

Please show **all** your work! Answers without supporting work will not be given credit.  
Write answers in spaces provided.

Name: \_\_\_\_\_

1. Evaluate the indefinite integral.

$$\int 18x \cos(x^2) dx$$

$$\int 18x \cos(x^2) dx = \underline{\hspace{10em}}$$

2. Evaluate the indefinite integral.

$$\int 11x^2 e^{-4x^3} dx$$

$$\int 11x^2 e^{-4x^3} dx = \underline{\hspace{10em}}$$

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3. A forestry company estimates that acres of forest available for logging will increase at a rate given by  $R'(t) = \frac{56}{\sqrt{t+7}}$  for  $0 \leq t \leq 20$  where  $R'(t)$  is the rate of new acreage becoming available in thousands of acres per year,  $t$  years after the current year. How many acres of forest will become available for logging over the first 5 years? Round your answer to the nearest thousand acres.

Answer: \_\_\_\_\_

4. Find the area under the curve  $y = 7 \cos(4x)$  for  $0 \leq x \leq \pi/2$ .

Area = \_\_\_\_\_

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5. Evaluate the indefinite integral

$$\int \frac{\ln(7x)}{x} dx$$

$$\int \frac{\ln(7x)}{x} dx = \underline{\hspace{10em}}$$

6. Evaluate the definite integral.

$$\int_1^e \frac{\ln(x)}{x} dx$$

$$\int_1^e \frac{\ln(x)}{x} dx = \underline{\hspace{10em}}$$

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7. Evaluate the indefinite integral.

$$\int x^3 \ln(9x) dx$$

$$\int x^3 \ln(9x) dx = \underline{\hspace{10em}}$$

8. Evaluate

$$\int 3x \ln(x^6) dx$$

$$\int 3x \ln(x^6) dx = \underline{\hspace{10em}}$$

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9. Evaluate the indefinite integral.

$$\int 20x \sin(2x) dx$$

$$\int 20x \sin(2x) dx = \underline{\hspace{10em}}$$

10. Evaluate the indefinite integral.

$$\int 18x \cos(3x) dx$$

$$\int 18x \cos(3x) dx = \underline{\hspace{10em}}$$

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11. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{7x - 5}{x^2(x^2 + 9)}$$

(A)  $\frac{A}{x} + \frac{B}{x} + \frac{Cx + D}{x^2 + 9}$

(B)  $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 9}$

(C)  $\frac{A}{x} + \frac{Bx + C}{x^2} + \frac{Dx + E}{x^2 + 9}$

(D)  $\frac{Ax + B}{x^2} + \frac{Cx + D}{x^2 + 9}$

(E)  $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 3} + \frac{D}{x - 3}$

(F)  $\frac{Ax + B}{x^2} + \frac{C}{x + 3} + \frac{D}{x - 3}$

12. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{2x}{(x - 1)^2(x^2 + 5)}$$

(A)  $\frac{A}{x - 1} + \frac{Bx + C}{(x - 1)^2} + \frac{D}{x^2 + 5}$

(B)  $\frac{A}{(x - 1)^2} + \frac{Bx + C}{x^2 + 5}$

(C)  $\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 5}$

(D)  $\frac{A}{x - 1} + \frac{Bx + C}{(x - 1)^2} + \frac{Dx + E}{x^2 + 5}$

(E)  $\frac{A}{x - 1} + \frac{Bx + C}{x^2 + 5}$

(F)  $\frac{Ax + B}{(x - 1)^2} + \frac{Cx + D}{x^2 + 5}$

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13. Evaluate the improper integral or state that it is divergent.

$$\int_3^{\infty} \frac{4}{x^5} dx$$

$$\int_3^{\infty} \frac{4}{x^5} dx = \underline{\hspace{10cm}}$$

14. Evaluate the following integral;

$$\int_0^{\infty} e^{-x/6} dx$$

$$\int_0^{\infty} e^{-x/6} dx = \underline{\hspace{10cm}}$$

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15. Evaluate the definite integral.

$$\int_2^{\infty} \frac{dx}{3x+1}$$

$$\int_2^{\infty} \frac{dx}{3x+1} = \underline{\hspace{10cm}}$$

16. Set up the integral that represents the **AREA** of the region bounded by the following curves:

$$y = \frac{1}{2}x^2, \quad y = -x^2 + 6$$

$$\text{Area} = \underline{\hspace{10cm}}$$



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17. Find the **AREA** of the region bounded by  $y = 2x - x^2$  and  $y = x^2$ .

Area = \_\_\_\_\_

18. Find the **VOLUME** of the solid that results by revolving the region enclosed by the following curves about the  $x$ -axis.

$$y = 3x, x = 1, x = 4, \text{ and } y = 0$$

Volume = \_\_\_\_\_

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19. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 2, \quad x = 0, \quad y = 6$$

around the y-axis

Volume = \_\_\_\_\_

20. **SET-UP using the washer method.** the **VOLUME** of the region bounded by

$$y = x^2, \quad y = 2x$$

around the x-axis

(A)  $\pi \int_0^2 (2x - x^2)^2 dx$

(B)  $\pi \int_0^2 (4x^2 - x^4) dx$

(C)  $\pi \int_0^2 (2x - x^2) dx$

(D)  $\pi \int_0^2 (x^2 - 2x) dx$

(E)  $\pi \int_0^2 (x^4 - 4x^2) dx$

(F)  $2\pi \int_0^2 (x^3 - 2x^2) dx$

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21. Set-up the definite integral that would calculate the **VOLUME** of the region bounded by the following curves when rotated about the  $x$ -axis.

$$y = 5x \text{ and } y = 15\sqrt{x}$$

VOLUME = \_\_\_\_\_

22. Find the **VOLUME** of the solid obtained by revolving the region bounded by the following curves about the  $y$ -axis.

$$x + 2y = 4, x = 0, \text{ and } y = 0$$

VOLUME = \_\_\_\_\_

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23. Find the general solution of the given differential equation.

$$\frac{dy}{dx} = \frac{4x}{y}$$

$y =$  \_\_\_\_\_

24. Find the particular solution to the given differential equation if  $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$y =$  \_\_\_\_\_

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25. Find the particular solution of the equation.

$$\frac{dy}{dx} - 5y = 0, y(1) = 7$$

$y =$  \_\_\_\_\_

26. The rate of change of the population  $N(t)$  of a sample of bacteria is directly proportional to the number of bacteria, so  $N'(t) = kN$ , where time  $t$  is measured in minutes. Initially, there are 270 bacteria present. If the number of bacteria after 7 minutes is 770, find the growth rate  $k$  in terms of minutes. Round to four decimal places.

$k =$  \_\_\_\_\_

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27. What is the **integrating factor** of the following differential equation?

$$y' + \left(\frac{2x+3}{x}\right)y = 10 \ln(x)$$

$$u(x) = \underline{\hspace{10cm}}$$

28. Find the general solution of the following differential equation.

$$\frac{dy}{dx} + \frac{4y}{x} = 2x + 20$$

$$y = \underline{\hspace{10cm}}$$

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29. Compute the following sum.

$$\sum_{n=0}^{\infty} \left( \frac{(-1)^n}{3^n} + \frac{2^{n+1}}{3^n} \right)$$

Answer: \_\_\_\_\_

30. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty} \frac{4(3)^{n-1}}{5^n}$$

Answer: \_\_\_\_\_

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31. Find the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

Answer: \_\_\_\_\_

32. Express  $f(x) = \frac{x}{3+x}$  as a power series.

$$\frac{x}{3+x} = \underline{\hspace{10em}}$$



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33. Express  $f(x) = \frac{x}{4 + 3x^2}$  as a power series.

$$\frac{x}{4 + 3x^2} = \underline{\hspace{10cm}}$$

34. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int e^{-3x} dx$$

$$\int e^{-3x} dx = \underline{\hspace{10cm}}$$

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35. Find the Maclaurin representation of the following:

$$\int \cos(\sqrt{x}) dx$$

$$\int \cos(\sqrt{x}) dx = \underline{\hspace{10cm}}$$

36. Given  $f(x, y) = 3x^3y^2 - x^2y^{1/3}$ , evaluate  $f(3, -8)$ .

$$f(3, -8) = \underline{\hspace{10cm}}$$

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37. What do the level curves for the following function look like?

$$f(x, y) = 12 \ln(6(x - 3)^2 + 6(y - 2)^2)$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

38. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2}$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

39. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x})$$

- (a) Increasing exponential functions
- (b) Rational Functions with x-axis symmetry
- (c) Natural logarithm functions
- (d) Decreasing exponential functions
- (e) Rational Functions with y-axis symmetry

40. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2)$$

- (a) Lines
- (b) Parabolas
- (c) Circles
- (d) Point at the origin
- (e) Ellipses
- (f) Hyperbolas

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41. Given  $f(x, y) = x \sin(xy^2)$ , evaluate  $f_y(3, 7)$ . Round to 4 decimal places.

$$f_y(3, 7) = \underline{\hspace{10cm}}$$

42. Find the first order partial derivatives of  $f(x, y) = xe^{xy}$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

$$f_y(x, y) = \underline{\hspace{10cm}}$$

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43. For the given function  $f(x, y)$ , find  $f_x(x, y)$ .

$$f(x, y) = 5 \cos(x^7 y^8)$$

$$f_x(x, y) = \underline{\hspace{10cm}}$$

44. Given the function  $f(x, y) = x^3 y^2 - 3x + 5y - 5x^2 y^3$ , compute  $f_{xx}(x, y)$

$$f_{xx}(x, y) = \underline{\hspace{10cm}}$$

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45. Given the function  $f(x, y) = x^3 \sin(y)$ , compute  $f_{xy}(2, 0)$

$$f_{xy}(2, 0) = \underline{\hspace{10cm}}$$

46. For the function  $f(x, y)$ , find  $f_{xy}(\pi, 2)$ .

$$f(x, y) = 8y^5 \sin(x)$$

$$f_{xy}(\pi, 2) = \underline{\hspace{10cm}}$$

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47. At what point  $(x, y)$  does the function  $f(x, y)$  have a local minimum?

$$f(x, y) = 7x^2 - xy + 5y^2 + 75x + 84y + 2$$

Local min occurs at \_\_\_\_\_

48. Given the information below, which critical point(s)  $(a, b)$  would be classified as a relative maximum?

$(a, b)$	$f_{xx}(a, b)$	$f_{yy}(a, b)$	$f_{xy}(a, b)$
$(7, 8)$	$-5$	$-5$	$10$
$(-8, -1)$	$-4$	$-7$	$-2$
$(1, 7)$	$-10$	$-1$	$6$

Answer: \_\_\_\_\_

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49. Given the table below,

$(a, b)$	$f_{xx}(a, b)$	$f_{yy}(a, b)$	$f_{xy}(a, b)$
$(9, 4)$	$-1$	$-1$	$-1$
$(-2, 2)$	$4$	$3$	$-4$
$(4, 5)$	$8$	$5$	$6$

Which statements are true?

- I.  $f(x, y)$  has exactly 1 saddle points
- II.  $f(x, y)$  has exactly 1 relative minimum
- III.  $f(x, y)$  has exactly 1 relative maximum
- IV.  $f(x, y)$  has exactly 1 inconclusive critical point

Answer: \_\_\_\_\_

50. The critical points for a function  $f(x, y)$  are  $(0,0)$  and  $(8,4)$ . Given that the partial derivatives of  $f(x, y)$  are

$$f_x(x, y) = 3x - 6y \quad f_y(x, y) = 3y^2 - 6x$$

Classify each critical point as a maximum, minimum, or saddle point.

$(0,0)$  is \_\_\_\_\_

$(8,4)$  is \_\_\_\_\_



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51. Classify the critical points of the function  $f(x, y)$  given the partial derivatives

$$f_x(x, y) = x - 2y \quad f_y(x, y) = y^2 - 2x$$

- A) 2 Local Minima
- B) 2 Saddle Points
- C) 1 Saddle Point and 1 Local Minimum
- D) 1 Saddle Point and 1 Local Maximum
- E) 1 Local Maximum and 1 Local Minimum
- F) 2 Local Maxima

52. Find the minimum of the function using LaGrange Multipliers of the function  $f(x, y) = x^2 + 2y^2$  subject to the constraint  $x^2 + y^2 = 1$ .

Minimum Value = \_\_\_\_\_

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53. Find the maximum value of the function  $f(x, y) = 18x - 19y^2$  subject to the constraint  $x^2 + 19y^2 = 81$ .

Maximum Value = \_\_\_\_\_

54. A factory can produce a chocolate bar with a weight of  $W(x, y) = \frac{xy}{100}$  with the weight  $W$  in ounces and  $x$  and  $y$  are the percentages of cocoa and sugar respectively. The percentage of cocoa and sugar are constrained to  $2x + y = 75$ . What is the weight, in ounces, of the largest chocolate bar that can be produced? Round to 2 decimal places.

Weight of Largest Chocolate Bar = \_\_\_\_\_

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55. Evaluate the double integral

$$\int_0^{\pi/2} \int_0^1 16y^3 \cos(x) \, dy \, dx$$

$$\int_0^1 \int_0^{\pi/2} 16y^3 \cos(x) \, dy \, dx = \underline{\hspace{2cm}}$$

56. Evaluate the double integral

$$\int_0^7 \int_0^{\pi/2} 20y^4 \cos(x) \, dx \, dy$$

$$\int_0^7 \int_0^{\pi/2} 20y^4 \cos(x) \, dx \, dy = \underline{\hspace{2cm}}$$

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57. Evaluate the definite integral.

$$\int_0^4 \int_3^x \frac{6x}{y^2} dy dx$$

$$\int_0^4 \int_3^x \frac{6x}{y^2} dy dx = \underline{\hspace{10em}}$$

58. Evaluate the definite integral.

$$\int_0^6 \int_2^x 30x dy dx$$

$$\int_0^6 \int_2^x 30x dy dx = \underline{\hspace{10em}}$$

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59. Find the bounds for the integral  $\iint_R f(x, y) dA$  where  $R$  is a triangle with vertices  $(0,0)$ ,  $(1,0)$ , and  $(1,2)$ .

Answer: \_\_\_\_\_

60. Switch the order of integration for the following integral

$$\int_0^1 \int_{9y}^9 f(x, y) dx dy$$

Answer: \_\_\_\_\_

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61. Switch the order of integration on the follow integral

$$\int_0^1 \int_{10y}^{10} f(x, y) dx dy$$

Answer: \_\_\_\_\_

62. Evaluate the double integral

$$\int_0^1 \int_y^1 2e^{x^2} dx dy$$

(Hint: Change the order of integration)

$$\int_0^1 \int_y^1 2e^{x^2} dx dy = \underline{\hspace{2cm}}$$

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63. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2} dy dx$$

(Hint: Change the order of integration)

$$\int_0^2 \int_x^2 4e^{y^2} dy dx = \underline{\hspace{10em}}$$

64. Evaluate the double integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx dy$$

(Hint: Change the order of integration)

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) dx dy = \underline{\hspace{10em}}$$