}$

$210 = C \rightarrow N = 210e^{kt}$

$N(5)=360$: $360 = 210e^{k\cdot 5}$

$\frac{12}{7} = e^{5k}$

$\ln\left(\frac{12}{7}\right) = 5k$

$k=$ $\boxed{\frac{1}{5}\ln\left(\frac{12}{7}\right)}$

10. Let y denote the mass of a radioactive substance at time t. Suppose this substance obeys the equation

$$y' = -18y$$

Assume that initially, the mass of the substance is $y(0) = 20$ grams. At what time t in hours does half the original mass remain? Round your answer to 3 decimal places.

$$y' = -18y \Rightarrow y = Ce^{-18t}$$
$$y(0) = 20 \Rightarrow 20 = Ce^{-18(0)}$$
$$20 = C \quad \Rightarrow y = 20e^{-18t}$$

We want solve $\frac{1}{2}(20) = y(t)$ for t.

$$10 = 20e^{-18t}$$
$$1/2 = e^{-18t}$$
$$\ln(1/2) = -18t$$
$$\frac{\ln(1/2)}{-18} = t$$

t = $\boxed{0.039}$

11. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{3x^2}{y}$$

Rewrite: $y\,dy = 3x^2\,dx$

$$\int y\,dy = \int 3x^2\,dx$$
$$\frac{y^2}{2} = x^3 + C$$
$$y^2 = 2x^3 + C$$
$$y = \pm\sqrt{2x^3 + C}$$

y = $\boxed{\pm\sqrt{2x^3 + C}}$

12. Find the general solution to the differential equation:

$$\frac{dy}{dx} = 5y$$

Rewrite $dy = 5y\,dx$

$$\frac{dy}{y} = 5dx$$

$$\int \frac{dy}{y} = \int 5dx$$

$$\ln|y| = 5x + c$$

$$|y| = e^{5x+c}$$

$$\pm y = e^c e^{5x}$$

$$y = \underbrace{\pm e^c}_{= c} e^{5x}$$

$$y = ce^{5x}$$

Or memorize

$$\frac{dy}{dx} = ky$$

$$\Rightarrow y = Ce^{kx}$$

y = $\boxed{Ce^{5x}}$

13. Find the general solution to the differential equation:

$$\frac{dy}{dx} = \frac{-x}{y}$$

Rewrite: $y\,dy = -x\,dx$

$$\int y\,dy = \int -x\,dx$$

$$\frac{y^2}{2} = \frac{-x^2}{2} + c$$

$$y^2 = -x^2 + c$$

$$y = \pm\sqrt{c - x^2}$$

y = $\boxed{\pm\sqrt{c - x^2}}$

14. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dt} - 15y = 0$$

Note there are 2 ways to do this problem.

1. Separation of Variables
2. First-Order Linear Eqn

By method 1,

$$\frac{dy}{dt} = 15y$$

$$\frac{dy}{y} = 15\,dt$$

$$\int \frac{dy}{y} = \int 15\,dt$$

$$\ln|y| = 15t + c$$

$$y = e^{15t+c}$$

$$y = e^{c}e^{15t}$$

$$y = Ce^{15t}$$

$y = \boxed{Ce^{15t}}$

15. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = \frac{3}{y}$$

$$y\,dy = 3\,dx$$

$$\int y\,dy = \int 3\,dx$$

$$\frac{y^2}{2} = 3x + c$$

$$y^2 = 6x + 2c$$

$$y^2 = 6x + C$$

$$y = \pm\sqrt{6x + C}$$

$y = \boxed{\pm\sqrt{6x+C}}$

16. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 3x^2 y$$

$$\frac{dy}{y} = 3x^2\,dx$$

$$\int \frac{dy}{y} = \int 3x^2\,dx$$

$$\ln|y| = x^3 + C$$

$$y = e^{x^3 + C}$$

$$y = e^C e^{x^3}$$

$$y = Ce^{x^3}$$

$y = \boxed{Ce^{x^3}}$

17. Find the general solution to the given differential question. Use C as an arbitrary constant.

$$\frac{dy}{dx} = 16e^{2x-y}$$

$$dy = 16e^{2x}\,e^{-y}\,dx$$

$$e^y\,dy = 16e^{2x}\,dx$$

$$\int e^y\,dy = \int 16e^{2x}\,dx$$

$$e^y = \frac{16}{2}e^{2x} + C$$

$$e^y = 8e^{2x} + C$$

$$y = \ln(8e^{2x} + C)$$

$y = \boxed{\ln(8e^{2x} + C)}$

18. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{3x+2}{2y} \quad \text{and} \quad y(0) = 4$$

$2y\,dy = (3x+2)\,dx$

$\int 2y\,dy = \int (3x+2)\,dx$

$y^2 = \frac{3x^2}{2} + 2x + C$

So $y^2 = \frac{3x^2}{2} + 2x + 16$

$y = \pm\sqrt{\frac{3x^2}{2} + 2x + 16}$

when $y(0) = 4$

$4^2 = 0 + 0 + C$

$16 = C$

$y = \boxed{\pm\sqrt{\frac{3x^2}{2} + 2x + 16}}$

19. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = \frac{5y}{6x+3} \quad \text{and} \quad y(0) = 1$$

$\frac{dy}{y} = \frac{5}{6x+3}\,dx$

$\int \frac{dy}{y} = \int \frac{5}{6x+3}\,dx$

$\ln|y| = \frac{5}{6}\ln|6x+3| + C$

$y = \exp\left[\frac{5}{6}\ln|6x+3| + C\right]$

$y = e^C \exp\left[\ln|6x+3|^{5/6}\right]$

$y = C \cdot |6x+3|^{5/6}$

When $y(0) = 1$

$1 = C \cdot |6(0)+3|^{5/6}$

$1 = C \cdot 3^{5/6}$

$C = 3^{-5/6}$

$y = \boxed{3^{-5/6} \cdot |6x+3|^{5/6}}$

20. Find the particular solution to the given differential equation.

$$\frac{dy}{dx} = 3x^2(9+y), \qquad y(0) = 5$$

$$\frac{dy}{9+y} = 3x\,dx$$

$$\int \frac{dy}{9+y} = \int 3x\,dx$$

$$\ln|9+y| = \frac{3}{2}x^2 + C$$

$$|9+y| = \exp\left[\frac{3}{2}x^2 + C\right]$$

$$\pm(9+y) = e^C \exp\left[\frac{3}{2}x^2\right]$$

$$9+y = \pm e^C \exp\left[\frac{3}{2}x^2\right]$$

$$9+y = C\exp\left[\frac{3}{2}x^2\right]$$

$$y = C\exp\left[\frac{3}{2}x^2\right] - 9$$

When $y(0) = 5$

$$y = C\exp\left[\frac{3}{2}x^2\right] - 9$$

$$5 = C\exp\left[\frac{3}{2}(0)^2\right] - 9$$

$$5 = C - 9$$

$$14 = C$$

$$\Rightarrow y = 14\exp\left[\frac{3}{2}x^2\right] - 9$$

$y = \boxed{14\exp\left[\frac{3}{2}x^2\right] - 9}$

21. Consider the following IVP:

$$\frac{dy}{dx} = 11x^2e^{-x^3} \quad \text{where} \ \ y = 10 \ \ \text{when} \ \ x = 2$$

Find the value of the integration constant, C.

$dy = 11x^2e^{-x^3}dx$

$\int dy = \underbrace{\int 11x^2e^{-x^3}dx}$

$u = -x^3$

$du = -3x^2dx$

$y = \int -\frac{11}{3}e^u du$

$y = -\frac{11}{3}e^{-x^3} + C$

When $y=10$ and $x=2$

$10 = -\frac{11}{3}e^{-2^3} + C$

$10 = -\frac{11}{3}e^{-8} + C$

$C = 10 + \frac{11}{3}e^{-8}$

$C =$ $\boxed{10 + \frac{11}{3}e^{-8}}$

22. Find the particular solution to the given differential equation if $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$y^2dy = xdx$

$\int y^2dy = \int xdx$

$\frac{y^3}{3} = \frac{x^2}{2} + C$

Find C. w/ $y(2)=3$

$\frac{3^3}{3} = \frac{2^2}{2} + C$

$9 = 2 + C$

$7 = C$

$\frac{y^3}{3} = \frac{x^2}{2} + 7$

$y^3 = \frac{3x^2}{2} + 21$

$y = \sqrt[3]{\frac{3x^2}{2} + 21}$

$y=$ $\boxed{\sqrt[3]{\frac{3x^2}{2} + 21}}$

23. Calculate the constant of integration, C, for the given differential equation.

$$\frac{dy}{dx} = \frac{7x^3}{6y}, \qquad y(1) = 2$$

Rewrite $6y\,dy = 7x^3\,dx$

$$\int 6y\,dy = \int 7x^3\,dx$$

$$3y^2 = \frac{7x^4}{4} + C$$

Note we want C when $y(1)=2$

$$3(2)^2 = \frac{7(1)^4}{4} + C$$

$$12 = \frac{7}{4} + C$$

$$C = 41/4$$

$C=$ $\boxed{\frac{41}{4}}$

24. The volume of an object $V(t)$ in cubic millimeter at any time t in seconds changes according to the model

$$\frac{dV}{dt} = \cos\left(\frac{t}{10}\right),$$

where $V(0) = 5$. Find the volume of the object at $t = 3$ seconds. Round to 4 decimal places.

Rewrite $dV = \cos\left(\frac{t}{10}\right)dt$

$$\int dV = \int \cos\left(\frac{t}{10}\right)dt$$

$$V = 10\sin\left(\frac{t}{10}\right) + C$$

Find C w/ $V(0)=5$

$$5 = 10\sin\left(\frac{0}{10}\right) + C$$

$$C = 5$$

So $V = 10\sin\left(\frac{t}{10}\right) + 5$

$$V(3) = 10\sin\left(\frac{3}{10}\right) + 5$$

$$\approx 7.9552$$

$V(3) =$ $\boxed{7.9552}$

25. What is the **integrating factor** of the following differential equation?

$$\frac{2y' + \left(\frac{6}{x}\right) y}{2} = \frac{10\ln(x)}{2}$$

$y' + \frac{3}{x}y = 5\ln x$

$P(x) = \frac{3}{x} \quad Q(x) = 5\ln x$

$u(x) = \exp\left[\int \frac{3}{x}dx\right]$

$= \exp[3\ln x]$

$= \exp[\ln x^3]$

$= x^3$

$u(x) =$ $\boxed{x^3}$

26. What is the **integrating factor** of the following differential equation?

$$y' + \left(\frac{2x+3}{x}\right) y = 10\ln(x)$$

$P(x) = \frac{2x+3}{x} \quad Q(x) = 10\ln(x)$

$u(x) = \exp\left[\int P(x)dx\right]$

$= \exp\left[\int \frac{2x+3}{x}dx\right]$

$= \exp\left[\int 2 + \frac{3}{x}dx\right]$

$= \exp[2x + 3\ln x]$

$= e^{2x+3\ln x}$

$= e^{2x} \cdot e^{3\ln x}$

$= e^{2x} \cdot e^{\ln x^3}$

$= x^3 e^{2x}$

$u(x) =$ $\boxed{x^3 e^{2x}}$

27. What is the **integrating factor** of the following differential equation?

$$\frac{x^8 y' - 14x^7 y}{x^8} = \frac{32e^{7x}}{x^8}$$

$$y' + \underbrace{\left(\frac{-14}{x}\right)}_{P} y = \underbrace{\frac{32e^{7x}}{x^8}}_{Q}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int -\frac{14}{x}\,dx\right]$$
$$= \exp\left[-14\ln x\right]$$
$$= \exp\left[\ln x^{-14}\right]$$
$$= x^{-14}$$
$$= \frac{1}{x^{14}}$$

$u(x) =$ $\boxed{1/x^{14}}$

28. What is the **integrating factor** of the following differential equation?

$$\frac{(x+1)\frac{dy}{dx} - 2(x^2+x)y}{(x+1)} = \frac{(x+1)e^{x^2}}{(x+1)}$$

$$\frac{dy}{dx} - \frac{2x(x+1)}{(x+1)}y = e^{x^2}$$

$$\frac{dy}{dx} + (-2x)\cdot y = e^{x^2}$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int -2x\,dx\right]$$
$$= \exp\left[-x^2\right]$$

$u(x) =$ $\boxed{e^{-x^2}}$

29. What is the **integrating factor** of the following differential equation?

$$y' + \cot(x) \cdot y = \sin^2(x)$$

$u(x) = \exp\left[\int P(x)\,dx\right]$

$= \exp\left[\int \cot x\,dx\right]$

$= \exp\left[\int \frac{\cos x}{\sin x}\,dx\right]$

$u = \sin x$

$du = \cos x\,dx$

$= \exp\left[\int \frac{du}{u}\right]$

$= \exp[\ln u]$

$u(x) = \exp[\ln \sin x]$

$= \sin x$

$u(x) =$ $\boxed{\sin x}$

30. What is the **integrating factor** of the following differential equation?

$$y' + \tan(x) \cdot y = \sec(x)$$

$u(x) = \exp\left[\int P(x)\,dx\right]$

$= \exp\left[\int \tan x\,dx\right]$

$= \exp\left[\int \frac{\sin x}{\cos x}\,dx\right]$

$u = \cos x$

$du = -\sin x\,dx$

$= \exp\left[-\int \frac{du}{u}\right]$

$= \exp[-\ln u]$

$u(x) = \exp[-\ln(\cos x)]$

$= \exp[\ln(\cos x)^{-1}]$

$= (\cos x)^{-1} = \sec x$

$u(x) =$ $\boxed{\sec(x)}$

Note there are 2 ways to do this problem.
① Separation of Variables
② First-Order Linear Eqn

31. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + (4x-1)y = 8x-2$$

$P(x) = 4x-1 \qquad Q(x) = 8x-2$

$u(x) = \exp\left[\int (4x-1)\,dx\right]$

$= \exp\left[2x^2-x\right]$

$= e^{2x^2-x}$

$y\,u(x) = \int Q(x)u(x)\,dx + C$

$y e^{2x^2-x} = \underbrace{\int (8x-2)e^{2x^2-x}\,dx}_{} + C$

$u = 2x^2-x$

$du = 4x-1\ dx$

$y e^{2x^2-x} = \int \frac{8x-2}{4x-1} e^u\,du + C$

$y e^{2x^2-x} = \int \frac{2\cancel{(4x-1)}}{\cancel{4x-1}} e^u\,du + C$

$y e^{2x^2-x} = \int 2e^u\,du + C$

$y e^{2x^2-x} = 2e^u + C$

$y e^{2x^2-x} = 2e^{2x^2-x} + C$

$y = \frac{2e^{2x^2-x} + C}{e^{2x^2-x}}$

$y = 2 + Ce^{-(2x^2-x)}$

$= 2 + Ce^{x-2x^2}$

$y = \boxed{2 + Ce^{x-2x^2}}$

32. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + \frac{6y}{x} = x + 10$$

$P(x) = \frac{6}{x}$ $Q(x) = x + 10$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int \frac{6}{x}\,dx\right]$$
$$= \exp\left[6 \ln x\right]$$
$$= \exp\left[\ln(x^6)\right]$$
$$= x^6$$

$$y \cdot u(x) = \int Q(x)u(x)\,dx + C$$
$$y x^6 = \int (x+10)x^6\,dx + C$$
$$y x^6 = \int (x^7 + 10x^6)\,dx + C$$
$$y x^6 = \frac{x^8}{8} + \frac{10x^7}{7} + C$$

$$y = \frac{x^2}{8} + \frac{10x}{7} + \frac{C}{x^6}$$

$y =$ $\boxed{\frac{x^2}{8} + \frac{10x}{7} + \frac{C}{x^6}}$

33. Find the particular solution to the differential equation.

$$\frac{dy}{dx} = 6x^2(y+4) \text{ and } y(0) = 3$$

$y' = 6x^2 y + 24x^2$

$y' - 6x^2 y = 24x^2$

$P(x) = -6x^2 \quad Q(x) = 24x^2$

$u(x) = \exp\left[\int -6x^2 dx\right]$

$= \exp\left[-2x^3\right]$

$= e^{-2x^3}$

$y \cdot u(x) = \int Q(x) u(x)\, dx + c$

$y e^{-2x^3} = \underbrace{\int 24x^2 e^{-2x^3} dx}_{} + c$

$u = -2x^3$

$du = -6x^2 dx$

$y e^{-2x^3} = \int -4e^u du + c$

$y e^{-2x^3} = -4e^u + c$

$y e^{-2x^3} = -4e^{-2x^3} + C$

$y = -4 + Ce^{2x^3}$

With $y(0) = 3$

$3 = -4 + Ce^{2 \cdot 0^3}$

$3 = -4 + C$

$7 = C$

So $y = -4 + 7e^{2x^3}$

$y = \boxed{-4 + 7e^{2x^3}}$

34. Solve the initial value problem.

$$x^4y' + 4x^3 \cdot y = 10x^9 \text{ with } f(1) = 23$$

$$\frac{x^4y' + 4x^3y}{x^4} = \frac{10x^9}{x^4}$$

$$y' + \frac{4}{x} \cdot y = 10x^5$$

$$P(x) = \frac{4}{x} \quad Q(x) = 10x^5$$

$$u(x) = \exp\left[\int P(x)\,dx\right]$$
$$= \exp\left[\int \frac{4}{x}\,dx\right]$$
$$= \exp[4\ln x]$$
$$= \exp[\ln x^4]$$
$$= x^4$$

$$y \cdot u(x) = \int Q(x)u(x)\,dx + C$$
$$y \cdot x^4 = \int 10x^5 x^4\,dx + C$$
$$y \cdot x^4 = \int 10x^9\,dx + C$$
$$y \cdot x^4 = x^{10} + C$$
$$y = \frac{x^{10}}{x^4} + \frac{C}{x^4}$$
$$y = x^6 + \frac{C}{x^4}$$

$$23 = 1 + \frac{C}{1}$$
$$22 = C$$
$$y = x^6 + \frac{22}{x^4}$$

$y = \boxed{x^6 + \frac{22}{x^4}}$

35. Find the general solution for the differential equation.

$$\frac{x\frac{dy}{dx}}{x} + 5y = \frac{12x^2}{x}$$

$$\frac{dy}{dx} + \frac{5}{x}y = 12x$$

$P(x) = \frac{5}{x}$ $\quad Q(x) = 12x$

$$u(x) = \exp\left[\int P(x)dx\right]$$
$$= \exp\left[\int \frac{5}{x}dx\right]$$
$$= \exp[5\ln(x)]$$
$$= \exp[\ln(x^5)]$$
$$= x^5$$

$$y \cdot x^5 = \int 12x(x^5)\,dx + C$$
$$y \cdot x^5 = \int 12x^6\,dx + C$$
$$y \cdot x^5 = \frac{12x^7}{7} + C$$
$$y = \frac{12}{7}x^2 + \frac{C}{x^5}$$

$y =$ $\boxed{y = \frac{12}{7}x^2 + \frac{C}{x^5}}$

36. Find the value of the constant of integration, C, for the particular solution to the following differential equation.

$$\frac{(x-5)y' + y}{(x-5)} = \frac{x^2 + 14}{(x-5)}, \qquad y(3) = 12$$

$$y' + \frac{1}{x-5}y = \frac{x^2+14}{x-5}$$

$P(x) = \frac{1}{x-5}$ $\quad Q(x) = \frac{x^2+14}{x-5}$

$$u(x) = \exp\left[\int \frac{1}{x-5}dx\right]$$
$$= \exp[\ln(x-5)]$$
$$= x-5$$

$$y(x-5) = \int \frac{x^2+14}{x-5}(x-5)\,dx + C$$
$$y(x-5) = \int x^2 + 14\,dx + C$$
$$y(x-5) = \frac{x^3}{3} + 14x + C$$

when $y(3) = 12$

$$12(3-5) = \frac{3^3}{3} + 14(3) + C$$

$C =$ $\boxed{-75}$

37. (a) Use summation notation to write the series in compact form.

$$1 - 0.6 + 0.36 - 0.216 + \ldots$$

$$= 1 - \frac{6}{10} + \frac{36}{100} - \frac{216}{1000} + \ldots$$

$$= 1 - \frac{6}{10} + \left(\frac{6}{10}\right)^2 - \left(\frac{6}{10}\right)^3 + \ldots$$

$$= \sum_{n=0}^{\infty} (-1)^n \left(\frac{6}{10}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n$$

Answer: $\boxed{\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n}$

(b) Use the sum from (a) and compute the sum.

$$\sum_{n=0}^{\infty} \left(\frac{-6}{10}\right)^n = \frac{1}{1-(-6/10)} = \frac{1}{1+6/10} = \frac{1}{16/10} = \frac{10}{16} = \frac{5}{8}$$

Answer: $\boxed{5/8}$

38. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Note $r = 3/2$ and

$\left|\frac{3}{2}\right| < 1$ is false

So the sum diverges

$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n =$ $\boxed{\text{diverges}}$

$$7 + \overbrace{0.3333}^{\text{sum}}$$

$||$

39. Express the repeating decimal 7.33333... as a sum.

$$0.3333 = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \cdots$$

$$= \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{100} + \cdots\right)$$

$$= \frac{3}{10} \sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$$

7.33333... = $7 + \frac{3}{10} \sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n$

40. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n$$

$$= \frac{6}{1-(-1/9)}$$

$$= \frac{6}{1+1/9}$$

$$= \frac{6}{10/9}$$

$$= 6 \cdot \frac{9}{10}$$

$$= 3 \cdot \frac{9}{5} = \frac{27}{5}$$

$\boxed{27/5}$

$\sum_{n=0}^{\infty} 6\left(-\frac{1}{9}\right)^n =$ ______

41. If the given series converges, then find its sum. If not, state that it diverges.

$$\sum_{n=0}^{\infty}\left(\frac{7}{4^n}\right)$$

$$= \sum_{n=0}^{\infty} 7\left(\frac{1}{4}\right)^n$$

$$= \frac{7}{1-1/4}$$

$$= \frac{7}{3/4}$$

$$= 7\cdot\frac{4}{3} = \frac{28}{3}$$

$\sum_{n=0}^{\infty}\left(\frac{7}{4^n}\right) =$ $\boxed{28/3}$

42. Compute

$$\sum_{n=1}^{\infty}\frac{5^{n+2}}{6^n}$$

$$= \frac{5^3}{6} + \frac{5^4}{6^2} + \frac{5^5}{6^3} + \cdots$$

$$= \frac{5^3}{6}\left(1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \cdots\right)$$

$$= \frac{125}{6}\sum_{n=0}^{\infty}\left(\frac{5}{6}\right)^n = \frac{125}{6}\cdot\frac{1}{1-5/6}$$

$$= \frac{125}{6}\cdot\frac{1}{1/6} = \frac{125}{6}\cdot\frac{6}{1} = 125$$

$\sum_{n=1}^{\infty}\frac{5^{n+2}}{6^n} =$ $\boxed{125}$

43. Compute

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n}{3 \cdot 3^{2n}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{3} \left(-\frac{2}{9}\right)^n$$

$$= \frac{1/3}{1-(-2/9)}$$

$$= \frac{1/3}{1+2/9}$$

$$= \frac{1/3}{11/9}$$

$$= \frac{1}{3} \cdot \frac{9}{11}$$

$$= 3/11$$

$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}} =$ $\boxed{3/11}$

44. Evaluate the sum of the following infinite series.

$$\sum_{n=0}^{\infty} (-1)^n \frac{5^{n+1}}{3^{2n}}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, 5^n \cdot 5^1}{(3^2)^n}$$

$$= \sum_{n=0}^{\infty} 5\left(-\frac{5}{9}\right)^n$$

$$= \frac{5}{1-(-5/9)}$$

$$= \frac{5}{14/9}$$

$$= \frac{5}{1} \cdot \frac{9}{14}$$

$$= \frac{45}{14}$$

Answer: $\boxed{45/14}$

45. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \frac{4(3)^{n-1}}{5^n}$$

$$= \frac{4(3)^0}{5^1} + \frac{4(3)^1}{5^2} + \frac{4(3)^2}{5^3} + \frac{4(3)^3}{5^4} + \dots$$

$$= \frac{4}{5}\left(1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \left(\frac{3}{5}\right)^3 + \dots\right)$$

$$= \frac{4}{5} \sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^n$$

$$= \frac{4}{5} \cdot \frac{1}{1 - 3/5}$$

$$= \frac{4}{5} \cdot \frac{1}{2/5} = \frac{4}{5} \cdot \frac{5}{2} = 2$$

Answer: $\boxed{2}$

46. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \left(\frac{3^{n-1}}{4^n} + \frac{(-1)^{n+1}}{9^n}\right)$$

$$= \sum_{n=1}^{\infty} \left(\frac{3^{-1}}{1} \cdot \frac{3^n}{4^n} + \frac{(-1)^1}{1} \cdot \frac{(-1)^n}{9^n}\right)$$

$$= \sum_{n=1}^{\infty} \left(\frac{1}{3}\left(\frac{3}{4}\right)^n - \left(\frac{-1}{9}\right)^n\right)$$

$$= \frac{1}{3}\left(\frac{3}{4}\right)^1 - \left(\frac{-1}{9}\right)^1$$

$$+ \frac{1}{3}\left(\frac{3}{4}\right)^2 - \left(\frac{-1}{9}\right)^2$$

$$+ \frac{1}{3}\left(\frac{3}{4}\right)^3 - \left(\frac{-1}{9}\right)^3$$

$$+ \dots$$

$$= \frac{1}{3}\left(\frac{3}{4}\right)\left[1 + \left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 + \dots\right]$$

$$- \left(\frac{1}{-9}\right)\left[1 + \left(-\frac{1}{9}\right) + \left(-\frac{1}{9}\right)^2 \dots\right]$$

$$= \frac{1}{4} \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n + \frac{1}{9} \sum_{n=0}^{\infty} \left(-\frac{1}{9}\right)^n$$

$$= \frac{1}{4} \cdot \frac{1}{1 - 3/4} + \frac{1}{9} \cdot \frac{1}{1 - (-1/9)}$$

Answer: $\boxed{1.1}$

47. Find the radius of convergence for the power series shown below.

$$f(x) = \frac{x}{6x+13} = \frac{x}{13+6x}$$

$$\frac{x}{13} \cdot \frac{1}{1+6/13x}$$

$$= \frac{x}{13} \cdot \frac{1}{1-(-6/13x)}$$

So $$\frac{1}{1-(-6/13x)} = \sum_{n=0}^{\infty} \left(-\frac{6}{13}x\right)^n \text{ where } \underbrace{\left|\frac{-6}{13}x\right|<1}_{\text{Note we only want this}}$$

$$|x| < \frac{13}{6}$$

$R =$ $\boxed{13/6}$

48. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3(-2x)^n$$

Remember

$$\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square} \text{ where } |\square| < 1$$

$$|-2x| < 1$$
$$|2x| < 1$$
$$2|x| < 1$$
$$|x| < 1/2 = R$$

$R =$ $\boxed{1/2}$

49. Find the radius of convergence for the power series shown below.

$$\sum_{n=0}^{\infty} 3\left(7x^2\right)^n$$

Remember

$$\sum_{n=0}^{\infty} \square^n = \frac{1}{1-\square} \text{ where } |\square| < 1$$

$$|7x^2| < 1$$

$$7|x^2| < 1$$

$$|x^2| < 1/7$$

$$-1/7 < x^2 < 1/7$$

By algebra

$$x^2 < 1/7$$

$$x < \pm\sqrt{1/7}$$

$$|x| < \sqrt{1/7}$$

$R =$ $\boxed{\sqrt{1/7}}$

50. Express $f(x) = \dfrac{3}{1+2x}$ as a power series and determine it's radius of converge.

$$\frac{3}{1+2x} = \frac{3}{1} \cdot \frac{1}{1+2x} = \frac{3}{1} \cdot \frac{1}{1-(-2x)}$$

$$\frac{1}{1-(-2x)} = \sum_{n=0}^{\infty} (-2x)^n \text{ where } |-2x| < 1$$

$$f(x) = \frac{3}{1-(-2x)} = 3\sum_{n=0}^{\infty} (-2x)^n \text{ where } 2|x| < 1$$

$$= \sum_{n=0}^{\infty} 3(-1)^n 2^n x^n \text{ where } |x| < 1/2$$

$\dfrac{3}{1+2x} =$ $\sum_{n=0}^{\infty} 3(-1)^n 2^n x^n$

$R =$ $1/2$

51. Express $f(x) = \dfrac{x}{4+3x^2}$ as a power series.

$$\frac{x}{4(1+3x^2/4)} = \frac{x}{4} \cdot \frac{1}{1-(-(3x^2/4))}$$

$$\frac{1}{1-(-3x^2/4)} = \sum_{n=0}^{\infty} \left(\frac{-3x^2}{4}\right)^n$$

$$f(x) = \frac{x}{4} \cdot \frac{1}{1-(-3x^2/4)} = \frac{x}{4} \sum_{n=0}^{\infty} \left(\frac{-3x^2}{4}\right)^n$$

$$f(x) = \frac{x}{4} \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n}}{4^n}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n \cdot x^{2n+1}}{4^{n+1}}$$

$$\frac{x}{4+3x^2} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n 3^n \cdot x^{2n+1}}{4^{n+1}}}$$

52. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int \sin(x^{3/2})\,dx$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\sin(x^{3/2}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x^{3/2})^{2n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}$$

$$\int \sin(x^{3/2})\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{3n+3/2}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{3n+3/2}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{3n+5/2}}{3n+5/2}$$

$$= \frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6\cdot(3+5/2)} + \frac{x^{17/2}}{5!\,(6+5/2)}$$

$$\int \sin(x^{3/2})\,dx = \boxed{\frac{x^{5/2}}{5/2} - \frac{x^{11/2}}{6\cdot(3+5/2)} + \frac{x^{17/5}}{5!\,(6+5/2)}}$$

53. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int e^{-3x}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-3x} = \sum_{n=0}^{\infty} \frac{(-3x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!}$$

$$\int e^{-3x}\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \int x^n\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \cdot \frac{x^{n+1}}{(n+1)}$$

$$= \frac{(-1)^0 3^0}{0!} \cdot \frac{x^1}{1} + \frac{(-1)^1 3^1}{1!} \cdot \frac{x^2}{2} + \frac{(-1)^2 3^2}{2!} \cdot \frac{x^3}{3}$$

$\int e^{-3x}\,dx =$ $\boxed{x - \frac{3}{2}x^2 + \frac{3}{2}x^3}$

54. Evaluate the following indefinite integral as a series. [Leave your answer as a sum.]

$$\int 5e^{5x^3}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{5x^3} = \sum_{n=0}^{\infty} \frac{(5x^3)^n}{n!} = \sum_{n=0}^{\infty} \frac{5^n x^{3n}}{n!}$$

$$5e^{5x^3} = 5\sum_{n=0}^{\infty} \frac{5^n x^{3n}}{n!} = \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} x^{3n}$$

$$\int 5e^{5x^3}\,dx = \int \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} x^{3n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \int x^{3n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \cdot \frac{x^{3n+1}}{(3n+1)}$$

$\int 5e^{5x^3}\,dx =$ $\boxed{\sum_{n=0}^{\infty} \frac{5^{n+1}}{n!} \cdot \frac{x^{3n+1}}{(3n+1)}}$

55. Use the first three terms of the powers series representation of the $f(x) = \dfrac{3x}{10+2x}$ to estimate $f(0.5)$. Round to 4 decimal places.

$$\frac{3x}{10\left(1+\frac{2}{10}x\right)} = \frac{3x}{16}\cdot\frac{1}{1-\left(-\frac{2}{10}x\right)}$$

$$\frac{1}{1-\left(-\frac{2}{10}x\right)} = \sum_{n=0}^{\infty}\left(-\frac{2}{10}x\right)^n$$

$$f(x) = \frac{3x}{10}\cdot\frac{1}{1-\left(-\frac{2}{10}x\right)} = \frac{3x}{10}\sum_{n=0}^{\infty}\left(-\frac{2}{10}x\right)^n$$

$$f(x) = \frac{3x}{10}\sum_{n=0}^{\infty}\frac{(-1)^n 2^n x^n}{10^n}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n 2^n \cdot 3^1 x^{n+1}}{10^{n+1}}$$

$$f(0.5) = \sum_{n=0}^{\infty}\frac{(-1)^n 2^n \cdot 3\cdot(0.5)^{n+1}}{10^{n+1}}$$

$$= \frac{3(0.5)}{10} - \frac{2\cdot 3(0.5)^2}{10^2} + \frac{2^2\cdot 3(0.5)^3}{10^3}$$

$$\approx 0.1365$$

$f(0.5) \approx$ $\boxed{0.1365}$

56. Evaluate the indefinite integral as a power series.

$$\int \frac{x}{3+5x^2}\,dx$$

$$\frac{x}{3}\cdot\frac{1}{1+5/3\,x^2} = \frac{x}{3}\cdot\frac{1}{\underbrace{1-(-5/3\,x^2)}_{\text{sum}}}$$

$$\frac{1}{1-(-5/3\,x^2)} = \sum_{n=0}^{\infty}\left(-\frac{5}{3}x^2\right)^n$$

$$\frac{x}{3}\cdot\frac{1}{1-(-5/3\,x^2)} = \frac{x}{3}\sum_{n=0}^{\infty}\left(-\frac{5}{3}x^2\right)^n$$

$$= \frac{x}{3}\sum_{n=0}^{\infty}\frac{(-1)^n 5^n x^{2n}}{3^n}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n 5^n x^{2n+1}}{3^{n+1}}$$

$$\int \frac{x}{3+5x^2}\,dx = \sum_{n=0}^{\infty}\frac{(-1)^n 5^n}{3^{n+1}}\cdot\int x^{2n+1}\,dx$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n 5^n}{3^{n+1}}\cdot\frac{x^{2n+2}}{2n+2}$$

Answer: $\boxed{\sum_{n=0}^{\infty}\frac{(-1)^n 5^n}{3^{n+1}}\cdot\frac{x^{2n+2}}{2n+2}}$

57. Use a power series to approximate the definite integral using the first 3 terms of the series.

$$\int_0^{0.24} \frac{x}{5+x^6}\, dx$$

$$\frac{x}{5+x^6} = \frac{x}{5-(-x^6)} = \frac{x}{5[1-(-x^6/5)]} = \frac{x}{5} \cdot \frac{1}{1-(-x^6/5)}$$

$$\frac{1}{1-(-x^6/5)} = \sum_{n=0}^{\infty} \left(-\frac{x^6}{5}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{5^n}$$

$$\frac{x}{5} \cdot \frac{1}{1-(-x^6/5)} = \frac{x}{5} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{5^n} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{5^{n+1}}$$

$$\int_0^{0.24} \frac{x}{5+x^6}\, dx = \int_0^{0.24} \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{5^{n+1}}\, dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}} \int_0^{0.24} x^{6n+1}\, dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+1}} \cdot \frac{x^{6n+2}}{(6n+2)} \Bigg]_0^{0.24}$$

$$= \left(\frac{1}{5} \cdot \frac{x^2}{2} - \frac{1}{5^2} \cdot \frac{x^8}{8} + \frac{1}{5^3} \cdot \frac{x^{14}}{14} \right) \Bigg]_0^{0.24}$$

$$\approx 0.00576$$

$\int_0^{0.24} \frac{x}{5+x^6}\, dx \approx$ $\boxed{0.00576}$

58. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx$$

$$\frac{1}{1+x^4} = \frac{1}{1-(-x^4)} = \sum_{n=0}^{\infty} (-x^4)^n = \sum_{n=0}^{\infty} (-1)^n x^{4n}$$

$$\int_0^{0.11} \frac{1}{1+x^4}\,dx = \int_0^{0.11} \sum_{n=0}^{\infty} (-1)^n x^{4n}\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \int_0^{0.11} x^{4n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{4n+1}\Big]_0^{0.11}$$

$$= \left(x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13}\right)\Big]_0^{0.11}$$

$\int_0^{0.11} \frac{1}{1+x^4}\,dx \approx$ $\boxed{0.11000}$

59. Use a power series to approximate the definite integral using the first 3 terms of the series. Round to 5 decimal places.

$$\int_0^{0.23} e^{-x^2}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}$$

$$\int_0^{0.23} e^{-x^2}\,dx = \int_0^{0.23} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^{0.23} x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{2n+1}}{2n+1}\Big]_0^{0.23}$$

$$= \left(\frac{x}{0!} - \frac{x^3}{1!(3)} + \frac{x^5}{2!(5)}\right)\Big]_0^{0.23}$$

$$= \left(x - \frac{x^3}{3} + \frac{x^5}{10}\right)\Big]_0^{0.23}$$

$\int_0^{0.23} e^{-x^2}\,dx \approx$ $\boxed{0.226}$

60. Use a power series to approximate the definite integral using the first 4 terms of the series. Round to 5 decimal places.

$$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

$$\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (x^{1/2})^{2n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$f(x) = 4x\cos(\sqrt{x}) = 4x \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1}$$

$$\int_0^{0.45} 4x\cos(\sqrt{x})dx = \int_0^{0.45} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4x^{n+1} dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\int_0^{0.45} x^{n+1} dx$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot 4\frac{x^{n+2}}{n+2}\Bigg]_0^{0.45}$$

$$= \left(\frac{4x^2}{0!(2)} - \frac{4x^3}{2!(3)} + \frac{4x^4}{4!(4)} - \frac{4x^5}{6!(5)}\right)\Bigg]_0^{0.45}$$

$$= \left(2x^2 - \frac{2x^3}{3} + \frac{x^4}{24} - \frac{x^5}{900}\right)\Bigg]_0^{0.45}$$

$\int_0^{0.45} 4x\cos(\sqrt{x})\,dx \approx$ $\boxed{0.34593}$

61. Use the first five terms of the Macluarin series for $f(x) = \ln(1+x)$ to evaluate $ln(1.44)$. Round to 5 decimal places.

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$$

Note $1.56 = 1 + 0.56$

$$\ln(1+0.56) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (0.56)^n = 0.56 - \frac{(0.56)^2}{2} + \frac{(0.56)^3}{3}$$

$\ln(1.44) \approx$ $\boxed{0.46174}$

62. Use the first 4 terms of the Macluarin series for $f(x) = \sin(x)$ to evaluate $\sin(0.75)$. Round to 5 decimal places.

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin(0.75) = \sum_{n=0}^{\infty} \frac{(-1)^n (0.75)^{2n+1}}{(2n+1)!} = \frac{0.75}{1!} - \frac{(0.75)^3}{3!} + \frac{(0.75)^5}{5!} - \frac{(0.75)^7}{7!}$$

$\sin(0.75) \approx$ $\boxed{0.74631}$

63. Given $f(x, y) = 3x^3y^2 - x^2y^{1/3}$, evaluate $f(3, -8)$.

$$f(3,-8) = 3(3)^3(-8)^2 - (3)^2(-8)^{1/3}$$

$f(3, -8)=$ $\boxed{5202}$

64. Find the domain of

$$f(x, y) = \frac{-5x}{\sqrt{x + 9y + 1}}$$

$$\frac{1}{\sqrt{?}} \rightarrow ? > 0$$

$$x + 9y + 1 > 0$$

Domain = $\{(x,y) \mid x + 9y + 1 > 0\}$

65. Find the domain of

$$f(x,y) = \frac{\sqrt{x+y-1}}{\ln(y-11)-9}$$

$\sqrt{?} \to ? \geq 0$

$\sqrt{x+y-1} \to x+y-1 \geq 0$

$x+y \geq 1$

$\ln(?) \to ? > 0$

$\ln(y-11) \to y-11 > 0$

$y > 11$

$\frac{1}{?} \to ? \neq 0$

$\ln(y-11)-9 \neq 0$

$\ln(y-11) \neq 9$

$y-11 \neq e^9$

$y \neq e^9+11$

Domain = $\boxed{\{(x,y) \mid x+y \geq 1,\ y > 11,\ y \neq 11+e^9\}}$.

66. Find the domain of

$$f(x,y) = \frac{\ln(x^2-y+3)}{\sqrt{x-6}}$$

$\ln(?) \to ? > 0$

$\ln(x^2-y+3) \to x^2-y+3 > 0$

$x^2+3 > y$

$\frac{1}{\sqrt{?}} \to ? > 0$

$\frac{1}{\sqrt{x-6}} \to x-6 > 0$

$x > 6$

Domain = $\boxed{\{(x,y) \mid x > 6,\ x^2+3 > y\}}$

67. Describe the indicated level curves $f(x, y) = C$

$$f(x, y) = \ln(x^2 + y^2) \;\; C = \ln(36)$$

$\ln(x^2+y^2)=\ln(36)$

$x^2+y^2=36$

$x^2+y^2=6^2$

(a) Parabola with vertices at $(0, 0)$
(b) Circle with center at $(0, \ln(36))$ and radius 6
(c) Parabola with vertices at $(0, \ln(36))$
(d) Circle with center at $(0, 0)$ and radius 6
(e) Increasing Logarithm Function

68. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

$\ln(y-e^{5x})=C$

$y-e^{5x}=e^C$

$y-e^{5x}=C$

$y=e^{5x}+C$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

69. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

$\sqrt{x^2+y^2}=C$

$x^2+y^2=C^2$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

70. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

$\cos(y+4x^2)=C$

$y+4x^2=\cos^{-1}(C)$

$y+4x^2=C$

$y=-4x^2+C$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

71. What do the level curves for the following function look like?

$$f(x, y) = \ln(8y - 5x^2)$$

(a) logarithmic curves
(b) lines
(c) a point at the origin
(d) circles
(e) hyperbolas
(f) parabolas

$\ln(8y-5x^2)=C$

$8y-5x^2=e^C=C$

$8y=C+5x^2$

72. For the following function $f(x, y)$, evaluate $f_y(-2, -3)$.

$$f(x, y) = 8x^4y^5 + 3x^3 - 12y^2$$

$f_y(x,y)=\frac{d}{dy}(8x^4y^5+3x^3-12y^2)$

$=8x^4\frac{d}{dy}(y^5)+3x^3\frac{d}{dy}(1)-\frac{d}{dy}(12y^2)$

$=(8x^4)(5y^4)+(3x^3)(0)-24y$

$=40x^4y^4-24y$

$f_y(-2,-3)=40(-2)^4(-3)^4-24(-3)$

$=51912$

$f_y(-2, -3) =$ $\boxed{51912}$

73. Compute $f_x(6,5)$ when

$$f(x,y) = \frac{(6x-6y)^2}{\sqrt{y^2-1}}$$

$$f_x(x,y) = \frac{d}{dx}\left(\frac{(6x-6y)^2}{\sqrt{y^2-1}}\right)$$

$$= \frac{1}{\sqrt{y^2-1}} \frac{d}{dx}\left((6x-6y)^2\right)$$

$$= \frac{1}{\sqrt{y^2-1}} \cdot 2(6x-6y)\frac{d}{dx}(6x+6y)$$

$$= \frac{1}{\sqrt{y^2-1}} \cdot 2(6x-6y)\cdot 6$$

$$= \frac{72x-72y}{\sqrt{y^2-1}}$$

$f_x(6,5)=$ $\boxed{72/\sqrt{24}}$

74. Given the function $f(x,y) = \sin(x^2y)$, evaluate $f_x\left(\frac{1}{2},\pi\right)$.

$$f_x = \cos(x^2y) \cdot 2xy$$

$$f_x\left(\frac{1}{2},\pi\right) = \cos\left(\frac{1}{4}\cdot\pi\right) \cdot 2\cdot\frac{1}{2}\cdot\pi$$

$$= \frac{\sqrt{2}}{2}\cdot\pi$$

$f_x\left(\frac{1}{2},\pi\right) =$ $\boxed{\frac{\sqrt{2}}{2}\pi}$

75. Find the first order partial derivatives of

$$f(x,y) = \frac{3x^2y^3}{(y-1)^2}$$

$f(x,y) = 3x^2 \cdot \frac{y^3}{(y-1)^2}$

$f_x(x,y) = \frac{d}{dx}\left(3x^2 \cdot \frac{y^3}{(y-1)^2}\right) = \frac{y^3}{(y-1)^2} \cdot \frac{d}{dx}(3x^2) = \frac{y^3}{(y-1)^2} \cdot 6x$

$f_y(x,y) = \frac{d}{dy}\left(3x^2 \cdot \frac{y^3}{(y-1)^2}\right) = 3x^2 \frac{d}{dy}\left(\frac{y^3}{(y-1)^2}\right) = 3x^2\left(\frac{3y^2(y-1)^2 - y^3 \cdot 2(y-1)}{(y-1)^4}\right)$

$= 3x^2\left(\frac{\cancel{(y-1)}[3y^2(y-1) - 2y^3]}{(y-1)^{\cancel{4}\,3}}\right) = \frac{3x^2(3y^3 - 3y^2 - 2y^3)}{(y-1)^3}$

$= \frac{3x^2(y^3 - 3y^2)}{(y-1)^3}$

$f_x(x,y) =$ $6xy^3/(y-1)^2$

$f_y(x,y) =$ $\frac{3x^2(y^3-3y^2)}{(y-1)^3}$

76. Find the first order partial derivatives of

$$f(x,y) = x\sin(xy)$$

$f_x(x,y) = \frac{d}{dx}(x\sin(xy)) = \frac{d}{dx}(x)\sin(xy) + x\frac{d}{dx}(\sin(xy))$

$= \sin(xy) + x\cos(xy)\frac{d}{dx}(xy)$

$= \sin(xy) + x \cdot y\cos(xy)$

$f_y(x,y) = \frac{d}{dy}(x\sin(xy)) = x\frac{d}{dy}(\sin(xy))$

$= x\cos(xy)\frac{d}{dy}(xy)$

$= x^2\cos(xy)$

$f_x(x,y) =$ $\sin(xy) + xy\cos(xy)$

$f_y(x,y) =$ $x^2\cos(xy)$

77. Find the first order partial derivatives of $f(x, y) = (xy - 1)^2$

$$f_x(x,y) = \frac{d}{dx}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dx}(xy-1)$$
$$= 2(xy-1)y$$
$$= 2xy^2 - 2y$$

$$f_y(x,y) = \frac{d}{dy}\left((xy-1)^2\right) = 2(xy-1)\frac{d}{dy}(xy-1)$$
$$= 2(xy-1)x$$
$$= 2x^2y - 2x$$

$f_x(x, y) =$ $\boxed{2xy^2 - 2y}$

$f_y(x, y) =$ $\boxed{2x^2y - 2x}$

78. Find the first order partial derivatives of $f(x, y) = xe^{x^2+xy+y^2}$

$$f_x(x,y) = \frac{d}{dx}(x)\, e^{x^2+xy+y^2} + x\frac{d}{dx}\left(e^{x^2+xy+y^2}\right)$$
$$= e^{x^2+xy+y^2} + x\left(e^{x^2+xy+y^2}\right)(2x+y)$$
$$= (1 + 2x^2 + xy)e^{x^2+xy+y^2}$$

$$f_y(x,y) = x\frac{d}{dy}\left(e^{x^2+xy+y^2}\right) = x\left(e^{x^2+xy+y^2}\right)(x+2y)$$
$$= (x^2 + 2xy)e^{x^2+xy+y^2}$$

$f_x(x, y) =$ $\boxed{(1 + 2x^2 + xy)e^{x^2+xy+y^2}}$

$f_y(x, y) =$ $\boxed{(x^2 + 2xy)e^{x^2+xy+y^2}}$

79. Find the first order partial derivatives of $f(x,y) = -7\tan(x^7y^8)$

$f_x(x,y) = -7\frac{d}{dx}(\tan(x^7y^8)) = -7\sec^2(x^7y^8)\frac{d}{dx}(x^7y^8)$

$= -7\cdot 7x^6y^8\sec^2(x^7y^8) = -49x^6y^8\sec^2(x^7y^8)$

$f_y(x,y) = -7\frac{d}{dy}(\tan(x^7y^8)) = -7\sec^2(x^7y^8)\frac{d}{dy}(x^7y^8)$

$= -7\cdot 8x^7y^7\sec^2(x^7y^8)$

$= -56x^7y^7\sec^2(x^7y^8)$

$f_x(x,y) =$ $-49x^6y^8\sec^2(x^7y^8)$

$f_y(x,y) =$ $-56x^7y^7\sec^2(x^7y^8)$

80. Find the first order partial derivatives of $f(x,y) = y\cos(x^2y)$

$f_x(x,y) = y\frac{d}{dx}(\cos(x^2y)) = y(-\sin(x^2y))\frac{d}{dx}(x^2y) = -y\sin(x^2y)[2xy]$

$= -2xy^2\sin(x^2y)$

$f_y(x,y) = \frac{d}{dy}(y)\cos(x^2y) + y\frac{d}{dy}(\cos(x^2y))$

$= \cos(x^2y) + y(-\sin(x^2y))\frac{d}{dy}(x^2y)$

$= \cos(x^2y) - y\sin(x^2y)[x^2]$

$= \cos(x^2y) - x^2y\sin(x^2y)$

$f_x(x,y) =$ $-2xy^2\sin(x^2y)$

$f_y(x,y) =$ $\cos(x^2y) - x^2y\sin(x^2y)$

81. Find the first order partial derivatives of $f(x,y) = xe^{xy}$

$$f_x = \frac{\partial}{\partial x}(xe^{xy}) = \frac{\partial}{\partial x}(x)e^{xy} + x\frac{\partial}{\partial x}(e^{xy})$$
$$= e^{xy} + xe^{xy}\frac{\partial}{\partial x}(xy)$$
$$= e^{xy} + xe^{xy}(y)$$
$$= e^{xy}(1+xy)$$

$$f_y = \frac{\partial}{\partial y}(xe^{xy}) = x\frac{\partial}{\partial y}(e^{xy})$$
$$= xe^{xy}\frac{\partial}{\partial y}(xy)$$
$$= xe^{xy}\cdot x$$
$$= x^2e^{xy}$$

$f_x(x,y) = \boxed{e^{xy}(1+xy)}$

$f_y(x,y) = \boxed{x^2e^{xy}}$