Please show **all** your work! Answers without supporting work will not be given credit. Write answers in spaces provided.

Name: Solutions

1. Evaluate the indefinite integral.

$$\int 18x\cos(x^2)\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int 18x\cos(u)\frac{du}{2x} = \int 9\cos(u)\,du = 9\sin(u) + C$$
$$= 9\sin(x^2) + C$$

$\int 18x\cos(x^2)\,dx =$ $9\sin(x^2) + C$

2. Evaluate the indefinite integral.

$$\int 11x^2 e^{-4x^3}\,dx$$

$u = -4x^3$

$du = -12x^2\,dx$

$\frac{du}{-12x^2} = dx$

$$\int 11x^2 e^u \frac{du}{-12x^2} = -\frac{11}{12}\int e^u\,du = -\frac{11}{12}e^u + C$$
$$= -\frac{11}{12}e^{-4x^3} + C$$

$\int 11x^2 e^{-4x^3}\,dx =$ $\frac{-11}{12}e^{-4x^3} + C$

3. A forestry company estimates that acres of forest available for logging will increase at a rate given by $R'(t) = \frac{56}{\sqrt{t+7}}$ for $0 \le t \le 20$ where $R'(t)$ is the rate of new acreage becoming available in thousands of acres per year, t years after the current year. How many acres of forest will become available for logging over the first 5 years? Round your answer to the nearest thousand acres.

$$\int_0^5 56(t+7)^{-1/2}\,dt \quad \frac{u=t+7}{du=dt} \quad \int 56u^{-1/2}\,du = 56u^{1/2}\cdot\frac{2}{1}$$

$$= 112(t+7)^{1/2}\Big]_0^5$$

$$\approx 92$$

Answer: 92

4. Find the area under the curve $y = 7\cos(4x)$ for $0 \le x \le \pi/2$.

$$\int_0^{\pi/2} 7\cos(4x)\,dx \quad \frac{u=4x}{du=4dx} \quad \int 7\cos(u)\frac{du}{4} = \frac{7}{4}\sin(u)$$

$$\frac{du}{4} = dx$$

$$= \frac{7}{4}\sin(4x)\Big]_0^{\pi/2} = \frac{7}{4}\sin\left(\frac{4\pi}{2}\right) - \frac{7}{4}\sin(4\cdot 0)$$

$$= 0 - 0$$

Area = 0

5. **Evaluate the indefinite integral**

$$\int \frac{\ln(7x)}{x}\, dx$$

$u = \ln(7x)$

$du = \frac{1}{7x} \cdot 7dx$

$du = \frac{1}{x} dx$

$x\,du = dx$

$$\int \frac{u}{x} \cdot x\,du = \int u\,du = \frac{u^2}{2} + C$$

$$= \frac{(\ln(7x))^2}{2} + C$$

$$\int \frac{\ln(7x)}{x}\, dx = \frac{(\ln(7x))^2}{2} + C$$

6. **Evaluate the definite integral.**

$$\int_1^e \frac{\ln(x)}{x}\, dx$$

$u = \ln(x)$

$du = \frac{1}{x} dx$

$x\,du = dx$

$$\int \frac{u}{x} \cdot x\,du = \int u\,du = \frac{u^2}{2}$$

$$= \left.\frac{(\ln(x))^2}{2}\right]_1^e$$

$$= \frac{(\ln(e))^2}{2} - \frac{(\ln(1))^2}{2}$$

$$\int_1^e \frac{\ln(x)}{x}\, dx = 1/2$$

7. Evaluate the indefinite integral.

$$\int x^3 \ln(9x)\,dx$$

$$u = \ln(9x) \qquad dv = x^3\,dx \qquad uv - \int v\,du = \frac{x^4 \ln(9x)}{4} - \int \frac{x^4}{4} \cdot \frac{1}{x}\,dx$$

$$du = \frac{1}{9x} \cdot 9\,dx$$

$$du = \frac{1}{x}\,dx \qquad v = \frac{x^4}{4}\,dx$$

$$= \frac{x^4 \ln(9x)}{4} - \int \frac{1}{4} x^3\,dx = \frac{x^4 \ln(9x)}{4} - \frac{1}{4} \cdot \frac{x^4}{4} + C$$

$$\int x^3 \ln(9x)\,dx = \frac{x^4 \ln(9x)}{4} - \frac{x^4}{16} + C$$

8. Evaluate

$$\int 3x \ln(x^6)\,dx$$

$$= \int 3x \cdot 6 \ln(x)\,dx = \int 18x \ln(x)\,dx$$

$$u = \ln(x) \qquad dv = 18x\,dx \qquad uv - \int v\,du = 9x^2 \ln(x) - \int 9x^2 \cdot \frac{1}{x}\,dx$$

$$du = \frac{1}{x}\,dx \qquad v = \frac{18x^2}{2}$$

$$v = 9x^2$$

$$= 9x^2 \ln(x) - \int 9x\,dx = 9x^2 \ln(x) - \frac{9x^2}{2} + C$$

$$\int 3x \ln(x^6)\,dx = 9x^2 \ln(x) - \frac{9x^2}{2} + C$$

9. **Evaluate the indefinite integral.**

$$\int 20x\sin(2x)\,dx$$

$u = 20x \quad dv = \sin(2x)dx \quad uv - \int v\,du$

$du = 20dx \quad v = -\frac{\cos(2x)}{2}$

$= 20x\left(-\frac{\cos(2x)}{2}\right) - \int\left(-\frac{\cos(2x)}{2}\right)20dx$

$= -10x\cos(2x) + \int 10\cos(2x)dx$

$= -10x\cos(2x) + \frac{10\sin(2x)}{2} + C$

$\int 20x\sin(2x)\,dx =$ $-10x\cos(2x) + 5\sin(2x) + C$

10. **Evaluate the indefinite integral.**

$$\int 18x\cos(3x)\,dx$$

$u = 18x \quad dv = \cos(3x)dx \quad uv - \int v\,du = 18x\left(\frac{\sin(3x)}{3}\right) - \int\left(\frac{\sin(3x)}{3}\right)18dx$

$du = 18dx \quad v = \frac{\sin(3x)}{3}$

$= 6x\sin(3x) - \int 6\sin(3x)dx$

$= 6x\sin(3x) + \frac{6\cos(3x)}{3} + C$

$\int 18x\cos(3x)\,dx =$ $6x\sin(3x) + 2\cos(3x) + C$

11. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{7x-5}{x^2(x^2+9)}$$

(A) $\frac{A}{x} + \frac{B}{x} + \frac{Cx+D}{x^2+9}$

(B) $\frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}$

(C) $\frac{A}{x} + \frac{Bx+C}{x^2} + \frac{Dx+E}{x^2+9}$

(D) $\frac{Ax+B}{x^2} + \frac{Cx+D}{x^2+9}$

(E) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3} + \frac{D}{x-3}$

(F) $\frac{Ax+B}{x^2} + \frac{C}{x+3} + \frac{D}{x-3}$

12. Which of the following is a partial fraction decomposition of the rational expression show? Do not explicitly solve for the constant.

$$f(x) = \frac{2x}{(x-1)^2(x^2+5)}$$

(A) $\frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{D}{x^2+5}$

(B) $\frac{A}{(x-1)^2} + \frac{Bx+C}{x^2+5}$

(C) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+5}$

(D) $\frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{Dx+E}{x^2+5}$

(E) $\frac{A}{x-1} + \frac{Bx+C}{x^2+5}$

(F) $\frac{Ax+B}{(x-1)^2} + \frac{Cx+D}{x^2+5}$

13. Evaluate the improper integral or state that it is divergent.

$$\int_3^\infty \frac{4}{x^5}\,dx$$

$$\lim_{N\to\infty} \int_3^N 4x^{-5}dx = \lim_{N\to\infty} \frac{4x^{-4}}{-4}\Big]_3^N = \lim_{N\to\infty} -\frac{1}{x^4}\Big]_3^N$$

$$= \lim_{N\to\infty} \left(-\frac{1}{N^4} - \left(\frac{1}{3^4}\right)\right) = \frac{1}{3^4} = \frac{1}{81}$$

$$\int_3^\infty \frac{4}{x^5}\,dx = \underline{\quad 1/81 \quad}$$

14. Evaluate the following integral;

$$\int_0^\infty e^{-x/6}dx$$

$$\lim_{N\to\infty} \int_0^N e^{-x/6}\,dx \quad \frac{u=-x/6}{du=-\frac{1}{6}dx} \quad \lim_{N\to\infty} \int -6e^u du$$

$$-6du = dx$$

$$= \lim_{N\to\infty} (-6e^u) = \lim_{N\to\infty} \left(-6e^{-x/6}\right)\Big]_0^N = \lim_{N\to\infty} \left(-6e^{-N/6} - (-6e^0)\right)$$

$$= 6$$

$$\int_0^\infty e^{-x/6}dx = \underline{\quad 6 \quad}$$

15. **Evaluate the definite integral.**

$$\int_2^\infty \frac{dx}{3x+1}$$

$$= \lim_{N\to\infty} \int_2^N \frac{dx}{3x+1} \quad \frac{u=3x+1}{du=3dx} \quad \lim_{N\to\infty} \int \frac{1}{u}\frac{du}{3} = \lim_{N\to\infty} \frac{1}{3}\ln|u|$$

$$\frac{du}{3} = dx$$

$$= \lim_{N\to\infty} \frac{1}{3}\ln|3x+1|\Big]_2^N = \lim_{N\to\infty}\left(\frac{1}{3}\ln|3\cdot N+1| - \frac{1}{3}\ln|3\cdot 2+1|\right)$$

$$= \infty - \frac{1}{3}\ln(7)$$

$$\int_2^\infty \frac{dx}{3x+1} = \underline{\infty \Rightarrow \text{diverges}}$$

16. Set up the integral that represents the **AREA** of the region bounded by the following curves:

$$y = \frac{1}{2}x^2, \qquad y = -x^2+6$$

Bounds: $\frac{1}{2}x^2 = -x^2+6$

$\frac{3}{2}x^2 = 6$

$x^2 = 6\cdot\frac{2}{3}$

$x^2 = 4$

$x = \pm 2$

Test Pt: $x=0$

$y=\frac{1}{2}x^2 \to y=0 \to$ Bottom

$y=-x^2+6 \to y=6 \to$ Top

Area $= \underline{\int_{-2}^{2}\left(-x^2+6-\frac{1}{2}x^2\right)dx}$

17. Find the **AREA** of the region bounded by $y = 2x - x^2$ and $y = x^2$.

Bounds: $2x - x^2 = x^2$

$2x - 2x^2 = 0$

$2x(1-x) = 0$

$x = 0, 1$

Test Pt: $x = \frac{1}{2}$

$y = 2x - x^2 \rightarrow y = 3/4 \rightarrow$ Top

$y = x^2 \rightarrow y = 1/4 \rightarrow$ Bottom

$$A = \int_0^1 (2x - x^2 - x^2)\,dx$$

$$= \int_0^1 (2x - 2x^2)\,dx$$

$$= \left(\frac{2x^2}{2} - \frac{2x^3}{3}\right)\Big]_0^1$$

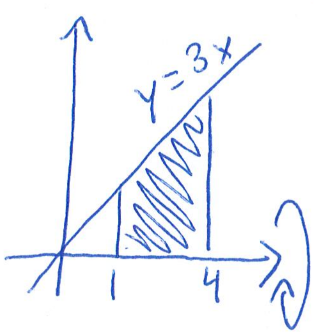
$$= \left(x^2 - \frac{2x^3}{3}\right)\Big]_0^1 = \frac{1}{3}$$

Area = 1/3

18. Find the **VOLUME** of the solid that results by revolving the region enclosed by the following curves about the x-axis.

$y = 3x, x = 1, x = 4,$ and $y = 0$

and



Disk Method $\Rightarrow dx$

$$V = \pi \int_1^4 (3x)^2\,dx = \pi \int_1^4 9x^2\,dx$$

$$= \pi \frac{9x^3}{3}\Big]_1^4 = 3\pi x^3\Big]_1^4$$

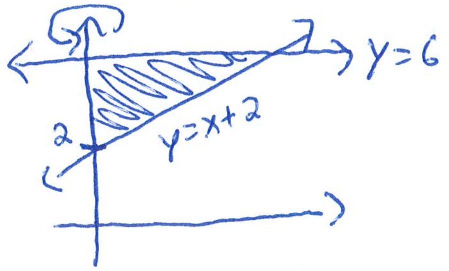
$$= 3\pi(4^3 - 1^3) = 189\pi$$

Volume = 189π

19. Find the **VOLUME** of the solid generated by rotating the region bounded by

$$y = x + 2, \quad x = 0, \quad y = 6$$

around the y-axis



y-axis + Disk $\Rightarrow$ dy

$\Rightarrow y = x + 2 \Longleftrightarrow x = y - 2$

$$V = \pi \int_2^6 (y-2)^2 \, dy \quad \frac{u = y-2}{du = dy} \quad \pi \int u^3 \, du$$

$$= \pi \frac{u^4}{4} = \frac{\pi}{4} (y-2)^2 \Big]_2^6$$

$$= \frac{\pi}{4}\left((6-2)^2 - (2-2)^2\right) = 4\pi$$

Volume = ______________________

20. **SET-UP using the washer method.** the **VOLUME** of the region bounded by

$$y = x^2, \quad y = 2x$$

around the x-axis

(A) $\pi \int_0^2 (2x - x^2)^2 \, dx$

(B) $\pi \int_0^2 (4x^2 - x^4) \, dx$

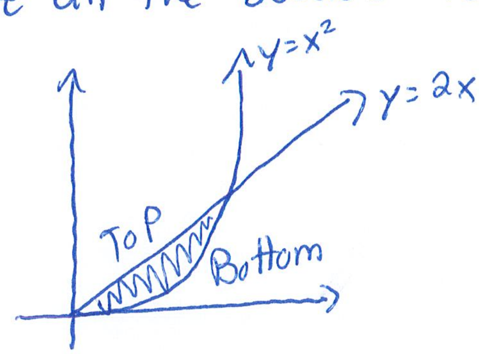
(C) $\pi \int_0^2 (2x - x^2) \, dx$

(D) $\pi \int_0^2 (x^2 - 2x) \, dx$

(E) $\pi \int_0^2 (x^4 - 4x^2) \, dx$

(F) $2\pi \int_0^2 (x^3 - 2x^2) \, dx$

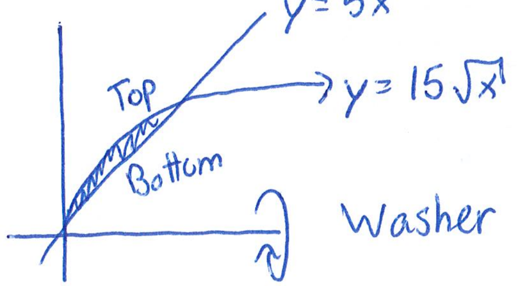
Note all the bounds are the same.



$$\pi \int_0^2 (2x)^2 - (x^2)^2 \, dx$$

21. Set-up the definite integral that would calculate the **VOLUME** of the region bounded by the following curves when rotated about the x-axis.

$$y = 5x \text{ and } y = 15\sqrt{x}$$



Washer

Bounds: $5x = 15\sqrt{x}$

$(5x)^2 = (15\sqrt{x})^2$

$25x^2 = 225x$

$25x^2 - 225x = 0$

$25x(x-9) = 0$

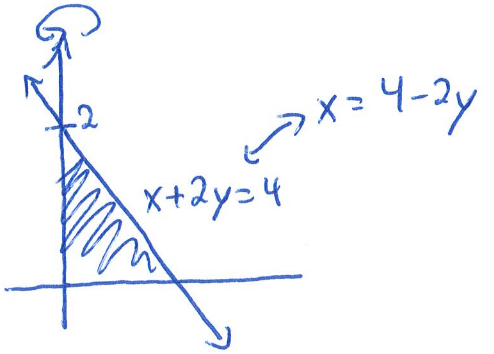
$x = 0, 9$

$V = \pi \int (15\sqrt{x})^2 - (5x)^2 dx$

VOLUME = $\pi \int_0^9 (225x - 25x^2)\, dx$

22. Find the **VOLUME** of the solid obtained by revolving the region bounded by the following curves about the y-axis.

$$x + 2y = 4, x = 0, \text{ and } y = 0$$



$2y = 4 - x$

$y = 2 - \frac{x}{2}$

Disk + y-axis ⇒ dy

$V = \pi \int_0^2 (4-2y)^2 dy$

$= \pi \int_0^2 (2y-4)^2 dy$

$u = 2y - 4$

$du = 2dy$

$\frac{du}{2} = dy$

$\pi \int u^2 \cdot \frac{du}{2}$

$= \frac{\pi}{2} \frac{u^3}{3} = \frac{\pi}{6}(2y-4)^3 \Big]_0^2$

$= \frac{\pi}{6}\left((2\cdot 2-4)^3 - (0-4)^3\right)$

VOLUME = $\frac{32\pi}{3}$

23. Find the general solution of the given differential equation.

$$\frac{dy}{dx} = \frac{4x}{y}$$

$y\,dy = 4x\,dx$

$\int y\,dy = \int 4x\,dx$

$\frac{y^2}{2} = \frac{4x^2}{2} + C$

$y^2 = 4x^2 + 2C$

$y^2 = 4x^2 + C$

$y = \pm\sqrt{4x^2 + C}$

$y =$ $\pm\sqrt{4x^2+C}$

24. Find the particular solution to the given differential equation if $y(2) = 3$

$$\frac{dy}{dx} = \frac{x}{y^2}$$

$y^2\,dy = x\,dx$

$\int y^2\,dy = \int x\,dx$

$\frac{y^3}{3} = \frac{x^2}{2} + C$

$y^3 = \frac{3x^2}{2} + 3C$

$y^3 = \frac{3x^2}{2} + C$

If $y(2) = 3$,

$3^3 = \frac{3(2)^2}{2} + C$

$27 = 6 + C$

$21 = C$

So $y^3 = \frac{3x^2}{2} + 21$

$y =$ $\sqrt[3]{\frac{3x^2}{2} + 21}$

25. Find the particular solution of the equation.

$$\frac{dy}{dx} - 5y = 0, y(1) = 7$$

$\frac{dy}{dx} = 5y \Rightarrow y = Ce^{5x}$

When $y(1) = 7$,

$7 = Ce^{5 \cdot 1}$

$7e^{-5} = C$

$y = 7e^{-5}e^{5x}$

$= 7e^{5x-6}$

$y=$ $7e^{5x-5}$

26. The rate of change of the population $N(t)$ of a sample of bacteria is directly proportional to the number of bacteria, so $N'(t) = kN$, where time t is measured in minutes. Initially, there are 270 bacteria present. If the number of bacteria after 7 minutes is 770, find the growth rate k in terms of minutes. Round to four decimal places.

$N' = kN \Rightarrow N(t) = Ce^{k \cdot t}$

$N(0) = 270 \Rightarrow 270 = Ce^{k \cdot 0}$

$270 = C \quad \Rightarrow N(t) = 270e^{k \cdot t}$

$N(7) = 770 \Rightarrow 770 = 270e^{k \cdot 7}$

$\frac{770}{270} = e^{7k}$

$\ln\left(\frac{77}{27}\right) = 7k$

$k=$ 0.1497

$k = \frac{1}{7}\ln\left(\frac{77}{27}\right)$

27. What is the **integrating factor** of the following differential equation?

$$y' + \underbrace{\left(\frac{2x+3}{x}\right)}_{P(x)} y = 10\ln(x)$$

$u(x) = \exp\left[\int P(x)\,dx\right]$

$= \exp\left[\int \frac{2x+3}{x}\,dx\right]$

$= \exp\left[\int 2 + \frac{3}{x}\,dx\right]$

$= \exp\left[2x + 3\ln(x)\right]$

$= e^{2x} e^{3\ln(x)}$

$= e^{2x} e^{\ln(x^3)}$

$= x^3 e^{2x}$

$u(x) =$ $x^3 e^{2x}$

28. Find the general solution of the following differential equation.

$P(x) = \frac{4}{x}$ $\quad Q(x) = 2x + 20$

$$\frac{dy}{dx} + \frac{4y}{x} = 2x + 20$$

$u(x) = \exp\left[\int P(x)\,dx\right]$

$= \exp\left[\int \frac{4}{x}\,dx\right]$

$= \exp\left[4\ln(x)\right]$

$= e^{\ln(x^4)}$

$= x^4$

$y \cdot u(x) = \int Q(x) \cdot u(x)\,dx + C$

$y \cdot x^4 = \int (2x+20)\,x^4\,dx + C$

$y \cdot x^4 = \int (2x^5 + 20x^4)\,dx + C$

$y \cdot x^4 = \frac{2x^6}{6} + \frac{20x^5}{5} + C$

$y = \frac{x^2}{3} + 4x + \frac{C}{x^4}$

$y =$ $\frac{x^2}{3} + 4x + \frac{C}{x^4}$

29. Compute the following sum.

$$\sum_{n=0}^{\infty}\left(\frac{(-1)^n}{3^n}+\frac{2^{n+1}}{3^n}\right)$$

$$= \sum_{n=0}^{\infty}\left(\frac{(-1)^n}{3^n}+\frac{2\cdot 2^n}{3^n}\right)$$

$$= \sum_{n=0}^{\infty}\left(\left(\frac{-1}{3}\right)^n + 2\left(\frac{2}{3}\right)^n\right)$$

$$= \frac{1}{1-(-1/3)} + \frac{2}{1-2/3}$$

$$= \frac{1}{4/3} + \frac{2}{1/3}$$

$$= \frac{3}{4} + 6$$

Answer: $27/4$

30. Evaluate the sum of the following infinite series.

$$\sum_{n=1}^{\infty}\frac{4(3)^{n-1}}{5^n}$$

$$= \frac{4(3)^{1-1}}{5^1} + \frac{4(3)^{2-1}}{5^2} + \frac{4(3)^{3-1}}{5^3} + \ldots$$

$$= \frac{4}{5} + \frac{4}{5^2}(3) + \frac{4(3)^2}{5^3} + \ldots$$

$$= \frac{4}{5}\left(1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \ldots\right)$$

$$= \frac{4}{5}\sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n$$

$$= \frac{4}{5}\cdot\frac{1}{1-3/5}$$

$$= \frac{4}{5}\cdot\frac{1}{2/5}$$

$$= \frac{4}{5}\cdot\frac{5}{2} = 2$$

Answer: 2

31. Find the sum of the following series:

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-2)^n}{3^{2n} \cdot 3^1} = \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-2)^n}{(3^2)^n} = \sum_{n=0}^{\infty} \frac{1}{3} \left(\frac{-2}{9}\right)^n$$

$$= \frac{1}{3} \cdot \frac{1}{1-(-2/9)} = \frac{1}{3} \cdot \frac{1}{11/9} = \frac{1}{3} \cdot \frac{9}{11} = \frac{3}{11}$$

Answer: $3/11$

32. Express $f(x) = \dfrac{x}{3+x}$ as a power series.

$$\frac{x}{1} \cdot \frac{1}{3+x} = \frac{x}{3} \cdot \frac{1}{1+x/3} = \frac{x}{3} \cdot \frac{1}{1-(-x/3)}$$

$$= \frac{x}{3} \sum_{n=0}^{\infty} \left(-\frac{x}{3}\right)^n = \frac{x}{3} \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{3^n}$$

$$\frac{x}{3+x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{3^{n+1}}$$

33. Express $f(x) = \dfrac{x}{4+3x^2}$ as a power series.

$$\frac{x}{1} \cdot \frac{1}{4+3x^2} = \frac{x}{4} \cdot \frac{1}{1+\frac{3}{4}x^2} = \frac{x}{4} \cdot \frac{1}{1-(-\frac{3}{4}x^2)}$$

$$= \frac{x}{4} \sum_{n=0}^{\infty} \left(\frac{-3}{4}x^2\right)^n = \frac{x}{4} \sum_{n=0}^{\infty} \frac{(-1)^n 3^n (x^2)^n}{4^n}$$

$$= \frac{x}{4} \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n}}{4^n}$$

$$\frac{x}{4+3x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n+1}}{4^{n+1}}$$

34. What are the first 3 non-zero terms of the Maclaurin series representation of the follow?

$$\int e^{-3x}\,dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{-3x} = \sum_{n=0}^{\infty} \frac{(-3x)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 3^n x^n}{n!}$$

$$\int e^{-3x}\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} x^n\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \int x^n\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{n!} \cdot \frac{x^{n+1}}{n+1} + C$$

$$= \frac{(-1)^0 3^0}{0!} \cdot \frac{x^1}{1} + \frac{(-1)^1 3^1}{1!} \cdot \frac{x^2}{2} + \frac{(-1)^2 3^2}{2!} \frac{x^3}{3}$$

$$\int e^{-3x}\,dx = \boxed{x - \frac{3}{2}x^2 + \frac{3}{2}x^3}$$

35. Find the Maclaurin representation of the following:

$$\int \cos(\sqrt{x})\,dx$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$\cos(\sqrt{x}) = \sum_{n=0}^{\infty} \frac{(-1)^n (x^{1/2})^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n$$

$$\int \cos(\sqrt{x})\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^n\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \int x^n\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \frac{x^{n+1}}{n+1}$$

$\int \cos(\sqrt{x})\,dx =$ $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \cdot \frac{x^{n+1}}{(n+1)}$

36. Given $f(x, y) = 3x^3y^2 - x^2y^{1/3}$, evaluate $f(3, -8)$.

$$f(3,-8) = 3(3)^3(-8)^2 - (3)^2(-8)^{1/3}$$

$$= 5184 + 18$$

$f(3, -8) =$ 5202

37. What do the level curves for the following function look like?

$$f(x, y) = 12\ln(6(x-3)^2 + 6(y-2)^2) = C$$

$$\ln(6(x-3)^2 + 6(y-2)^2) = \frac{C}{12} = C$$

$$6(x-3)^2 + 6(y-2)^2 = e^C = C$$

$$(x-3)^2 + (y-2)^2 = \frac{C}{6} = C$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

38. What do the level curves for the following function look like?

$$f(x, y) = \sqrt{y + 4x^2} = C$$

$$y + 4x^2 = C^2 = C$$

$$y = -4x^2 + C$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

39. What do the level curves for the following function look like?

$$f(x, y) = \ln(y - e^{5x}) = C$$

$$y = e^{5x} + C$$

(a) Increasing exponential functions
(b) Rational Functions with x-axis symmetry
(c) Natural logarithm functions
(d) Decreasing exponential functions
(e) Rational Functions with y-axis symmetry

40. What do the level curves for the following function look like?

$$f(x, y) = \cos(y + 4x^2) = C$$

$$y + 4x^2 = \cos^{-1}(C) = C$$

$$y = -4x^2 + C$$

(a) Lines
(b) Parabolas
(c) Circles
(d) Point at the origin
(e) Ellipses
(f) Hyperbolas

41. Given $f(x, y) = x\sin(xy^2)$, evaluate $f_y(3, 7)$. Round to 4 decimal places.

$$f_y(x,y) = \frac{\partial}{\partial y}(x\sin(xy^2))$$
$$= x\frac{\partial}{\partial y}(\sin(xy^2))$$
$$= x\cos(xy^2)\frac{\partial}{\partial y}(xy^2)$$
$$= x\cos(xy^2) \cdot x\frac{\partial}{\partial y}(y^2)$$
$$= x^2\cos(xy^2)\cdot(2y)$$
$$= 2x^2y\cos(xy^2)$$

$f_y(3, 7) =$ -99.9352

42. Find the first order partial derivatives of $f(x, y) = xe^{xy}$

$$f_y = \frac{\partial}{\partial y}(xe^{xy})$$
$$= x\frac{\partial}{\partial y}(e^{xy})$$
$$= xe^{xy}\frac{\partial}{\partial y}(xy)$$
$$= xe^{xy}\cdot x$$

$$f_x = \frac{\partial}{\partial x}(xe^{xy})$$
$$= \frac{\partial}{\partial x}(x)e^{xy} + x\frac{\partial}{\partial x}(e^{xy})$$
$$= e^{xy} + xe^{xy}\frac{\partial}{\partial x}(xy)$$
$$= e^{xy} + xye^{xy}$$
$$= (1+xy)e^{xy}$$

$f_x(x, y) =$ $(1+xy)e^{xy}$

$f_y(x, y) =$ x^2e^{xy}

43. For the given function $f(x,y)$, find $f_x(x,y)$.

$$f(x,y) = 5\cos(x^7y^8)$$

$$f_x = \frac{\partial}{\partial x}(5\cos(x^7y^8))$$
$$= -5\sin(x^7y^8)\frac{\partial}{\partial x}(x^7y^8)$$
$$= -5\sin(x^7y^8)\cdot y^8\frac{\partial}{\partial x}(x^7)$$
$$= -5y^8\sin(x^7y^8)\cdot 7x^6$$

$f_x(x,y) =$ $-35x^6y^8\sin(x^7y^8)$

44. Given the function $f(x,y) = x^3y^2 - 3x + 5y - 5x^2y^3$, compute $f_{xx}(x,y)$

$$f_x = \frac{\partial}{\partial x}(x^3y^2 - 3x + 5y - 5x^2y^3)$$
$$= 3x^2y^2 - 3 + 0 - 5\cdot 2xy^3$$
$$= 3x^2y^2 - 3 - 10xy^3$$

$$f_{xx} = \frac{\partial}{\partial x}(3x^2y^2 - 3 - 10xy^3)$$
$$= 3\cdot 2xy^2 + 0 - 10y^3$$

$f_{xx}(x,y) =$ $6xy^2 - 10y^3$

45. Given the function $f(x,y) = x^3 \sin(y)$, compute $f_{xy}(2,0)$

$$f_x = \frac{\partial}{\partial x}(x^3 \sin(y))$$
$$= \sin(y)\frac{\partial}{\partial x}(x^3)$$
$$= \sin(y) \cdot 3x^2$$

$$f_{xy}(2,0) = 3(2)^2 \cos(0)$$
$$= 12$$

$$f_{xy} = \frac{\partial}{\partial y}(\sin(y) \cdot 3x^2)$$
$$= 3x^2 \frac{\partial}{\partial y}(\sin(y))$$
$$= 3x^2 \cos(y)$$

$f_{xy}(2,0) =$ 12

46. For the function $f(x,y)$, find $f_{xy}(\pi, 2)$.

$$f(x,y) = 8y^5 \sin(x)$$

$$f_x = \frac{\partial}{\partial x}(8y^5 \sin(x))$$
$$= 8y^5 \frac{\partial}{\partial x}(\sin(x))$$
$$= 8y^5 \cos(x)$$

$$f_{xy}(\pi/2) = 40\cos(\pi) \cdot 2^4$$
$$= -640$$

$$f_{xy} = \frac{\partial}{\partial y}(8y^5 \cos(x))$$
$$= \cos(x)\frac{\partial}{\partial y}(8y^5)$$
$$= \cos(x) \cdot 40y^4$$

$f_{xy}(\pi, 2) =$ −640

47. At what point (x, y) does the function $f(x, y)$ have a local minimum?

$$f(x,y) = 7x^2 - xy + 5y^2 + 75x + 84y + 2$$

$f_x = 14x - y + 0 + 75 + 0 + 0$

$= 14x - y + 75$

$f_y = 0 - x + 10y + 0 + 84$

$= -x + 10y + 84$

$\begin{cases} 14x - y + 75 = 0 & ① \\ -x + 10y + 84 = 0 & ② \end{cases}$

Multiply ① by 10 and add w/ ②

$140x - \cancel{10y} + 750 = 0$

$+ \quad -x + \cancel{10y} + 84 = 0$

$139x + 834 = 6$

$x = 6$

Plug $x=6$ into ①.

$14(6) - y + 75 = 0$

$84 - y + 75 = 0$

$159 - y = 0$

$y = 159$

$f_{xx} = 14 \quad f_{xy} = -1$

$f_{yy} = 10 \quad \Rightarrow D > 0 + f_{xx} < 0$ for all pts

Local min occurs at $(6, 159)$

48. Given the information below, which critical point(s) (a, b) would be classified as a relative maximum?

(a, b)	$f_{xx}(a, b)$	$f_{yy}(a, b)$	$f_{xy}(a, b)$
$(7, 8)$	-5	-5	10
$(-8, -1)$	-4	-7	-2
$(1, 7)$	-10	-1	6

$D = f_{xx} f_{yy} - (f_{xy})^2$

$D(7,8) = (-5)(-5) - 10^2 < 0 \Rightarrow$ saddle pt

$\left.\begin{matrix} D(-8,-1) = (-4)(-7) - (-2)^2 > 0 \\ f_{xx}(-8,-1) < 0 \end{matrix}\right\} \Rightarrow$ rel max

$D(1, 7) = (-10)(-1) - (6)^2 < 0 \Rightarrow$ saddle pt

Answer: $(-8, -1)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

49. Given the table below,

(a,b)	$f_{xx}(a,b)$	$f_{yy}(a,b)$	$f_{xy}(a,b)$
$(9,4)$	-1	-1	-1
$(-2,2)$	4	3	-4
$(4,5)$	8	5	6

Which statements are true?

I. $f(x,y)$ has exactly 1 saddle points ✓

II. $f(x,y)$ has exactly 1 relative minimum ✓

III. $f(x,y)$ has exactly 1 relative maximum

IV. $f(x,y)$ has exactly 1 inconclusive critical point ✓

$D(9,4) = (-1)(-1) - (-1)^2 = 0 \rightarrow$ inconclusive

$D(-2,2) = (4)(3) - (-4)^2 < 0 \rightarrow$ saddle

$\left.\begin{array}{l} D(4,5) = (8)(5) - 6^2 > 0 \\ f_{xx}(4,5) = 8 > 0 \end{array}\right\}$ rel min

Answer: I, II, IV

50. The critical points for a function $f(x,y)$ are (0,0) and (8,4). Given that the partial derivatives of $f(x,y)$ are

$$f_x(x,y) = 3x - 6y \qquad f_y(x,y) = 3y^2 - 6x$$

Classify each critical point as a maximum, minimum, or saddle point.

Critical Pts: $(0,0)$, $(8,4)$

$f_x = 3x - 6y$ $\quad f_y = 3y^2 - 6x$

$f_{xx} = 3$ $\quad f_{yy} = 6y$

$f_{xy} = -6$

$D(0,0) = -36 < 0$ saddle

$\left.\begin{array}{l} D(8,4) = 18(4) - 36 > 0 \\ f_{xx} = 3 > 0 \end{array}\right\}$ rel min

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 3(6y) - (-6)^2$

$= 18y - 36$

(0,0) is saddle pt

(8,4) is relative min

51. Classify the critical points of the function $f(x, y)$ given the partial derivatives

$$f_x(x,y) = x - 2y \qquad f_y(x,y) = y^2 - 2x$$

A) 2 Local Minima
B) 2 Saddle Points
C) 1 Saddle Point and 1 Local Minimum
D) 1 Saddle Point and 1 Local Maximum
E) 1 Local Maximum and 1 Local Minimum
F) 2 Local Maxima

$\begin{cases} x-2y=0 & (1) \\ y^2-2x=0 & (2) \end{cases}$

Solve (1) for x.
$x=2y$
Plug $x=2y$ into (2)
$y^2-2(2y)=0$
$y^2-4y=0$
$y(y-4)=0$
$y=0,4$

Plug $y=0,4$ into $x=2y$
$\underline{y=0}$: $x=0 \to (0,0)$
$\underline{y=4}$: $x=8 \to (8,4)$

$f_x=x-2y$
$f_{xx}=1$
$f_{xy}=-2$

$f_y=y^2-2x$
$f_{yy}=2y$

$D=f_{xx}f_{yy}-(f_{xy})^2$
$=1(2y)-(-2)^2$
$=2y-4$

$D(0,0)=-4<$ saddle
$D(8,4)>0$ + $f_{xx}=1>0 \Rightarrow$ rel min

52. Find the minimum of the function using LaGrange Multipliers of the function $f(x,y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$.

$f(x,y)=x^2+2y^2$ $\quad g(x,y)=x^2+y^2$
$f_x=2x$ $\quad g_x=2x$
$f_y=4y$ $\quad g_y=2y$

$\begin{cases} 2x=\lambda 2x & (1) \\ 4y=\lambda 2y & (2) \\ x^2+y^2=1 & (3) \end{cases}$

Solve (1).
$2x=\lambda 2x$
$2x-\lambda 2x=0$
$2x(1-\lambda)=0$
$x=0, \lambda=1$

Plug $\lambda=1$ into (2)
$4y=2y$
$2y=0$
$y=0$

Plug $y=0$ into (3)
$x^2=1$
$x=\pm 1$
Critical Pts: $(1,0), (-1,0)$

$f(x,y)=x^2+2y^2$
$f(\pm1,0)=1$
$f(0,\pm1)=2$

Plug $x=0$ into (3)
$y^2=1$
$y=\pm 1$
Critical Pts: $(0,1)$, $(0,-1)$

Minimum Value = 1

53. Find the maximum value of the function $f(x,y) = 18x - 19y^2$ subject to the constraint $x^2 + 19y^2 = 81$.

$f(x,y) = 18x - 19y^2$

$f_x = 18$

$f_y = -38y$

$g(x,y) = x^2 + 19y^2$

$g_x = 2x$

$g_y = 38y$

Plug $y=0$ into ③

$x^2 = 81$

$x = \pm 9$

Critical Pt: $(9,0), (-9,0)$

$$\begin{cases} 18 = \lambda 2x & ① \\ -38y = \lambda 38y & ② \\ x^2 + 19y^2 = 81 & ③ \end{cases}$$

Solve ②:

$38y - \lambda 38y = 0$

$38y(1+\lambda) = 0$

$y = 0, \lambda = -1$

Plug $\lambda = -1$ into ①

$18 = -2x$

$x = -9$

Plug $x = -9$ into ③

$(-9)^2 + 18y^2 = 81$

$18y^2 = 0$

$y = 0$

Critical Pt

$(-9,0)$

$f(x,y) = 18x - 19y^2$

$f(9,0) = 162$

$f(-9,0) = -162$

Maximum Value = 162

54. A factory can produce a chocolate bar with a weight of $W(x,y) = \frac{xy}{100}$ with the weight W in ounces and x and y are the percentages of cocoa and sugar respectively. The percentage of cocoa and sugar are constrained to $2x + y = 75$. What is the weight, in ounces, of the largest chocolate bar that can be produced? Round to 2 decimal places.

$W(x,y) = \frac{xy}{100}$

$W_x = y/100$

$W_y = x/100$

$g(x,y) = 2x + y$

$g_x = 2$

$g_y = 1$

$$\begin{cases} \frac{y}{100} = 2\lambda & ① \\ \frac{x}{100} = \lambda & ② \\ 2x + y = 75 & ③ \end{cases}$$

Plug ② into ①

$\frac{y}{100} = 2\left(\frac{x}{100}\right)$

$y = 2x$

Plug $y = 2x$ into $2x + y = 75$

$2x + 2x = 75$

$4x = 75$

$x = 75/4$

Hence $y = 75/2$

$W\left(\frac{75}{4}, \frac{75}{2}\right) \approx 7.03$

Weight of Largest Chocolate Bar = 7.03

55. Evaluate the double integral

$$\int_0^{\pi/2}\int_0^1 16y^3\cos(x)\,dy\,dx$$

$$\int_{x=0}^{x=\pi/2}\left(\int_{y=0}^{y=1} 16y^3\cos(x)dy\right)dx$$

$$= \int_{x=0}^{x=\pi/2} \cos(x)\left(\int_{y=0}^{y=1} 16y^3 dy\right)dx$$

$$= \int_{x=0}^{x=\pi/2} \cos(x)\left(\frac{16y^4}{4}\Big]_{y=0}^{y=1}\right)dx$$

$$= \int_{x=0}^{x=\pi/2} \cos(x)\cdot 4\,dx$$

$$= 4\sin(x)\Big]_{x=0}^{x=\pi/2}$$

$$= 4\sin\left(\frac{\pi}{2}\right) - 4\sin(0)$$

$\int_0^1\int_0^{\pi/2} 16y^3\cos(x)\,dy\,dx =$ 4

56. Evaluate the double integral

$$\int_0^7\int_0^{\pi/2} 20y^4\cos(x)\,dx\,dy$$

$$\int_{y=0}^{y=7}\left(\int_{x=0}^{x=\pi/2} 20y^4\cos(x)dx\right)dy$$

$$= \int_{y=0}^{y=7} 20y^4\left(\int_{x=0}^{x=\pi/2}\cos(x)dx\right)dy$$

$$= \int_{y=0}^{y=7} 20y^4\left(\sin(x)\Big]_{x=0}^{x=\pi/2}\right)dy$$

$$= \int_{y=0}^{y=7} 20y^4\cdot 1\,dy$$

$$= \frac{20y^5}{5}\Big]_{y=0}^{y=7}$$

$$= 4(7)^5$$

$\int_0^7\int_0^{\pi/2} 20y^4\cos(x)\,dx\,dy =$ 67228

57. **Evaluate the definite integral.**

$$\int_0^4 \int_3^x \frac{6x}{y^2}\, dy\, dx$$

$$\int_{x=0}^{x=4}\left(\int_{y=3}^{y=x} 6xy^{-2}\,dy\right)dx$$

$$= \int_{x=0}^{x=4} 6x\left(\int_{y=3}^{y=x} y^{-2}\,dy\right)dx$$

$$= \int_{x=0}^{x=4} 6x\left(\frac{y^{-1}}{-1}\Big]_{y=3}^{y=x}\right)dx$$

$$= \int_{x=0}^{x=4} 6x\left(-\frac{1}{y}\Big]_{y=3}^{y=x}\right)dx$$

$$= \int_{x=0}^{x=4} 6x\left(-\frac{1}{x}+\frac{1}{3}\right)dx$$

$$= \int_{x=0}^{x=4} (-6+2x)\,dx$$

$$= \left(-6x+\frac{2x^2}{2}\right)\Big]_{x=0}^{x=4}$$

$$= (x^2-6x)\Big]_{x=0}^{x=4}$$

$$= -8$$

$\int_0^4 \int_3^x \frac{6x}{y^2}\, dy\, dx =$ -8

58. **Evaluate the definite integral.**

$$\int_0^6 \int_2^x 30x\, dy\, dx$$

$$\int_{x=0}^{x=6}\left(\int_{y=2}^{y=x} 30x\,dy\right)dx$$

$$= \int_{x=0}^{x=6} 30x\left(\int_{y=2}^{y=x} dy\right)dx$$

$$= \int_{x=0}^{x=6} 30x\left(y\Big]_{y=2}^{y=x}\right)dx$$

$$= \int_{x=0}^{x=6} 30x(x-2)\,dx$$

$$= \int_{x=0}^{x=6} (30x^2-60x)\,dx$$

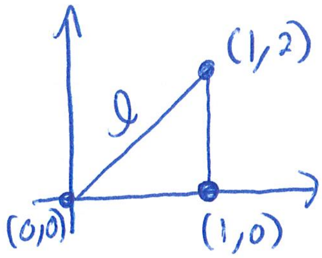
$$= \left(\frac{30x^3}{3}-\frac{60x^2}{2}\right)\Big]_{x=0}^{x=6}$$

$$= (10x^3-30x^2)\Big]_{x=0}^{x=6}$$

$$= 1080$$

$\int_0^6 \int_2^x 30x\, dy\, dx =$ 1080

59. Find the bounds for the integral $\iint_R f(x, y)\, dA$ where R is a triangle with vertices (0,0), (1,0), and (1,2).



Equation of l: $m = \frac{2-0}{1-0} = 2 \Rightarrow y = 2x$

$$\underbrace{\int_{x=0}^{x=1} \int_{y=0}^{y=2x} f(x,y)\, dy\, dx}$$

Answer: ____________________

60. Switch the order of integration for the following integral

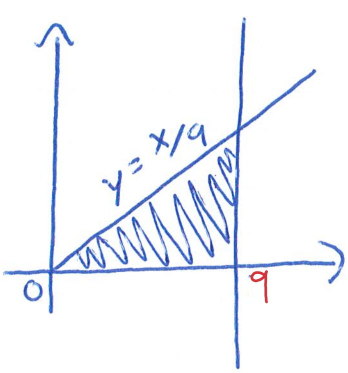
dydx ⇒ Top + Bottom

$$\int_0^1 \int_{9y}^{9} f(x, y)\, dx\, dy$$

$$\int_{y=0}^{y=1} \int_{x=9y}^{x=9} f(x,y)\, dx\, dy$$

Region is $y=0, y=1, x=9y, x=9$

$x = 9y \Leftrightarrow y = \frac{x}{9}$



Answer: $\int_{x=0}^{x=9} \int_{y=0}^{y=x/9} f(x,y)\, dy\, dx$

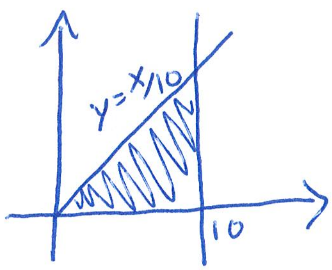
61. Switch the order of integration on the follow integral

$$\int_0^1 \int_{10y}^{10} f(x,y)\,dx\,dy$$

$$\int_{y=0}^{y=1} \int_{x=10y}^{x=10} f(x,y)\,dx\,dy$$

Region $y=0, y=1, x=10y, x=10$

$x=10y \Leftrightarrow y = x/10$



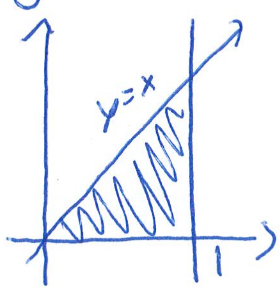
Answer: $\int_{x=0}^{x=10} \int_{y=0}^{y=x/10} f(x,y)\,dy\,dx$

62. Evaluate the double integral

$$\int_0^1 \int_y^1 2e^{x^2}\,dx\,dy = \int_{y=0}^{y=1} \int_{x=y}^{x=1} 2e^{x^2}\,dx\,dy$$

(Hint: Change the order of integration)

Region $y=0, y=1, x=y, x=1$



$$\int_{x=0}^{x=1} \left(\int_{y=0}^{y=x} 2e^{x^2}\,dy \right) dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} \left(\int_{y=0}^{y=x} dy \right) dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} \left(y \Big]_{y=0}^{y=x} \right) dx$$

$$= \int_{x=0}^{x=1} 2e^{x^2} \cdot x\,dx$$

$\dfrac{u = x^2}{du = 2x\,dx}$ $\int e^u\,du = e^u$

$$= e^{x^2} \Big]_{x=0}^{x=1}$$

$$= e^1 - e^0$$

$\int_0^1 \int_y^1 2e^{x^2}\,dx\,dy =$ $e-1$

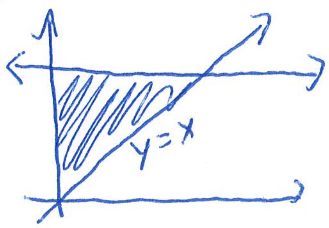
$dx\,dy \Rightarrow$ Right + Left

63. Evaluate the double integral

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx = \int_{x=0}^{x=2} \int_{y=x}^{y=2} 4e^{y^2}\, dy\, dx$$

(Hint: Change the order of integration)

Region $x=0, x=2, y=x, y=2$



$$\int_{y=0}^{y=2}\left(\int_{x=0}^{x=y} 4e^{y^2}\, dx\right) dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2}\left(\int_{x=0}^{x=y} dx\right) dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2}\left(x \Big]_{x=0}^{x=y}\right) dy$$

$$= \int_{y=0}^{y=2} 4e^{y^2} \cdot y\, dy$$

$$\frac{u = y^2}{du = 2y\,dy} \quad \int 4y\, e^u \frac{du}{2y} = \int 2e^u\, du$$

$$\frac{du}{2y} = dy \quad = 2e^u = 2e^{y^2}\Big]_{y=0}^{y=2}$$

$$\int_0^2 \int_x^2 4e^{y^2}\, dy\, dx = \underline{\quad 2e^4 - 2 \quad}$$

64. Evaluate the double integral

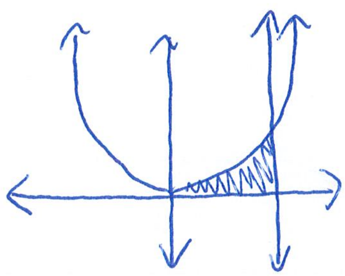
$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy = \int_{y=0}^{y=1} \int_{x=\sqrt{y}}^{x=1} \sin(x^3)\, dx\, dy$$

(Hint: Change the order of integration)

Region $y=0, y=1, x=\sqrt{y}, x=1$

$$\updownarrow$$

$$y = x^2$$



$$\int_{x=0}^{x=1}\left(\int_{y=0}^{y=x^2} \sin(x^3)\, dy\right) dx$$

$$= \int_{x=0}^{x=1} \sin(x^3)\left(\int_{y=0}^{y=x^2} dy\right) dx$$

$$= \int_{x=0}^{x=1} \sin(x^3)\left(y \Big]_{y=0}^{y=x^2}\right) dx$$

$$= \int_{x=0}^{x=1} x^2 \sin(x^3)\, dx$$

$$\frac{u = x^3}{du = 3x^2\,dx} \quad \int x^2 \sin(u) \frac{du}{3x^2}$$

$$\frac{du}{3x^2} = dx$$

$$= \frac{1}{3}\int \sin(u)\, du = -\frac{1}{3}\cos(u)$$

$$= -\frac{1}{3}\cos(x^3)\Big]_{x=0}^{x=1}$$

$$\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3)\, dx\, dy = \underline{\qquad\qquad}$$

$$\hookrightarrow -\frac{1}{3}\cos(1) + \frac{1}{3}$$